Math "coffin" submitted by others

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The problems

- **Problem 1.** Submitted by Evguenia Kaganova. This problem was given to her in 1973.

  Calculate: tan (pi/7) * tan (3pi/7) * tan (5pi/7). Here I used the notation pi instead of the corresponding Greek letter.

- **Problem 2.** Submitted by Maxim Umansky. This problem was given to his friend in the early 70s.

  Let p, q be two irrational numbers. Can p^q be rational? Can p^q be irrational?

  You can find my solution, a solution submitted by Alexey Radul, and a solution submitted by Frederick Lewis here.

- **Problem 3.** Submitted by Michael Shtilman. This problem was given to an acquaintance of his in 1969.

  The graph of a monotonically increasing function is cut off with two horizontal lines. Find a point on the graph between intersections such that the sum of the two areas bounded by the lines, the graph and the vertical line through this point is minimum.

- **Problem 4.** Submitted by Michael Shtilman. This problem was given to another acquaintance of his in 1982.

  Given finite sequence of zeros and ones, a new sequence is created so that zeros are substituted with ones and vice versa. This new sequence is added at the end of the first one: If the first sequence was 0110, the new one will be 1001, and the final one will be 01101001. Starting with a one figure sequence the process is repeated infinitely. Now we consider this infinite sequence as decimals: 0.01101001... . Prove that this number is irrational.

- **Problem 5.** Submitted by Michael Shtilman. This problem was given to yet a third acquaintance of his in 1974.

  The radii of the circumscribed and inscribed circles for a triangle are R and r respectively. What are the maximum and minimum possible distances between the centers of these circles?

  Comment: This problem is similar to problem 29 of the main list. In this wording, the problem becomes confusing, as the distance between the centers is uniquely specified by the radii.

- **Problem 6.** Submitted by Dima Barboy. This problem was given to him in 1982.

  Given an irregular octahedron, a sphere is built on each edge in such a way that the edge is a diameter of the corresponding sphere. Prove that the spheres cover the inside of the octahedron completely.

- **Problem 7.** This problem is from a discussion on LiveJournal, submitted by the user "arbat". The solution is there too.

  Take a cyclic quadrilateral. Build a perpendicular from the center of each side to the opposite side. Prove that all these perpendiculars intersect in one point.

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