BOOK OF ABSTRATCS

MAT8o

XIII Workshop on Dynamical Systems

Celebrating the 80th birthday of Marco Antonio Teixeira

IMECC/Unicamp – November 25-29, 2024
Campinas/SP – Brazil
http://www.ime.unicamp.br/~mat80/

Confirmed speakers

Antonio Teruel (Universitat de les Illes Balears)
Armengol Gasull (Universitat Autònoma de Barcelona)
Clodoaldo Grotta Ragazzo (USP)
Colin Christopher (University of Plymouth)
Daniel Panazzolo (Université de Haute-Alsace)
Enrique Ponce (Universidad de Sevilla)
Freddy Dumortier (Hasselt University)
Jaume Llibre (Universitat Autònoma de Barcelona)
Jeroen Lamb (Imperial College)
Ketty Abaroa de Rezende (Unicamp)
Lorenzo Díaz (PUC-Rio)
Mike Jeffrey (University of Bristol)
Mirian Manoel (USP)
Tere Seara (Universitat Politècnica de Catalunya)

Mini-course

Fernando Fernández Sánchez (Universidad de Sevilla)

Scientific Committee
Alain Jacquemard (Université de Bourgogne)
Jaume Llibre (Universitat Autònoma de Barcelona)
Ketty Abaroa de Rezende (Unicamp)
Mike Jeffrey (University of Bristol)
Paulo Ricardo da Silva (Unesp)
Ronaldo Garcia (UFG)
Tere Seara (Universitat Politècnica de Catalunya)
Victoriano Carmona (Universidad de Sevilla)

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Local Committee
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Helena Costa
Letícia Cândido
Luana Ascoli
Lucas Arakaki
Luiz Fernando Gouveia
Murilo Zigart

Financial support

FAPESP IMECC UNICAMP

Welcome

It is a pleasure to welcome you to the XIII Workshop on Dynamical Systems. In this conference, we will celebrate the 80th birthday of Marco Antonio Teixeira, focusing on the major contributions of his work in dynamical systems. The conference covers topics ranging from Reversible Systems, Polynomial Vector Fields, Bifurcations, Non Smooth Dynamical Systems, and related areas. This is the thirteenth edition in a series of workshops aimed at fostering collaboration among the growing community of researchers working on dynamical systems.

Scientific Committee

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Jaume Llibre (Universitat Autònoma de Barcelona - Spain)
Ketty Abaroa de Rezende (Universidade Estadual de Campinas - Brazil)
Mike Jeffrey (University of Bristol - United Kingdom)
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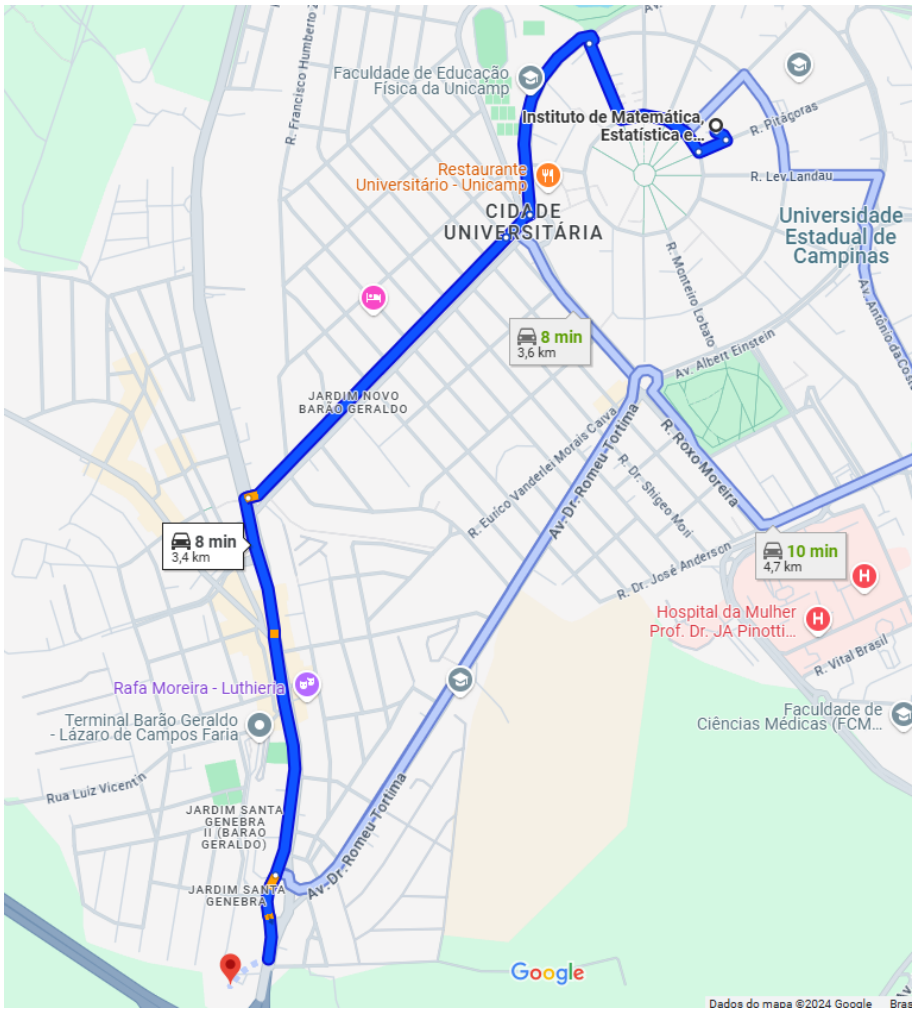
General Information

Location

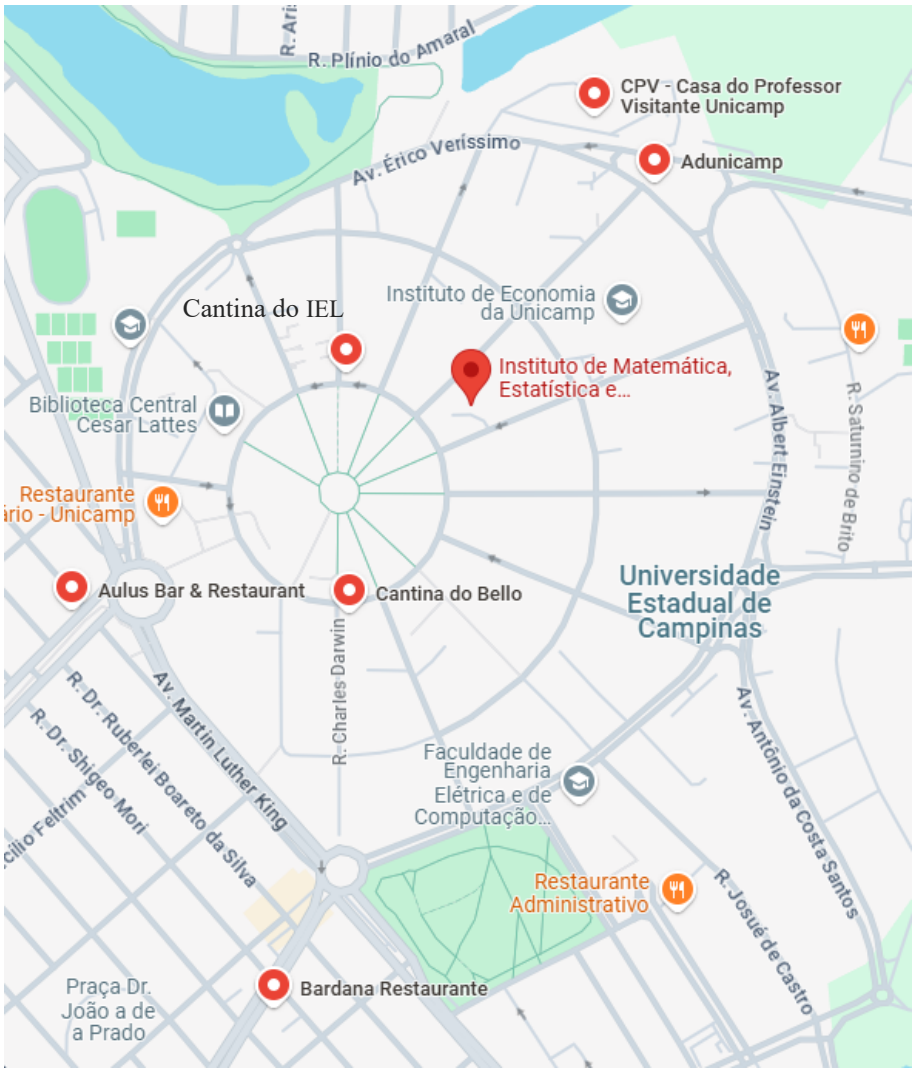
The conference will be held at IMECC - Unicamp at 651, Sérgio Buarque de Holanda St. The rooms **123** and **124** are reserved for discussions, meetings, etc. To connect to Wifi, access IMECC-EVENTOS using the password: **20Imecc24**

Dinner

The dinner will take place at Churrascaria Estância Grill - 271, Albino J. B. de Oliveira Ave. - Jardim Santa Genebra II - Barão Geraldo, on Wednesday (November 27th) at 7:30 pm.



Closest Restaurants



- Cantina do IEL.
- Cantina do Bello.
- Aulus Bar and Restaurant.
- Bardana Restaurant.
- CPV Restaurant.
- Adunicamp Restaurant.

Certificates

The certificates will be sent by email to all speakers and poster presenters. The certificates for attendees will be provided upon request.

Program

	Monday	Tuesday	Wednesday	Thursday	Friday
	25/11	26/11	27/11	28/11	29/11
9:00 - 9:15	Registration + welcome coffee	Mirian Manoel	No activities*	Antonio Teruel	Clodoaldo Ragazzo
9:15 - 9:30					
9:30 - 9:45					
9:45 - 10:00		Mike Jeffrey		Enrique Ponce	Jeroen Lamb
10:00 - 10:15	Opening				
10:15 - 10:30					
10:30 - 10:45	Ketty	Poster presentation - group 1 (+ coffee + discussion)		Poster presentation - group 3 (+ coffee + discussion)	Poster presentation - group 4 (+ coffee + discussion)
10:45 - 11:00					
11:00 - 11:15					
11:15 - 11:30	Jaume Llibre				
11:30 - 11:45		Diego Rodrigues	Poster presentation - group 2 (+ coffee + discussion)	Sônia de Carvalho	Daniel Gomes
11:45 - 12:00					Renata Possobon
12:00 - 12:15	Francisco Braun	Regilene		Luis F. Mello	Daniel Panazzolo
12:15 - 12:30					
12:30 - 12:45	Lunch	**Group picture!**	Lunch	Lunch	
12:45 - 13:00		Lunch			Ending
13:00 - 13:15					
13:15 - 13:30					
13:30 - 13:45					
13:45 - 14:00					
14:00 - 14:15	Gabriel Rondón	Mini-course: Fernando Fernández Sánchez	Mini-course: Fernando Fernández Sánchez	Mini-course: Fernando Fernández Sánchez	
14:15 - 14:30					
14:30 - 14:45	Claudio Pessoa				
14:45 - 15:00					
15:00 - 15:15	Tere Seara	Armengol Gasull	Colin Christopher	Freddy Dumortier	
15:15 - 15:30					
15:30 - 15:45					
15:45 - 16:00	Poster presentation - group 1 (coffe+discussion)	Poster presentation - group 2 (+ coffee + discussion)	Poster presentation - group 3 (+ coffee + discussion)	Poster presentation - group 4 (+ coffee + discussion)	
16:00 - 16:15					
16:15 - 16:30					
16:30 - 16:45					
16:45 - 17:00	Matheus	Kamila Andrade	Paulo Santana	Ygor	
17:00 - 17:15	Neemias Martins		Lorenzo Díaz	Pedro Mattos	
17:15 - 17:30		Otávio Gomide		Douglas Coates	
17:15 - 17:45					
17:45 - 18:00					
18:00 - 18:15					
19:30 - 23:00			Dinner		

* Reserved for the ceremony in which Prof. Marco Antonio Teixeira will receive the title of professor emeritus of Unicamp.
Access to the ceremony by invitation.

Minicourse

Integral characterization of Poincaré half-maps and its applications to limit cycles of planar piecewise linear systems

Fernando Fernandez

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Universidad de Sevilla, Spain

In this minicourse, we introduce a novel integral characterization of Poincaré half-maps for linear differential systems and explore its applications to various problems concerning the periodic behavior of planar piecewise linear systems. Key topics include the existence of a uniform upper bound for the maximum number of limit cycles in planar piecewise linear differential systems with two zones separated by a straight line—a problem that has remained unresolved for over 30 years since Lum-Chua's seminal work, despite considerable research efforts. We also address the uniqueness and stability of limit cycles in these systems when there are no sliding regions, as well as characterize the existence of period annuli in systems with a straight line of non-smoothness. Additional applications and related open questions will also be discussed.

Plenaries

The Algebra behind the Dynamics of Isolated Invariant Sets

Ketty A. de Rezende

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Universidade Estadual de Campinas, Brazil

In this talk we exhibit homotopical algebraic invariants, such as the Conley index, that capture some dynamical features of isolated invariant sets. We also present algebraic tools to comprehend how certain algebraic invariants indicate changes to the flow within its phase space under homotopical deformation. Changes such as cancellation phenomena between invariant sets, death and birth of connections of the flow within its phase space are presented.

Hopf bifurcation of limit cycles surrounding a node

Jaume Llibre

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Universitat Autònoma de Barcelona, Spain

We consider the planar polynomial differential systems in $\mathbb{R}^2$

$$\begin{aligned} \dot{x} &= \mu x + P(x, y, \mu), \\ \dot{y} &= \mu y + Q(x, y, \mu), \end{aligned}$$

where the polynomials P and Q have neither constant terms nor linear terms, satisfying that when the parameter $|\mu| \neq 0$ and small, the origin is a node, and for $\mu = 0$ the origin is either a node or a focus. The main theorem characterizes the Hopf bifurcation from the equilibrium point at the origin of this system. We illustrate this result with several examples. As a consequence, we can give a lower bound for the number of small limit cycles surrounding a node depending only on the degree of the polynomial differential system. In summary, we extend the Hopf bifurcation that usually studies the bifurcation of limit cycles from a focus to the bifurcation of limit cycles from a node.

In this talk the node at the origin is an star node, but the results that I will present for the star node also work for the standard node $\dot{x} = \mu x$, $\dot{y} = \lambda y$ with $\mu\lambda > 0$, or for the non-diagonalizable node $\dot{x} = \mu x + y$, $\dot{y} = \mu y$.

Unstable motions in the three body problem

Tere Seara

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Universitat Politècnica da Catalunya - Spain

Since the works of Johannes Kepler (1571-1630), and later the works of Isaac Newton (1642-1727) about the universal laws of gravitation, it is well known that the possible motion of two bodies moving under their mutual Newtonian gravitational forces can be on ellipses, parabolas or hyperbolas. In this talk we will recall Kepler results about the motion of two bodies and we will talk about the possible motions when one considers the more realistic model of three bodies. This problem, already considered by Henri Poincaré (1854-1912), is still far for being understood, and one can found many works proving the existence of different type of solutions for the three body problem like periodic orbits, asymptotic orbits to them, among others. In this talk we will give some light about the possible motions that three bodies can have if we wait enough time and we will try to answer questions like: will the orbits be bounded or can the bodies fall apart (and therefore going to "infinity")? Can one find chaotic motion in the three body problem and therefore unpredictability? These and related questions will be the main goals of this talk, where we will see how the mathematical tools coming from Dynamical Systems can be used to deal with this complex problem. In particular, we will prove the existence of oscillatory motions, that is, orbits which leave every bounded region but which return infinitely often to some fixed bounded region.

Involutions at the crossroads of singularities and dynamics

Miriam Manoel
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Universidade de São Paulo - Brazil

An involution is a map that is its own inverse. This intriguing condition leads us naturally into the singularities, as involutions act as symmetries of folds, and into discrete or continuous dynamical systems, where they act as antisymmetries of reversible diffeomorphisms or vector fields. Around 20 years ago, Marco Teixeira introduced us to an inspiring path in these directions. In this talk, I will present the developments in both directions we've made since then. This journey has been undertaken in collaboration with Marco Teixeira, Solange Mancini, Patricia Baptistelli, Isabel Labouriau, and Marcelo Salazar.

Catastrophe theory for modern mathematical modelling

Mike Jeffrey
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University of Bristol - United Kingdom

Many of us are familiar with the basic singularities or bifurcations: folds or 'saddle-nodes', cusps, Hopf bifurcations, and so on. And the list goes on: swallowtails, butterflies, wigwams, stars,... But did you know that there are low order bifurcations of vector fields that have never (seemingly) been classified? I'll show you a singularity where 4 equilibria of a vector field collide that seems to have eluded traditional classifications. And then, given a complicated physical or biological model, how do you discover if and where any given singularity occurs in its system of equations? Did you know that finding a 4-parameter

bifurcation in 4 dimensions could take at least 95 million calculations, to find a 5-parameter bifurcation in 5 dimensions could take 3×10^{30} calculations! It turns out that while traditional singularity theory is great at classifying all the bifurcations that are possible (well, not 'all' as we've discovered), it is terrible if you want to find where a given singularity or bifurcation happens. But it turns out that there is an alternative: instead of the singularity or bifurcation itself, you look for its "underlying catastrophe". The underlying catastrophe doesn't care whether you're studying a map or a vector field or even a PDE, it doesn't care how many dimensions your system has ... to locate an r-parameter singularity or bifurcation will take precisely r calculations, and no more. Since I proposed this preposterous idea 3 years ago, and applied it to find a butterfly singularity in a biological reaction diffusion related to plant growth, I have been working to understand how it connects to the deep singularity theory set down by Thom, Arnold, Mather, with some wonderful ideas and almost 'lost' bifurcations you may not have seen from such characters as Dumortier and Sotomayor. I'll introduce you to the idea of underlying catastrophes, show you some singularities and bifurcations you (probably) have never seen, and show you new ways to investigate them in contexts (like vector fields and PDEs) where they couldn't be investigated before, opening the door to studying their wider role in applications to pattern formation and phase transitions.

Different viewpoints about Hilbert's 16th problem

Armengol Gasull
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Universitat Autònoma de Barcelona, Spain

In Hilbert's 16th problem the main goal is to get (good) upper and lower bounds, for the number of limit cycles of a planar polynomial differential system in terms of its degree. In this talk we face the same question but in terms of the number of monomials of the planar polynomial differential equation. This number of monomials can be counted either in the usual real notation or in the complex notation, $\dot{z} = F(z, \bar{z})$, and both points of view are, not at all, equivalent.

Liouvillian Integrability of Higher Dimensional Systems

Colin Christopher
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University of Plymouth, United Kingdom

We give a summary of joint work with Chara Pantazi and Sebastian Walcher which describes systems in three or more dimensions with Liouvillain first integrals, generalising the work of Singer for two dimensional systems.

A conservative dilemma: to be or not to be (hyperbolic)

Lorenzo Díaz
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Pontifícia Universidade Católica de São Paulo - Brazil

In dimension three, we present three conservative partially hyperbolic settings (derived from Anosov systems, time one-maps of geodesic flows on negatively curved surfaces, and skew-product with circle fibers) where the following dichotomy holds: the diffeomorphism is either Anosov or it supports and (ergodic) nonhyperbolic measure. Join work with J, Yang (UFF, Niterói, Brazil) J. Zhang (Beihang, Beijing, China)

Slow passage through a homoclinic bifurcation by PWL differential systems

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The phenomenon of slow passage through a bifurcation, also known as dynamic bifurcations, arises when a slow dynamics is added to the bifurcation parameter in a differential system that exhibits a bifurcation. Specifically, if the bifurcation parameter varies slowly with a speed $0 < \varepsilon \ll 1$, the system gradually crosses the bifurcation point. In general terms, the dynamics of the resulting slow-fast system is governed by Fenichel's Theory. This theory establishes that the compact, normally hyperbolic invariant manifolds of the fast subsystem (when $\varepsilon = 0$) persist as slow invariant manifolds of the full system when $0 < \varepsilon \ll 1$, maintaining their stability properties. However, at the bifurcation point, when some invariant set of the fast subsystem

loses normal hyperbolicity, effects not predicted by Fenichel's Theory arise. Specifically, when the bifurcation involves the loss of stability of an equilibrium, delay phenomena in the loss of stability are observed. These phenomena have been extensively documented in the cases of the Hopf bifurcation and the transcritical bifurcation. Recently, in [P Carter, Spike-adding canard explosion in a class of square-wave bursters, Journal of Nonlinear Science, 2020], the phenomenon of slow passage through a homoclinic bifurcation has been studied, introducing considerable additional complexity since one of the invariant manifolds involved is two-dimensional.

Piecewise linear (PWL) differential systems have proven to be valuable tools in complementing the existing understanding of nonlinear dynamics, both in phase space analysis and in parameter space analysis, within a more accessible framework. This is particularly true in the context of slow-fast dynamics, where PWL systems have the advantage of possessing canonical slow manifolds that are unique, explicit, and affine in structure. As a result, in previous works, we have addressed the phenomenon of slow passage through Hopf bifurcations and transcritical bifurcations using PWL systems, providing relevant qualitative and quantitative insights. In the present work, we extend the analysis to the phenomena of slow passage through a homoclinic bifurcation using PWL systems. For this initial approach, we employ a PWL version of the Morris-Lecar neuronal model, which we refer to as PWL-ML. Although the nature of the homoclinic bifurcation in the PWL-ML model differs from that of the original model, some dynamic aspects obtained in the analysis of the slow passage through this bifurcation are relevant for understanding the spike-adding phenomenon

observed in the Morris-Lecar model.

On complex normal forms for the double boundary focus

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Universidad de Sevilla - Spain

We consider planar piecewise polynomial systems with a straight line as separation boundary between the two vector fields. It is assumed that the origin, which belongs to the boundary, is an isolated equilibrium of center/focus type for both vector fields. Thus, we deal with the so called FF case. Working in the complex setting, firstly we study which are the possible normal forms, and secondly, how to exploit these normal forms in computing the Lyapunov constants that characterize the cyclicity of the origin. To illustrate the usefulness of the approach, some examples regarding piecewise quadratic systems are considered; in particular, we show how a piecewise quadratic system with an attractive weak focus from both sides can produce a repulsive weak focus. This is a joint work with Marina Esteban, Emilio Freire, and Francisco Torres.

A systematic study of multi-layer canard cycles

Freddy Dumortier
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Hasselt University, Belgium

Given a slow-fast system containing several layers, we study the canard cycles transiting through these layers. All the canard cycles we consider are topological circles and in between the layers we restrict to generic Hopf breaking mechanisms and generic jump breaking mechanisms. There is a difference in between transiting a terminal layer or a transiting a dodging layer, but more important reveals to be the precise way in which succesive layers are connected to each other. At such a breaking mechanism we have to compare a transition map coming from an attracting sequence on one side to a transition map coming (in negative time) from a repelling sequence on the other side. The comparison can be side-preserving or side-reversing and this makes a major difference in the calculations, affecting strongly the results on the number and the possible bifurcations of the limit cycles. We provide an easy criterion to see whether at a breaking mechanism the comparison is side-preserving or side-reversing and we introduce a "connection diagram" associated to a multi-layer canard cycle. We show that all possible abstract connection diagrams can be realized by a multi-layer canard cycle of a planar polynomial system. We also precisely state the conditions under which a connection diagram can be realized by a multi-layer canard cycle of a (polynomial) Liénard equation. The connection diagram presents in a clear way all the available information that is necessary to get the system of equations describing the bifurcating limit cycles. We provide some general results concerning these equations, leading already to precise consequences on the cyclicity and

the occurring bifurcations for all possible types of 2-layer canard cycles.

Global normalizations for centers of planar vector fields

Clodoaldo Ragazzo
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Universidade de São Paulo, Brazil

In this talk we answer a question posed by Carmen Chicone on whether an analytic vector field with a non-degenerate global center can be transformed into a classical Newtonian equation $\ddot{x} = -V'(x)$. Additionally, we establish a global Poincaré normal form for planar centers.

Bifurcations in random dynamical systems

Jeroen Lamb
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Imperial College, United Kingdom

Random dynamical systems have equations of motion that depend in part on a stochastic process or random variable. They are relevant in many modelling situations in the sciences and engineering. The challenge is to extend the successful theory of deterministic dynamical systems to dynamical systems "with noise". We discuss recent results on the development of bifurcation theory in this setting, emphasizing the importance of probabilistic and topological aspects.

Solutions of the equation $a_n + (a_{n_1} + \ldots + (a_2 + (a_1 + x^{r_1})^{r_2} \ldots)^{r_n} = bx$

Daniel Panazollo

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Université de Haute Alsace, France

We will discuss some problems in dynamical systems where the given equations arise and introduce a novel upper bound for their real solutions. Our approach involves a generalized derivation-division algorithm, which not only provides the bound but also leads to a new set of Chebyshev functions specifically designed for this context.

Talks

Surjectivity of vector fields x injectivity of maps

Francisco Braun

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Universidade Federal de São Carlos, Brazil

The seek for conditions enabling global injectivity of local diffeomorphisms between Euclidean spaces remounts to the beginning of the 20st Century and is still a very active research topic in various areas of Mathematics. In this talk I will discuss conditions related to partial differential equations, singularity theory and dynamics. The results are taken from joint papers with Dias, Santos, Venato-Santos and Teixeira.

Lower Bounds for the Number of Limit Cycles in Piecewise Polynomial Holomorphic Systems

Gabriel Rondón
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Universidade Estadual Paulista, Brazil

In this work, we focus on establishing lower bounds for the number of limit cycles in piecewise polynomial holomorphic systems that have a straight line of discontinuity. Our approach involves several perspectives: analyzing the number of zeros in the first and second-order averaging functions, controlling limit cycles arising from a monodromic equilibrium point through a degenerate Andronov–Hopf bifurcation, and finally considering sliding effects. Additionally, we apply the Poincaré–Miranda theorem to construct an explicit piecewise linear holomorphic system with three limit cycles, result that improves the known examples in the literature that had a single limit cycle.

Nilpotent centers on center manifolds like limits of Hopf centers

Claudio Pessoa
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Universidade Estadual Paulista, Brazil

We prove that nilpotent centers from analytical systems on center manifolds are limits of Hopf-type centers.

Accessing the Pharmacokinetics of Magnetic Nanoparticles in Cirrhosis-Associated Hepatocarcinogenesis by Ordinary Differential Equation Modeling and AC Biosusceptometry

Diego Samuel Rodrigues
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Universidade Estadual de Campinas - Brazil

Pharmacokinetic studies using mathematical modeling examine the distribution of substances in multiple organs and compartments over time. Accordingly, this paper investigates the effect of cirrhosis-associated hepatocarcinogenesis on the pharmacokinetics of magnetic nanoparticles (MNPs) by using an *in vivo* model of cancer progression. To this end, a multichannel AC biosusceptometry system was used to record the transit of MNPs, in the heart and the liver, in two groups of animals: control (SAL), and within the diethylnitrosamine/thioacetamide-induced model of hepatocarcinogenesis (DEN/TAA). The evolution of MNPs concentration is then robustly described as a compartmental model, considering the transfer rate of nanoparticles from the heart to the liver (k_1), their return from the liver to the heart through the circulatory system (k_2), and their irreversible uptake by Kupffer cells within a liver subcompartment (k_3). Our nonlinear mixed-effects parameter estimation modeling shows that k_2 and k_3 change between SAL and DEN/TAA groups, but not k_1, indicating that cirrhosis-associated hepatocarcinogenesis is likely to affect mainly liver pharmacokinetics. Correspondingly, this is precisely reported here by also presenting a covariance analysis and an inter-individual variation of the transfer rates of the magnetic nanoparticles, contributing to their development in theranostic applications and tailored drug delivery systems.

Topological equivalence at infinity of some planar vector fields and its principal part via Newton polytope

Regilene Delazari dos Santos Oliveira

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Universidade de São Paulo - Brazil

Let X be a planar polynomial vector field with a fixed Newton polytope Γ. In this talk, we shall present recent results about the dynamics at infinity of some planar vector fields X and its relation with their Newton polytopes Γ. More specifically, we show that the monomials associated to the upper boundary of Γ satisfying some non-degeneracy conditions determine, under topological equivalence, the phase portrait of X in a neighborhood of the boundary of the Poincaré–Lyapunov disc. We also present vector fields of chief interest which do not satisfy such non-degeneracy conditions but still their phase portraits are determined by the monomials associated to the upper boundary of Γ in the Newton polytope of X. These results are part of works in collaboration with Thais Dalbelo (UFSCar), Otávio Perez (ICMC-USP) and Claudia Valls (IST-Lisboa).

Bifurcation of a boundary hyperbolic singularity in $\mathbb{R}^3$

Kamila da Silva Andrade
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In this work, we consider a two-parameter family of piecewise smooth vector fields in $\mathbb{R}^3$ that perturbs a linear-regular piecewise smooth vector field with a hyperbolic boundary singularity. More specifically, we demonstrate that for small parameter values, a sliding Shilnikov connection curve emerges on the bifurcation diagram of this family.

An overview of polycycles in Filippov Systems

Otávio Marçal Leandro Gomide
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Universidade Federal de Goiás, Brazil

Generally speaking, a graphic Γ of a smooth system X is a set made by the union of a finite number of isolated singular points and regular orbits of X which is an S^1-immersion oriented by increasing time along the regular orbits. If it is possible to define a first return map induced by the orbits of X around Γ, then such graphic is said to be a **polycycle**.

This talk is devoted to present an extension of such concept to the nonsmooth world of Filippov systems. We also provide a discussion on some results on generic bifurcation around some polycycles in Filippov systems of dimensions 2 and 3.

Optical Billiards

Sônia Pinto de Carvalho
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Universidade Federal de Minas Gerais - Brazil

Lignières and Georget, in 2009, presented an asymptotic analysis of high-frequency acoustic modes in rapidly rotating stars, based on acoustic ray dynamics. They remark that, as the star rotates, its boundary is a deformation of a sphere on the equatorial plan, i.e. it is a 4-periodic symmetric ovoid. Then they show that, using the symmetries of rotation, the acoustic rays can be described in two ways: either as trajectories of a particle under a classical 2-degree of freedom Hamiltonian depending on the frequency of rotation of the star, with a potential tending to infinity when approaching the boundary of the star; or as trajectories of optical rays in an isotropic 2-dimensional medium, with medium index depending on the distance to the center of the star.

This analysis produces a new kind of billiard problem, called optical billiard, where an optical ray travels along the geodesics of a Riemannian metric associated to the medium index, which depends only on the distance to the origin, performing elastic reflections at the impacts with the boundary, where elastic means angle of incidence equal angle of reflection when measured by the internal product induced by the Riemannian metric.

The motion is then completely determined by the point of reflection at the boundary and the direction of movement immediately after each reflection. So, a parameter θ, which locates the point of reflection, and the angle φ between the direction of motion and the tangent to the boundary at the reflection point, may be used to describe the system.

The billiard model defines then a bidimensional map, which associates to each impact coordinate and direction of motion (θ_0, φ_0) the next impact and direction (θ_1, φ_1). If the boundary is a C^k closed geodesically convex curve then the billiard map is a C^{k-1} Twist reversible and conservative diffeomorphism, having some of the generic properties as the usual convex billiard in the plane.

This is a joint work with Cássio H. V. Morais (UFES), Rafael da Costa Pereira (UFMG) , Mário J. Dias Carneiro (UFMG) and Sylvie Oliffson Kamphorst (UFMG).

Planar current fields

Luis Fernando Mello

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Universidade Federal de Itajubá, Brazil

Given a complex function $\psi : D \subset \mathbb{C} \longrightarrow \mathbb{C}$, $\psi(x+iy) = u(x,y) + iv(x,y)$, called wave function, we define the current field $\mathcal{J} : D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ as follows

$$\mathcal{J}(x,y) = u(x,y)\nabla v(x,y) - v(x,y)\nabla u(x,y),$$

where ∇ stands for the gradient operator. We analyze the phase portraits of $\mathcal{J}$ when the wave functions are holomorphic or meromorphic with emphasis on the special cases in which the wave function is polynomial or rational. Studies related to this subject are being developed in collaboration with D.C. Braga, A.F. Fonseca and R.M. Ribeiro.

Natural measures and statistical properties for non-statistical dynamical systems

Douglas Coates
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Universidade de São Paulo, Brazil

I will present recent joint work with I. Melbourne (University of Warwick, UK) and A. Talebi (Sharif University, Iran) in which we show that a large class maps that do not admit physical measures nevertheless exhibit strong statistical properties. In particular, we give sufficient conditions for existence of a distinguished natural measure ν such that the push forwards of any absolutely continuous probability measure converge to ν. Moreover, we obtain a distributional limit law for empirical measures. We also extend existing results on the characterization of the set of almost sure limit points for empirical measures. Our results apply to various intermittent maps with multiple neutral fixed points preserving an infinite σ-finite absolutely continuous measure.

Communications

On the Hausdorff dimension and Cantor set structure of invariant sliding Shilnikov sets

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We will study sliding Shilnikov connections, a recently introduced object of Filippov systems that implies chaotic behavior on an invariant subset of the system. In this talk, we conduct a local analysis on the first return map associated with a Shilnikov sliding connection, which reveals a conformal iterated function system (CIFS) structure. Utilizing this theory, we estimate the Hausdorff dimension of the local invariant set of the first return map, demonstrating that it is strictly greater than zero and strictly less than one, and its one-dimensional Lebesgue measure is zero. Furthermore, we prove that the closure of the local invariant set is a Cantor set with the same Hausdorff dimension and Lebesgue measure as the original invariant set. Additionally, it is defined as the invariant set augmented with the set of all pre-images of the regular-fold point.

Entropy of (m,l)-Bernoulli transformations

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The zip shift map extends the bilateral shift map into a two-alphabet symbolic dynamical system and it is an ergodic and mixing map with a chaotic behavior. For two alphabets with m and l symbols, we define the (m,l)-Bernoulli transformations as maps isomorphically mod 0 to a zip shift. In this sense, these maps extend the Bernoulli transformations into a locally invertible context.

In this talk, we present an example of such dynamical systems - the n-to-1 baker's map, which represents a non-invertible model of deterministic chaos, and we show how its folding and Kolmogorov-Sinai entropies are related. We use these results to study the isomorphism problem of (m,l)-Bernoulli transformations following Ornstein's techniques.

On the cyclicity of hyperbolic polycycles

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Let $\mathcal{H}(n)$ be the maximum number of limit cycles that a planar polynomial vector field of degree n can have. We prove that $\mathcal{H}(n)$ is realizable by structurally stable vector fields with only hyperbolic limit cycles and that it is a strictly increasing function whenever it is finite. Coauthored with Armengol Gasull.

Partially Hyperbolic Geodesic Flows

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In this talk we are going to present a construction of Partially Hyperbolic Geodesic flows via conformal metric deformations and a criterion for robust transitivity inside the class of the geodesic flows. Partially hyperbolicity is a topic that has been widely studied during the last 30 years and appears as a generalization of Anosov Systems. It is a classical result of dynamical systems that the geodesic flow is a natural example of an Anosov system when the Riemannian metric has negative sectional curvatures. Many of the techniques that are known nowadays to study Anosov systems were developed during the study of geodesic flows in negative curved manifolds such as the classic Hopf's Argument used to prove ergodicity. The presence of a center bundle implies many difficulties in generalizing techniques used to study uniform hyperbolic cases. For instance, Hopf's Argument can not be applied in general once strong stable and unstable foliations are not transversal for non-Anosov partially hyperbolic systems. For geodesic flows such difficulties can be even harder to overcome once many techniques rely on local perturbation of the system, which is essentially not possible in this setting once a small perturbation on the Riemannian metric can implies big changes in the dynamics of the geodesic flow. Our construction shows the existence of partial hyperbolicity in a new setting and also the existence of systems that are not Anosov but still can present similar interesting behaviors with the presence of a non-trivial center bundle. We are able to give positive answers to questions attributed to Enrique Pujals and Keith Burns on the existence of ergodic partially

hyperbolic geodesic flows that are not Anosov and transitive geodesic flows on Riemannian manifolds with conjugate points.

Entropy and Non-Deterministic Chaos for Piecewise Smooth Vector Fields

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Our context is piecewise smooth vector fields (PSVF) defined on two-dimensional manifolds having a finite number of tangency points. We prove that topological transitivity is a necessary and sufficient condition for the occurrence of non-deterministic chaos when the PSVF system has non-empty sliding or escaping regions.

A fundamental result for continuous flows is the equivalence of topological transitivity and existence of a dense orbit. We prove in our setting that topological transitivity for PSVF systems is indeed equivalent to the existence of a dense orbit, although, in contrast to the continuous case, we are not able to garantee that the dense orbit implies the existence of a residual set of dense orbits.

Finally we prove that, in this context, topological transitivity implies strictly positive topological entropy for the PSVF system. This calculation is made using techniques similar to those from symbolic dynamics.

Dynamics of Composition Operators

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We present about dynamical properties of composition operators acting on spaces of measurable functions. We characterize composition operators satisfying Kitai's Criterion and give an example of a composition operator that is mixing but does not satisfy Kitai's Criterion. (This is a joint work with Karl-G. Grosse-Erdmann, from Université de Mons, Belgium).

Geometric properties of disintegration of measures

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The disintegration of a measure over a partition of the space on which it is defined is a method of rewriting this measure as a combination of probability measures, which are concentrated on the elements of the partition. There are several reasons to study such possible combinations of measures. In Ergodic Theory, for example, the disintegration of a measure is directly related to the ergodic decomposition of invariant measures, which are crucial objects encoding the asymptotic behaviour of dynamical systems. In this talk, we will explore a connection between disintegration of measures and geometric properties of probability spaces. In particular, we will address disintegration from the perspective of an optimal transport problem: by analyzing the disintegration of transport plans, we define disintegration maps, and with

these objects, we will study the regularity and absolute continuity of disintegration of measures.

Poster

A note on a recent attempt to solve the second part of Hilbert's 16th Problem

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For a given natural number n, the second part of Hilbert's 16th Problem asks whether there exists a finite upper bound for the maximum number of limit cycles that planar polynomial vector fields of degree n can have. This maximum number of limit cycle, denoted by $H(n)$, is called the nth Hilbert number. It is well-established that $H(n)$ grows asymptotically as fast as $n^2 \log n$. A direct consequence of this growth estimation is that $H(n)$ cannot be bounded from above by any quadratic polynomial function of n. Recently, the authors of the paper [Exploring limit cycles of differential equations through information geometry unveils the solution to Hilbert's 16th problem. Entropy, 26(9), 2024] affirmed to have solved the second part of Hilbert's 16th Problem by claiming that $H(n) = 2(n-1)(4(n-1)-2)$. Since this expression is quadratic in n, it contradicts the established asymptotic behavior and, therefore, cannot hold. In this work, we further explore this issue by discussing some counterexamples.

Global Dynamics Analysis of a Class of Quadratic Polynomial Differential Systems with an Invariant Plane in $\mathbb{R}^3$

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We study a class of quadratic differential systems in $\mathbb{R}^3$. More precisely, for three distinct families of parameter sets, such differential systems exhibit the property of having an invariant plane. The first family exhibits a first integral in the form of $H(x, y) = ax + by$ ensures the invariance of every plane $ax + by =$ constant. Consequently, we describe the phase portraits of the system on each of such planes in the Poincaré disc. The second family has a Darboux invariant of the form $I(x, y, t) = (d_0 + d_1x + d_2y)\, e^{-k_0 t}$ which will allow us to describe the phase portraits on the Poincaré ball. Unlike the first two families, the third family lacks both a first integral and a Darboux invariant. Nevertheless, we present a detailed analysis of the phase portraits of these systems on the invariant plane using the Poincaré disc.

Dinâmica de um modelo de HIV usando campos vetoriais suaves por partes

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Os sistemas dinâmicos buscam entender o comportamento futuro ou passado dos estados de um sistema para o qual existe uma regra determinística de evolução. Este ramo de pesquisa tem desfrutado de um enorme avanço nos últimos anos e grandes centros de pesquisa têm se destacado no cenário nacional. Além disso, é conhecido que muitos problemas em Física, Engenharia, Economia, Biologia, etc, podem ser vistos como sistemas dinâmicos, cuja dinâmica pode ser entendida mediante a teoria que iremos estudar, principalmente quando o modelo é submetido a algum tipo de operação do tipo liga-desliga onde um campo vetorial X modela o estado ligado e outro campo vetorial Y modela o estado desligado. Será estudado então, a formação de regiões de escape e deslize, intersecção de trajetórias dos campos X e Y, com a região de onde ocorre a mudança de um campo para o outro, definição de conjuntos minimais para este contexto. Será estudado, também, uma aplicação da teoria à modelagem da dinâmica de evolução de células doentes em pacientes diagnosticados com HIV.

Centers of cubic polynomial differential systems

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An equilibrium point p of a differential system in the plane $\mathbb{R}^2$ is a center if there exists a neighborhood U of p such that $U \setminus \{p\}$ is filled with periodic orbits. A difficult classical problem in the qualitative theory of differential systems in the plane is the problem of distinguishing between a focus and a center.

In this work we study conditions under which the origin of coordinates is a center of the following differential systems

$$\dot{x} = -y, \qquad \dot{y} = x + a_1x^2 + a_2xy + a_3y^2 + A,$$

where A is a nonzero monomial of degree 3. Moreover we provide all topologically different phase portraits when $A = a_4x^3$.

Limit cycles for piecewise hybrid Hamiltonian differential systems

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In this work we study two class of hybrid differential systems formed by two linear Hamiltonian systems separated by a straight line or a circle, which we named piecewise hybrid Hamiltonian systems. We investigate an upper bound for the maximum number of limit cycles that these differential systems can exhibit when they have finitely many limit cycles. Moreover we present some examples where this upper bound is reached. In other words we solve the extension of the 16th Hilbert problem to this class of hybrid differential systems.

Pesadelo de Fubini - Fenômeno Contraintuitivo de Teoria da Medida.

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O fenômeno do Pesadelo de Fubini tem implicações no estudo de folheações. O artigo publicado por John Milnor mostra um exemplo através da construção de uma folheação "patológica" no qual suas folhas intersectam um conjunto de medida positiva em no máximo um ponto e no qual a medida desse conjunto calculado através da folheação pelo Teorema de Fubini resulta em uma falha. Isso motiva a procura de propriedades mais fortes de folheações, como folheações absolutamente continua e folheações transversalmente absolutamente continua, por exemplo. O objetivo é falar um pouco de como foi construído esse exemplo de uma folheação no qual ocorre o fenômeno do Pesadelo de Fubini, que propriedades decorrem da absoluta continuidade de folheações e expor alguns resultados, envolvendo o aparecimento de Pesadelos de Fubini no estudo de Sistemas Dinâmicos.

Conexão de Shilnikov Em Sistemas Lineares Por Partes em $\mathbb{R}^3$.

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Neste trabalho, estudamos a existência de conexões de Shilnikov deslizante em sistemas de equações diferenciais ordinárias suaves por partes da forma

$$\dot{x} = \begin{cases} Ax + n^+, & \text{se} \quad h(x) > 0, \\ Ax + n^-, & \text{se} \quad h(x) < 0, \end{cases}$$

onde $x \in \mathbb{R}^3$, a função h é expressa por $h(x) = z$ e $\dot{x}$ denota a derivada em relação ao tempo t. Além disso, $A = (a_{ij})_{3\times 3}$ para $i, j \in \{1, 2, 3\}$ e $n^{\pm} = (n_1^{\pm}, n_2^{\pm}, n_3^{\pm})$ são a matriz e o vetor de parâmetros, respectivamente, com cada $a_{ij}, n_i^{\pm} \in \mathbb{R}$.

Este trabalho é financiado pela Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG).

Centers of cubic polynomial differential systems

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An equilibrium point p of a differential system in the plane $\mathbb{R}^2$ is a center if there exists a neighborhood U of p such that $U \setminus \{p\}$ is filled with periodic orbits. A difficult classical problem in the qualitative theory of differential systems in the plane is the problem of distinguishing between a focus and a center.

In this talk we study conditions under which the origin of coordinates is a center of the following differential systems

$$\dot{x} = -y, \qquad \dot{y} = x + a_1x^2 + a_2xy + a_3y^2 + A,$$

where A is a nonzero monomial of degree 3. Moreover we provide all topologically different phase portraits when $A = a_4x^3$.

Towards a conjecture by Arnol'd

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The uniqueness of an unbounded component for each semilevel set of pseudoperiodic functions is conjectured by Arnol'd and proved for the real analytic setup and for the case when the function comes from $M = \mathbb{T}^2$.

We furnish weaker versions of this conjecture and also give a proof of it for the case when M is a closed orientable surface.

Modelagem com Equações Diferenciais: competição e coexistência em modelos populacionais

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Sistema de equações diferenciais Lotka-Volterra para predatismo e competição interes-pecífica. Para os dois primeiros modelos apresentaremos as soluções e a análise gráfica a partir dos respectivos campos de direções. Além do uso dos principais resultados sobre equações diferenciais, estudaremos dois resultados clássicos da teoria de sistemas dinâmicos sendo eles o Teorema da Forma Canônica de Jordan e o Teorema de Hartman–Grobman. O primeiro resultado classifica os sistemas de equações diferenciais lineares contribuindo para sua resolução, e o Teorema de Hartman-Grobman nos permite fazer uma análise de sistemas de EDO's não lineares uma vez que afirma que o comportamento qualitativo de tais sistemas é topologicamente equivalente ao comportamento de um sistema de EDO's linear. Os Teoremas citados serão fundamentais no desenvolvimento do trabalho visto que os sistemas de equações diferenciais Lotka-Volterra consistem em sistemas não lineares.

Limit cycles bifurcation in multi-parametric differential systems having a center

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We propose a new closed expression for the first-order Taylor Series of the First Melnikov Function to study limit cycles in multi-parametric differential systems with a center. In some cases, this new approach leads to identifying more limit cycles, up to a first-order analysis, while avoiding some of the typical computational obstructions of higher-order calculations.

Evolutionary Stable Strategies and Cubic Vector Fields

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The introduction of concepts of Game Theory and Ordinary Differential Equations into Biology gave birth to the field of Evolutionary Stable Strategies, with applications in Biology, Genetics, Politics, Economics and others. In special, the model composed by two players having two pure strategies each results in a planar cubic vector field with an invariant octothorpe. In this poster we provide the global phase portraits of the generic systems with a singularity at the central region of the octothorpe.

Generic singularities of relay systems

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In this paper, we present a classification of relay systems (on $\mathbb{R}^n$), categorizing generic and codimension one phenomena. We establish the openness and density of these families, providing an unfolding for normal forms and a description of their dynamics establishing stable conditions for one-parameter families of such systems.

The focus-center-limit cycle bifurcation in Buck Converter with PI controller

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A Buck converter is a DC-DC power converter that produces a lower output voltage from a given input voltage. The ideal dynamic model of the system in continuous driving mode is given by two-dimensional piecewise linear systems, where its evolution is controlled by a switch. However, the resistive load R is a variable parameter of the system, whose variation changes the position of the operating point (output voltage). To regulate the output voltage in relation to the disturbance of the resistive load R, a proportional-integral (PI) controller was introduced, transforming the system into a three-dimensional piecewise-linear dynamic system with two switching boundaries and whose control depends on two parameters, K_p and K_i. In this scenario, the system has a single equilibrium point $\mathbf{p}$ located within the central region, where it is assumed to be of focus type and K_p and K_i give us

conditions to obtain stability. Our aim is to describe qualitatively and quantitatively the focus-center-limit cycle bifurcation that arises from changes in the stability of the equilibrium point. Specifically, we aim to explore the bifurcation associated with the crossing of two eigenvalues of the system's linear matrix through the imaginary axis. Where, in the imaginary axis, we will have a center configuration which will be limited by the switching boundaries. Then, the question is whether a limit cycle can bifurcate from the most external periodic orbit of this center. This limit cycle, which exhibits a large amplitude, is a crossing limit cycle that intersects one or both switching boundaries (in the symmetric case). Such behavior is common in control problems where the control variable experiences saturation. Finally, numerical simulation results are presented to illustrate these dynamics.

One-dimensional Piecewise Smooth Rational Degree Maps

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We consider a class of continuous maps characterized by a singularity of order $x^{q/p}$ (with $p, q \in \mathbb{N}$, $p > q$, and $(p, q) = 1$) on one side of the discontinuity boundary Σ and a linear behaviour on the other side. Such maps arise naturally in the study of grazing bifurcations of hybrid and piecewise flows. In this context the boundary collision of a fixed point of the map with Σ corresponds to a grazing bifurcation of the flow. We will start by studying one-dimensional maps, and our main result is a classification of all bifurcation scenarios, including: period doubling and robust chaos.

Matching cross-cap singularities of Gutierrez-Sotomayor flows

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We investigate how singular flow lines of a Gutierrez-Sotomayor flow are allowed to travel along the ambient singular two-manifold, monitoring their trajectories as they pass through isolating blocks which compose the manifold. The idea is to register the topological obstructions that may occur depending on what singularities make up the α-limit and ω-limit sets of the singular flow lines. Here we put this approach to test, considering only singular flow lines of cross-cap singularities. We obtain a more general realizability theorem for Lyapunov graphs labeled with regular singularities, cones and cross-caps. Surprisingly, the ambient manifolds for all topologically non equivalent flows associated to one such graph can be constructed having the same level sets.

Normal Forms, Blow-Up and Nilpotent Singularities

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In this poster, we apply the formal normal form theorem to vector fields given by $X = A + f$, on $\mathbb{R}^n$ with A linear, and f a C^∞ function such that $f(0) = Df(0) = 0$. Next, we will then classify all nilpotent singularities at the origin of normal form of this vector fields and display their phase portraits.

Hopf-like bifurcation in the Holling-Tanner model with discontinuous harvesting action

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In this work we consider a model of interaction between a pair of biological species, where one of the species preys upon the other, suggested by Tanner and known in the literature as the Holling-Tanner model. This model has motivated several studies that prove the occurrence of two Hopf bifurcations (subcritical and supercritical) and show the coexistence of two limit cycles. In the present work, we introduce to the Holling-Tanner model a discontinuous harvesting action on the predatory specie with the aim of controlling the number of prey at a desired operating point, thus obtaining a planar piecewise smooth system with a straight line as its switching boundary. Based on the Poincaré map defined in a neighborhood of the desired operating point, we give explicit conditions on the system parameters so that the desired operating point is locally asymptotically stable. In addition, we prove the occurrence of two Hopf-like bifurcations: (i) *supercritical*, giving rise to a stable crossing limit cycle; (ii) *subcritical*, giving rise to an unstable crossing limit cycle. Finally, from numerical simulation results we show that these two crossing limit cycles can coexist and that a Saddle-Node bifurcation between them also occurs.

Shadowing lemma for piecewise Smooth vector fields

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Since the Shadowing Theorem is a classical result for discrete and smooth dynamical systems, the purpose of this paper is to obtain a version of it to the piecewise smooth scenario. An example where the result can not be obtained using the same hypotheses of the smooth case is exhibited. So, using suitable hypotheses, we prove a Like Shadowing Theorem for our context. Moreover, some extensions of the main result were obtained in the sense to reveal typical situations observed for piecewise smooth vector fields.

Principal Configuration near a hyperbolic singular point of a linear system

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In this work we analyze the principal configuration of a plane distribution Δ_η associated with liner vector field $\eta = (\eta_1, \eta_2, \eta_3)$ which has a hyperbolic singularity. The implicit differential equations

$$2(D\eta(dr), dr, \eta) + \langle \mathrm{rot}(\eta), \eta \rangle \cdot \langle dr, dr \rangle = 0, \quad \langle \eta, dr \rangle = 0,$$

that characterize the principal linear of the plane distribution Δ_η can be extended analytically to the singularity of a linear vector field η with a hyperbolic singularity. We show that the principal configuration in

a neighborhood of a regular partially umbilic point of a linear field η is always of type D_1.

We will show the principal configuration for two examples of hyperbolic linear vector fields. One whose the plane distribution Δ_η is completely integrable and another whose the plane distribution is not completely integrable.

In the first case we will see that the partially umbilical set is constituted of two straight lines that converge at the origin and their main configurations are as in the triply orthogonal system of Dupin surfaces. In the second case, we will show that the partially umbilic set is a pair of straight lines crossing through the origin.

Period function for a class of piecewise Hamiltonian systems in the plane

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In this work, we investigate the monotonicity and the number of critical periods of the period function associated with a family of piecewise planar continuous potential systems. Specifically, we detail the bifurcation diagram of the period function, identifying the regions in the parameter space where it exhibits monotonic increase or decrease, and where it has at most one simple critical period. To establish the main result, we use the fact that this function can be expressed in terms of the two period functions of the uncoupled planar Hamiltonian systems joined by the separation line.

Local cyclicity for double-reversible cubic systems with three aligned centers

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In 1900, Hilbert proposed a list of mathematical problems for the 20th century. The discussion about a uniform upper bound of limit cycles of a polynomial planar system depending on its degree is a part of the well-known 16th Hilbert's problem. In this work, we are interested in the limit cycles bifurcating from reversible centers. The first step is to determine conditions for the existence of such centers, that is, it is necessary to solve the center-focus problem for such class of systems. We study the systems which has a reversible symmetry with respect to the x-axis and to the y-axis, being then called double-reversible systems. The reversible symmetry systems appear in many applied areas such as physics, thermodynamics and quantum mechanics. Moreover, under the mathematical point of view, reversible symmetry is an interesting characteristic of a system because it provides the answer to the Center-Focus Problem when a monodromic critical point belongs to the curve of symmetry, in short, it ensures that such a singularity can not be a focus. In this work, we provide normal forms of the double-reversible homogeneous cubic systems having three aligned singularities of center type and we study their local cyclicity which depends on the parameters of the systems.

Holomorphic Systems

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In this work, we will study local and global properties of complex first order differential equations $\dot{z} = f(z)$, where f is a function defined on an open set $U \subset \mathbb{C}$, that except for poles, f is holomorphic in U. We will also study piecewise holomorphic systems, in particular, we provide conditions to ensure the existence of limit cycles of these systems.

Generic one-parameter families of 3-dimensional Filippov Systems

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This presentation addresses openness, density and structural stability conditions of one-parameter families of 3D piecewise smooth vector fields (PSVF) defined around typical singularities. Our treatment is local and the switching set, M, is a $2D$ surface embedded in $\mathbb{R}^3$. In short, we analyze the robustness and normal forms of certain codimension one singularities that occur in PSVF. The main machinery used in this paper involves the theory of contact between a vector field and M, Bifurcation Theory and the Topology of Manifolds. Our main result states robust mathematical statements resembling the classical Kupka-Smale Theorem in the sense that we establish the openness and density of a large class of PSVF presenting generic and quasi-generic singularities.

As an application, we present results involving PSVF that are a combination of linear and quadratic vector fields. Due to the lack

of uniqueness of certain solutions associated with PSVF, we employ Filippov's theory as the basis of our approach throughout the paper.

Some aspects of generic bifurcation theory in this context are fairly discussed. More precisely, the codimension one bifurcation emerging from the lack of structural stability is formally proved as codimension one sub-manifolds within the space of non-smooth vector fields. The background of the presentation lies in the context of structural stability, inspired by the works of mathematicians such as Andronov, Peixoto, and Sotomayor. In particular, some classical results from bifurcations theory, necessary to prove the results of the manuscript, are revisited for the sake of completeness.

The central problem of the local bifurcation theory for vector fields is to establish suitable conditions for the stability of trajectories near singularities. Such vector fields generate systems of differential equations, locally represented by equations of the form $\dot{x} = f(x, \lambda)$, where λ represents the bifurcation parameter. For background information on bifurcation theory.

It is worth to say that, the study of local bifurcation phenomena in PSVF for dimensions greater than 2 is poorly developed. Moreover, in general, the analysis of piecewise smooth bifurcations reveals an alarming proliferation of cases, subcases, and sub-subcases as the dimension of the phase space increases. One finds in real life and in various branches of science distinguished phenomena whose mathematical models are expressed by PSVF and deserve a systematic analysis. For example, in problems of control theory, electric and electronic systems, in biological models, models that propose an alternative protocol to treat diseases.

In this presentation we employ well establish results from the smooth and piecewise smooth bifurcation theory to study local bifurcations in 3D PSVF. We set a co-dimension one surface M given by $h^{-1}(0)$, where 0 is a regular value of $h : \mathbb{R}^3 \to \mathbb{R}$. The surface M split $\mathbb{R}^3$ into two half-spaces, the "+" half space corresponding to the points of $x \in \mathbb{R}^3$ such that $h(x) \geq 0$ and the "-" half space defined analogously. We denote Ω^r the set of PSVF, we call $X = (X^+, X^-)$, endowed with the product topology. The vector fields X^+ and X^- are defined on ^{3} but on Ω^r they are assumed to be restricted to the "+" and "-" half spaces, respectively.

Concerning the bifurcation theory for smooth vector fields, there exist several works providing a detailed analysis of the bifurcation events, normal forms and unfolding, with a precise characterization of their codimensions. However, in the context of PSVF, there exist only scarce results. In the two-dimensional case, there are some results addressing the classification of subsets of codimension zero, one and two, normal forms, explicit construction of topological equivalence between a PSVF and its respectively normal form.

In the three-dimensional context, some subsets of PSVF that are of codimension zero (locally structurally stable). One of the main obstacles to developing a systematic approach to characterize codimension one phenomenon in PSVF is that in dimension three a new kind of singularity appears, the so-called T-singularity or two-singularity. In the presence of that singularity, the PSVF may present a chaotic behavior, invariant cones, modulo infinite of instability.

Acknowledgments. Durval José Tonon is partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq

Grant 310362/2021-0)

Bounded Kukles systems of degree three

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One of the most important problems in the Qualitative Theory of Differential Equations in the plane is the study of global asymptotic stability: an equilibrium point that is globally attractive. It is known that if an equilibrium point of a planar vector field is globally asymptotically stable then the vector field is bounded. A planar vector field is said to be bounded if the forward orbit of every point enters and remains in a compact set. In this paper we study cubic Kukles systems that are bounded, that is, the first step towards the characterization of cubic Kukles systems that are globally asymptotically stable. We obtain a total of 25 cubic Kukles subsystems that are bounded and we provide 11 phase portraits of such subsystems in a neighbourhood of infinity.

Analytic study of two limit cycles bifurcating from a zero–Hopf equilibrium

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In this work we provide sufficient conditions for the existence of two limit cycles bifurcating from the unique zero–Hopf equilibrium of the differential system

$$\dot{x} = y + a, \quad \dot{y} = -x + z, \quad \dot{z} = -bx^2 + z^2 + c,$$

where a, b and c are real arbitrary parameters. Our study uses the averaging theory. This differential system has been studied previously for some authors because it can exhibit chaotic motion when it has no equilibrium points.

Conley index of low codimension singularities of Filippov vector fields

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Let U be a connected, open and bounded subset of $\mathbb{R}^2$, and let $K = \bar{U}$ be its closure. Consider the set $C^r(K, \mathbb{R}^2)$ as the collection of all C^r vector fields $X : K \to \mathbb{R}^2$. Additionally, let $\Omega_h^r(K, \mathbb{R}^2)$ be the space of piecewise vector fields given by:

$$Z(x) = \begin{cases} X(x) & \text{if } h(x) > 0, \\ Y(x) & \text{if } h(x) < 0, \end{cases} \tag{1}$$

where X, $Y \in C^r(K, \mathbb{R}^2)$, and $h : K \to \mathbb{R}$ is a differentiable function with zero as a regular value. The usual notation for Equation 1 is

$Z = (X, Y)$, and it is referred to as a **Filippov system**. The set $\Sigma = h^{-1}(0)$ represents the switching surface of codimension 1.

It is worth noting that $C^r(K, \mathbb{R}^2)$ is equipped with the C^r topology, while $\Omega_h^r(K, \mathbb{R}^2)$ is endowed with the product topology. To understand the behavior of this kind of vector field, we introduce the concept of **solution** of a Filippov vector field using differential inclusions.

In this context, C. Thieme has defined the Conley index for an isolated invariant set S of a Filippov vector field in terms of a pair of subsets (P, Q) that captures the behaviour of the orbits on an isolating neighborhood of S.

In this work, we calculate the Conley index for low codimension singularities of Filippov vector fields. We aim to establish a connection between the Conley index of these singularities and the index of a singularity in a Filippov vector field, as a generalization of the Poincaré-Hopf Theorem.

Estrutura topológica do retrato de fase para campos vetoriais lineares em $\mathbb{R}^3$ e aplicações

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Neste trabalho apresentamos uma análise do sinal da parte real das raízes de um polinômio de grau 3 em função de seus coeficientes. O tratamento do problema é feito principalmente do ponto de vista analítico, sabendo que do ponto de vista algébrico o problema já vem sendo tratado na literatura sem avanços significativos. Como aplicação dos resultados obtidos, caracterizaremos completamente a dinâmica e a estrutura topológica do retrato de fase para campos lineares em

$\mathbb{R}^3$. Além disso, apresentamos aplicações no estudo qualitativo local de pontos de equilíbrio para campos vetoriais em $\mathbb{R}^3$ definidos a um ou mais parâmetros.

On the boundedness of solutions of a forced discontinuous oscillator

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By understanding the behavior of solutions of discontinuous differential equations, we can gain insight into various phenomena in both natural and engineering domains, where systems exhibit sudden changes. For instance, such knowledge can be applied to model the population growth of species when a new predator or prey is introduced or to model chemical reactions that lead to the formation of solids.

In this work, we perform a qualitative study in the discontinuous undamped oscillator

$$\ddot{x} + \operatorname{sgn}(x) = \varepsilon\, p(t), \tag{2}$$

where sgn stands for the standard sign function, $\varepsilon \geq 0$ is a small real parameter and

$$p(t) = a_0 + \sum_{k\geq 1}(a_k \cos(kt) + b_k \sin(kt))$$

is real analytic and $2\pi-$periodic in t. More specifically, we determine the existence of infinitely many nested invariant tori of (2) with large amplitude, which in turn involves the boundedness all solutions of (2).

The strategy in showing such result is to consider the impact map defined in a subset of the discontinuous set $\Sigma = \{(t, x, \dot{x}) \in \mathbb{R}^3 : x =$

0}. In good coordinates, this map turns out to be exact sympletic close to an integrable twisting map. Then, by means of the parametrization method, we obtain a version of the KAM theorem for a general class of exact sympletic twisting maps that, applied to our case, provides an uniform upper bound for ε that depends only on p. This result allows to determine the persistence of invariant tori carrying quasi-periodic solutions of large amplitude.

Periodic orbits in the most general perturbed Hénon-Heiles cubic Hamiltonian system

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We study analytically the families of periodic orbits that bifurcate from the equilibrium point localized at the origin of coordinates of the most general cubic Hamiltonian system that is a perturbation of the Hénon-Heiles Hamiltonian system.

Global Dynamics and Bifurcation of Periodic Orbits and Invariant Tori in a Modified Nosé-Hoover Oscillator

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We perform a global dynamical analysis of a modified Nosé-Hoover oscillator, obtained as the perturbation of an integrable differential system. Using this new approach for studying such an oscillator, in the integrable case we give a complete description of the solutions in the phase space, including the dynamics at infinity via the Poincaré compactification. Then using the averaging theory, we prove analytically the existence of a linearly stable periodic orbit which bifurcates from one of the infinite periodic orbits which exist in the integrable case. Moreover, by a detailed numerical study, we show the existence of nested invariant tori around the bifurcating periodic orbit. Finally, starting with the integrable cases and increasing the parameter values, we show that chaotic dynamics may occur, due to the break of such an invariant tori, leading to the creation of chaotic seas surrounding regular regions in the phase space.

Hyperbolicity of renormalization for dissipative gap mappings

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A gap mapping is a discontinuous interval mapping with two strictly increasing branches that have a gap between their ranges. They are one-dimensional dynamical systems, which arise in the study of certain higher dimensional flows, for example the Lorenz flow and the Cherry flow. In this work we prove hyperbolicity of renormalization acting on C^3 dissipative gap mappings, and show that the topological conjugacy classes of infinitely renormalizable gap mappings are C^1 manifolds.

A Melnikov analysis for a class of piecewise smooth differential systems with nonsmooth perturbations

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In this work, we provide a Melnikov-like function for the following family of piecewise smooth differential systems

$$\begin{cases} x' &= y + \lambda f(x,y), \\ y' &= \eta x - \alpha sgn(x) + \lambda g(x,y), \end{cases} \tag{3}$$

where $\lambda \in \mathbb{R}$ is a sufficiently small perturbation parameter and $f, g : \mathbb{R}^2 \to \mathbb{R}$ are piecewise smooth functions given by

$$f(x,y) = \begin{cases} f^-(x,y) & \text{if } x \leqslant 0, \\ f^+(x,y) & \text{if } x \geqslant 0, \end{cases}$$

and

$$g(x,y) \;=\; \begin{cases} g^{-}(x,y) & \text{if } \; x \leqslant 0, \\ g^{+}(x,y) & \text{if } \; x \geqslant 0. \end{cases}$$

Using the switching line $\Sigma = \{(x,y) \in \mathbb{R}^2 : x = 0\}$ as a Poincaré section, a displacement function can be constructed, which quantifies the distance between the positive forward flow and the negative backward flow where both intersect Σ. Then, a Melnikov-like function is obtained by expanding this displacement function into a Taylor series. For λ small enough, this function governs the existence of crossing limit cycles bifurcating from the period annulus of the unperturbed system, which means a region composed by a family of periodic orbits.

It is worth noting that the work aims to extend the analysis made in [Novaes, D. D., Silva, L. V. M. F. *A Melnikov analysis on a family of second order discontinous differential equations.* São Paulo J. Math. Sci., 2024]. However, differently from the situation expressed in the system (3), in their work the system considered is not autonomous and, furthermore, the perturbation considered is smooth and occurs only in the y' component.

Relaxation oscillation in piecewise smooth fast-slow systems

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In this poster we provide a geometric analysis of relaxation oscillations in the context of planar fast-slow systems with discontinuous right-hand side. We give conditions which guarantee the existence of a stable crossing limit cycle Γ_ε when the singular perturbation parameter ε is positive and small enough. Moreover, in the singular limit $\varepsilon \to 0$, the cycle Γ_ε converges to a piecewise smooth closed singular trajectory. Our approach uses tools in geometric singular perturbation theory.

The study of polynomial submersions via their bifurcation set

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We say that two functions $f, g : \mathbb{R}^2 \to \mathbb{R}$ are *topologically equivalent* if there exist homeomorphisms $h_2 : \mathbb{R}^2 \to \mathbb{R}^2$ and $h_1 : \mathbb{R} \to \mathbb{R}$ such that $h_1 \circ f = g \circ h_2$. On the other hand, if $\mathscr{F}$ and $\mathscr{G}$ are regular foliations of open subsets $U, V \subset \mathbb{R}^2$, respectively, we say they are *topologically equivalent* if there exists a homeomorphism $h : U \to V$ such that each leaf of $\mathscr{F}$ is carried onto a leaf of $\mathscr{G}$.

Given $f, g : \mathbb{R}^2 \to \mathbb{R}$ two topologically equivalent C^∞ submersions, it is clear that the foliations induced by them are topologically equivalent. However, the converse is not true even in the polynomial case.

In this poster we will present the connections between these well known aforementioned concepts and the *separatrix configurations* of

the foliations and the phenomena of *vanishing* and *splitting* at infinity when a value in the *bifurcation set* of the function is approached.

Classificação de pontos hiperbólicos, semi-hiperbólicos e nilpotentes em sistemas diferenciais planares.

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O estudo de Sistemas Dinâmicos é de grande importância, não apenas para matemática, mas também para as áreas como engenharia, física e outras ciências aplicadas, pois aborda fenômenos naturais que evoluem ao longo do tempo, os quais são geralmente modelados por equações diferenciais ordinárias (E.D.O.). Um dos focos dessa área é a Teoria Qualitativa das E.D.O., iniciada por Poincaré no final do século XIX. A teoria qualitativa não estará na busca direta por expressões exatas para resolver o problema, mas se concentra na extração de propriedades (estrutura geométrica e topológica) das soluções por meio de uma análise das equações.

Este trabalho tem como objetivo principal estudar pontos singulares hiperbólicos e semi-hiperbólicos, conhecidos como pontos singulares elementares. Nesse sentido, apresentaremos dois teoremas práticos que nos ajudam a determinar o comportamento desses pontos. Além disso, introduzimos uma ferramenta fundamental para o estudo de singularidades não elementares: o método de blow-ups, uma técnica de mudança de coordenadas que ""expande"" uma singularidade em uma curva com finitas singularidades. Essa técnica é particularmente útil para classificar as singularidades nilpotentes, aquelas com autovalores nulos, mas cuja parte linear não é identicamente nula.

O blow-up permite analisar cada singularidade isoladamente, e, se necessário, aplicar sucessivamente novas transformações até que restem apenas singularidades elementares. Finalmente, através do processo inverso, o blow-down, obtemos o retrato de fase local em torno do ponto singular degenerado, proporcionando uma visão completa do comportamento dinâmico na vizinhança desse ponto. Dessa forma, exploraremos exemplos e os principais teoremas que classificam essas singularidades.

Singularidades de codimensão um de Campos Vetoriais 3D Suaves por Partes Simétricos

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Neste trabalho, apresentamos os tipos topológicos de todas as singularidades de codimensão um de Campos Vetoriais 3D Suaves por Partes Simétricos (CV3DSP simétricos). Trabalhamos com duas classes de CV3DSP simétricos, a saber, os reversíveis e os equivariantes. Embora haja simetria em ambos os casos, eles diferem bastante entre si, uma vez que no caso reversível o campo deslizante Z^s não está definido (não apresenta região de deslize nem de escape) e no caso equivariante Z^s está definido. Essa diferença é bastante significativa, pois na análise de codimensão de singularidades de CV3DSP reversíveis precisamos considerar, devido a simetria, apenas um dos campos que definem o campo $Z = (X, Y)$. Enquanto que no caso CV3DSP equivariante, precisamos considerar os campos X (ou Y) e Z^s, pois nesse caso a codimensão da singularidade é no mínimo a somas das codimensões nos campos X (ou Y) e Z^s. Tal diferença, exclui a possibilidade de pontos de

tangência de ordem maior que dois na origem serem singularidades de codimensão um no caso equivariante, enquanto que no caso reversível esses são singularidades de codimensão um. No caso reversível, as singularidades de codimensão um na origem são: two-lips, que é o caso em que a origem é uma singularidade lips de ambos os campos X e Y; two-beak to beak, que é o caso em que a origem é uma singularidade beak to beak de ambos os campos X e Y; two-swallowtail, que é o caso em que a origem é uma singularidade swallowtail de ambos os campos X e Y; singularidade no bordo, que é o caso em que a origem é uma singularidade hiperbólica, do mesmo tipo, de ambos os campos X e Y, que se encontra sobre a variedade de comutação. No caso equivariante, as singularidades de codimensão um na origem são: singularidade no bordo, caso análogo ao reversível; singularidade sela-nó, que é o caso em que os campos X e Y são transversais à variedade de comutação na origem e a origem é uma singularidade do tipo sela-nó do campo deslizante Z^s; singularidade Hopf, que é o caso em que os campos X e Y são transversais à variedade de comutação na origem e a origem é uma singularidade do tipo Hopf do campo deslizante Z^s. Nosso resultado principal, afirma que o conjunto de todos os campos em CV3DSP reversíveis que tem uma singularidade de codimensão um na origem é aberto e denso conjunto dos CV3DSP reversíveis e que o conjunto de todos os campos em CV3DSP equivariantes que tem uma singularidade de codimensão um na origem é aberto e denso conjunto dos CV3DSP equivariantes. Para provar tal resultado, apresentamos, em cada um dos casos, as submersões que fornecem os desdobramentos de todas a singularidades de codimensão um citadas acima em ambos os casos, reversível e equivariante.

The local bifurcations in a class of piecewise smooth system with a nonregular switching curve via a nonlinear double-regularization process

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We are interested in the study of piecewise smooth vector fields, in the case where the edges of the smooth parts of the fields are not regular via a regularization process. More precisely, we are interested in an analysis of the preservation of the bifurcations according to a regularization process, which approximates such a class by a continuous vector field.

Continuous planar piecewise linear systems with three zones without equilibrium points

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Consider planar piecewise linear systems with three zones separated by two straight lines and without equilibrium points, neither real nor virtual as follows

$$\mathcal{X}_L : \begin{cases} \dot{x} &= a_L x + b_L y + \gamma_L \\ \dot{y} &= \lambda_L a_L x + \lambda_L b_L y + \delta_L, \quad x \leq -1, \end{cases}$$

$$\mathcal{X}_C : \begin{cases} \dot{x} &= a_C x + b_C y + \gamma_C \\ \dot{y} &= \lambda_C a_C x + \lambda_C b_C y + \delta_C, \quad -1 \leq x \leq 1, \end{cases}$$

$$\mathcal{X}_R : \begin{cases} \dot{x} &= a_R x + b_R y + \gamma_R \\ \dot{y} &= \lambda_R a_R x + \lambda_R b_R y + \delta_R, \quad x \geq 1. \end{cases}$$

where $\delta_k \neq \lambda_k\gamma_k$, $k \in \{L, C, R\}$. A way for these systems to be continuous on the separation lines is obtained. In addition, we analyze whether there can be periodic orbits and for some cases on the coefficients of the system we prove that there are no periodic orbits. We conjecture that there are no periodic orbits in any case.

Dynamic study of the "simplest" memristor-based chaotic circuit

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The study of chaotic differential systems intensified in 1963, when Edward Lorenz discovered the first chaotic attractor in a three-dimensional autonomous system. Since then, several chaotic differential systems have been reported in the literature. In 2010, Muthuswamy and Chua presented an electrical circuit consisting of only three components in series—an inductor, a capacitor, and a memristor—described by a three-dimensional differential system. Although the Muthuswamy-Chua system is simple in its formulation, its dynamics proved to be quite complex. We present results related to the existence of first integrals, invariant algebraic surfaces, zero-Hopf equilibrium points, and, finally, periodic orbits, which lead to the formation of a chaotic attractor.

Crossing limit cycles for discontinuous piecewise linear differential centers separated by two circles

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In recent years, there has been growing interest in the study of planar discontinuous piecewise differential systems due to their significant applications in modeling real-world phenomena. Understanding the dynamics of these systems presents several challenges, particularly in the investigation of their limit cycles. In this work, we study the existence of crossing limit cycles in discontinuous planar piecewise differential systems formed by linear centers and separated by two concentric circles. We establish that the upper bound for the number of limit cycles for this class of systems is 2 and provide examples showing that the maximum number of limit cycles can be reached.

Symmetric periodic solutions in ODE

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We investigate the existence of several families of symmetric periodic solutions as continuation of circular solution of a ODE with perturbations. For now, we present the planar and spatial case.

An Ordinary Differential Equations Approach to Solving Polynomials

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It is well known that polynomial equations of degree $n \geqslant 5$ cannot be solved by radicals. However, their roots can be expressed in terms of elliptic or hyperelliptic functions, their inverses, or other transcendental functions.

In this poster, we explore the types of results on polynomial equations that can be obtained using ordinary differential equations as the main tool, based on the recent article published by Gasull and Giacomini (2021). Specifically, we consider the polynomial equation of degree n given by $P(x) = R(x) - q = 0$, where $q \in \mathbb{R}$ and $R(x)$ is a real monic polynomial of degree n, satisfying $R(0) = 0$ and $R'(0) \neq 0$. We are interested in finding an explicit local expression for the analytic solution $x(q)$ of this polynomial equation such that $\lim_{q \to 0} x(q) = 0$, which exists and is unique by the Implicit Function Theorem, since $R'(0) \neq 0$. We present a result that states $x(q)$ can be expressed in terms of hyperelliptic functions and their inverses. To illustrate, we apply the results to lower degree cases; in particular, when $n = 2$ and $n = 3$, we obtain the Babylonian and Cardano's formulas, respectively.

Acknowledgment: This work was carried out with the support of the Coordination for the Improvement of Higher Education Personnel - Brazil (CAPES) - Funding Code 001.

Hopf-Hopf Bifurcation Analysis in an Inductively Coupled Oscillator Circuit

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Inductively coupled electrical oscillators are widely used in near-field wireless power transferring (WPT). WPT systems can be used to recharge biomedical implants, in E-Mobility charging interfaces to recharge electrical vehicles or to recharge cell phones and IoT devices in sensor networks. In this class of systems, the power transfer is carry-out through the inductive coupling of two circuit resonators. In general, a high-power oscillator is employed as excitation source and it is connected to one of the two circuit resonators. The other is connected to the load. The transferred power and the efficiency depend on the distance and alignment between the two coils. A coupling factor can be defined to model and to measure the efficiency of the transferred power. The dynamics of this coupled system mainly depends on the system load and its coupling factor, that decreases for a larger distance between the two resonators or when they are misaligned. This paper address the bifurcation analysis of this class of coupled inductor system operating as an autonomous nonlinear dynamical system. In this case, a Hopf-Hopf bifurcation appears as an organizing point of the dynamics that allows explaining the cause of interactions between different oscillatory modes. Analytical and numerical results obtained by continuation methods and bifurcation analysis are presented.

Global phase portraits of a class of cubic Kolmogorov systems with a Darboux invariant

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The classical Lotka-Volterra systems are the differential systems of the form $\dot{x} = xP(x, y)$, $\dot{y} = yQ(x, y)$, where P and Q are polynomials of degree one. If P and Q are polynomials of degree greater than one, then these differential systems are called Kolmogorov systems. Both differential systems has been intensively studied due to their multiple applications. Using the Poincaré compactification we obtain the global phase portrait in the Poincaré disc of a class of planar cubic Kolmogorov systems which admit a Darboux invariant, such systems depend on eight real parameters.

Bifurcations of symmetrical two-zone piecewise vector fields

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This study is based on Smale's Program and aims to characterize the sets of two-zone vector fields with symmetry (reversible or equivariant) that are structurally stable and codimension one. Moreover, we provide a list of normal forms for the elements of these sets.

It is a joint work with Ubirajara Castro.

Local and global analysis of the displacement map for some near integrable systems

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In this work we introduce an alternative for applying averaging theory for orders 1 and 2 in the plane. We present a new way to make the first averaging function equal to zero, by using Taylor expansions of it, allowing us to study the second averaging function. One knows that to get second order results with averaging theory, it is necessary that the order 1-function is identically zero. So when working with Taylor expansions of the averaging functions, we usually can no guarantee they are identically zero. Anyway, in this paper we were able to prove that the annihilation of some coefficients of the Taylor series of the first averaging function makes it identically zero. We present our reasons in several concrete examples: a quadratic Lodka–Volterra system, a quadratic Hamiltonian system, the entire family of quadratic isochronous differential systems and a cubic system. For this last one, we also conclude a previous analysis contained in the literature is not correct. In none of the examples we need to calculate precisely the averaging functions.

Ordinary Differential Equations for Competition Between Normal and Tumor Cells: Growth Dynamics and Parameter Estimation

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In the field of oncology, the application of ordinary differential equations has been demonstrated to be a valuable tool for elucidating the temporal dynamics of tumor growth. In this context, our research is focused on examining the behavior of an in vitro cell coculture that mimics the progression of melanoma cells with human metastatic potential (SK-MEL-147) when in contact with immortalized human epithelial cells (HaCaT). Our study is guided by experimental data time series, which allow us to solve an inverse problem and perform parameter estimation of the ordinary differential equation model developed by Robert Gatenby. The primary focus of our study is to quantify the distinctive interspecific competition between HaCaT cells and SK-MEL-147 cells. For that, it is assumed that each population grows logistically, with the number of HaCaT normal cells denoted by $N_1(t)$ and the number of SK-MEL-147 tumor cells represented by $N_2(t)$. The interspecific competition between these cell lines is modeled as the product of the number of these two cell populations, resulting in a negative effect on $N_1(t)$ and $N_2(t)$ with competition coefficients respectively given by α_{12} and α_{21}. The estimation of these latter parameters is the key aspect of our work. To do that, we account for the variability in cell growth by using a nonlinear mixed-effects model for the raw data which contains several repeated noise measurements over time. As a result, the solution curves exhibited a reasonable degree of proximity to the mean of the experimental data for both cell types. The estimation yielded

values which indicate that HaCaT cells (N_1) exert a more pronounced inhibitory effect on SK-MEL-147 (N_2) than vice versa. This suggests that normal cells tend to impede the growth of tumor cells when in contact. Statistical analysis revealed significant insights into the obtained results, which may contribute to advancements in cancer research.

Asymptotic behavior of the Hilbert Number in planar piecewise polynomial vector fields

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In the context of planar polynomial vector fields, Christopher and Lloyd showed, from a polynomial perturbations sequence of a particular Hamiltonian center and a coordinate transformations that quadruple the number of limit cycles at each iteration, that $H(n)$ (the maximum number of limit cycles of a planar polynomial vector field of degree n) grows at least as fast as $n^2 \log(n)$.

In this work, we study the method introduced by Christopher and Lloyd, as well as the associated challenges. Motivated by their methodology, we investigate the possibility of extending this approach to analyze the asymptotic behavior of the Hilbert Number in planar piecewise polynomial vector fields.

Inverse Jacobi multipliers and the nilpotent center problem in $\mathbb{R}^3$

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Consider analytical three-dimensional differential systems having a singular point at the origin such that its linear part is $y\partial_x - \lambda z\partial_z$ for some $\lambda \neq 0$. The restriction of such systems to a Center Manifold has a nilpotent singular point at the origin. We prove that if the restricted system is analytic and has a nilpotent center at the origin, with Andreev number 2, then the three-dimensional system admits a formal inverse Jacobi multiplier. We use this result to solve the center problem for some three-dimensional systems without restricting the system to a parametrization of the center manifold.

Revealing Canard-Type Invariant Tori in a Modified Nosé–Hoover Oscillator

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In this study, we rigorously investigate the dynamics of a modified Nosé–Hoover oscillator, a three-dimensional dissipative system governed by two parameters. By introducing a small anti-damping term, we analytically establish the existence of a small periodic orbit that bifurcates into a stable invariant torus near the origin. This torus is characterized by canard-type oscillations, a phenomenon typically observed in singularly perturbed systems. Our results provide analytical proofs for the complex behaviors previously observed numerically, including the coexistence of multiple invariant manifolds. This research

advances the understanding of canard-induced dynamics in dissipative systems, with a particular focus on the modified Nosé–Hoover oscillator.

Um estudo qualitativo para a equação do Pêndulo

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Sistemas dinâmicos é uma área da matemática que estuda os sistemas físicos e matemáticos que evoluem e podem sofrer alterações em sua configuração com o tempo. O estudo de equações diferenciais tem sua relevância, uma vez que podem ser aplicadas em diversas áreas da ciência, podendo ser usadas para modelar fenômenos biológicos, problemas em Astronomia, Mecânica, Eletrônica, entre outros. A maioria das equações e/ou sistemas de equações diferenciais não podem ou são extremamente difíceis de serem resolvidas analiticamente, e assim, é importante buscarmos informações qualitativas que podem ser obtidas dessas equações, sem de fato resolvê-las. Neste trabalho, foi realizado um estudo acerca de um modelo matemático associado à equação de um pêndulo oscilatório por meio da teoria de sistemas dinâmicos. Neste sentido, considerando o fato que pontos de equilíbrio de sistemas denominados quase lineares, podem ter seu tipo e estabilidade dada por um sistema linear associado, exibimos as possibilidades, bem como um plano de fase referente ao modelo trabalhado.

Crossing limit cycles of the planar discontinuous piecewise linear Hamiltonian system

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In this paper, we study discontinuous piecewise linear differential systems which are separated by the line Σ. The line of separation Σ is either the straight line $x = 0$, or the nonregular line formed by the boundary of a sector of angle α in $(0, \pi)$ with vertex at the origin of coordinates, or a singular and irreducible cubic algebraic curve.

Moduli of continuity of Lyapunov exponents

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The Lyapunov exponents for i.i.d. random matrix products are continuous functions with respect to the compactly supported probability distribution. We give further their moduli of continuity by analyzing the corresponding random walk on the projective space.

Polynomial Decay of Correlations via Renewal Theory for Operators

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This work aims to present the use of renewal theory for operators, introduced by O. Sarig. This method allowed establishing polynomial lower bounds for the decay of correlations for transformations with countable Markov partition. Will be present a general result obtained by O. Sarig referring to the renewal sequences of bounded operators acting on Banach spaces, which applies to the iterations of the transfer operator.

Many-fold Learning

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Machine learning (ML) has been profitably leveraged across a wide variety of problems in recent years. Empirical observations show that ML models lying in suitable functional spaces are capable of adequately efficient learning across a wide variety of disciplines. In this work, we build the foundations for a generic perspective on ML optimization dynamics. Specifically, we prove that under variants of gradient descent, "well-initialised" models solve sufficiently well-posed problems at *a priori* or *in situ* determinable rates. Notably, these results are obtained for a wider class of problems, loss functions, and models than the standard mean squared error and large width regime that is the focus of conventional Neural Tangent Kernel (NTK) analysis. The ν

- Tangent Kernel (νTK), a functional analytic object reminiscent of the NTK, emerges naturally as a key object in our analysis and its properties contribute to control over learning.

We exemplify the power of our proposed perspective by showing that it applies to diverse practical problems solved using real ML models, such as classification tasks, data/regression fitting, differential equations, shape observable analysis, etc. We end with a small discussion of the numerical evidence, and the role our novel features like the νTKs may play in characterizing the search phase of optimization, which leads to the "well-initialised" models that are the crux of this work.

On the closure of irregular orbits of the horocyclic flow of infinite finness

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The topological dynamics of the horocyclic flow hR on the unit tangent bundle of a geometrically finite hyperbolic surface is well known. In particular on such a surface the flow hR is minimal or the minimal sets are the periodic orbits. When the surface is geometrically infinite, the situation is more complex and the presence of possible irregular orbits makes the description of minimal sets complicated. In this talk, through examples we study the intersection between the irregular orbits with those of the geodesic flow in order to write the minimal hR-sets.

Construção de Conjuntos Fractais Através de Sistemas de Funções Iteradas

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Este projeto tem o objetivo de apresentar uma maneira unificada de construir uma ampla gama de conjuntos fractais, por meio de uma aplicação recorrente de um conjunto finito de contrações em um espaço métrico completo: $(f_i : M \to M | i = 1, \ldots, n)$. A este conjunto, juntamente com o espaço M, damos o nome de sistema de funções iteradas (IFS, do inglês). Nesse sentido, mostramos que grande parte dos conjuntos fractais podem ser vistos como atratores para o operador de Hutchinson, definido sobre os subconjuntos compactos de M. Ainda, vemos como este método facilita o cálculo da dimensão de Hausdorff dos fractais obtidos, via Teorema de Morán. Por fim, discutimos a possibilidade de estender tal problema para o domínio dos sistemas dinâmicos aleatórios.

Existence and nonexistence of Puiseux inverse integrating factors and Puiseux first integrals

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The main goal of this presentation is to show a criteria for the existence and non-existence of Puiseux inverse integrating factors V and Puiseux first integrals H for planar vector fields with a monodromic singularity at the origin. One proves that neither H nor V are able to characterize degenerate centers but the existence of a Puiseux first integral H is a sufficient center condition.

The presented results are part of [García I.A., Giné J., Rodero A.L., *Existence and nonexistence of Puiseux inverse integrating factors in analytic monodromic singularities*, Stud Appl Math., 2024], developed jointly with Prof. Isaac A. García and Prof. Jaume Giné.

Ana Livia Rodero is supported by the grants 2023/05686-0 and 2021/12630-5, São Paulo Research Foundation (FAPESP).

Random Dynamical System and Iterated Function System

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We study random dynamical systems from the perspective of Iterated Function Systems (IFS). An Iterated Function System is a finite collection of transformations acting on a space, usually a metric space. For that, we consider an IFS with independent transformation such that the composition is chosen randomly, to model this process we use the concept of skew product systems. In our study, we explore significant results related to the behavior of these systems, such as the Coding Map. Additionally, we examine the conditions for the random composition of IFS define a homogeneous Markov Chain, which allows us to utilize probabilistic properties to determine an invariant measure for the systems. Finally, we provide examples to illustrate the applications of our study.

Network Structure as Exotic Symmetry

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Symmetry is well-known to cause all kinds of unusual dynamical phenomena, such as flow-invariant subspaces, degenerate eigenvalues in Jacobians and elaborate bifurcation scenarios. The same is observed in network dynamical systems, even when no symmetry is present. We show that many forms of network structure, as well as a wide plethora of connectivity properties can still be seen as symmetry, provided one allows for a wider class of algebraic objects. In many cases, this explains the aforementioned dynamical phenomena, and leads in particular to a full classification of generic bifurcations.

Ciclos limite de grande amplitude em sistemas lineares definidos por partes

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Neste trabalho tratamos de uma família de sistemas lineares planares e suaves por partes com duas zonas, e estudamos sobre o número máximo de ciclos limite que podem ser obtidos ao estudar as órbitas no infinito. Partimos de uma forma canônica com 12 parâmetros e reduzimos para até 5 o número de parâmetros para a análise. Investigamos questões relacionadas a estabilidade, caracterização das órbitas e outras propriedades que podem ser obtidas ao estudar a família de sistemas considerada. Este trabalho tem como objetivo encontrar uma

cota inferior para o número máximo de ciclos limite que são obtidos ao perturbar centro ou foco fraco partindo de um sistema linear por partes no plano.

O Problema de Sitnikov e a Existência de Conjuntos Invariantes do Tipo "Ferradura de Smale"

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O problema de Sitnikov é um caso particular do problema de três corpos restrito, no qual os corpos primários possuem massas idênticas e o terceiro corpo tem massa desprezível em comparação com os corpos primários. Neste modelo, os dois corpos de massas idênticas, descrevem trajetórias elípticas em um plano. O terceiro corpo, de massa insignificante, descreve uma trajetória perpendicular a este plano, passando pelo centro de massa dos corpos primários. Este estudo visa explorar esse problema, desde a formulação inicial até a conclusão sobre a existência de conjuntos invariantes do tipo "ferradura de Smale".

Poincaré-Mel'nikov Like Integrals in the Distribution Sense

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In general, Mel'nikov like integrals are used to show the persistence of periodic, homoclinic or heteroclinic orbits in a perturbation of integrable dynamical system.

In this work we show that, in some conditions, these integrals exist in sense of distributions.

A Study on the Global Phase Portraits of Nilpotent Centers with Cubic Nonlinearities

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A real planar analytic differential system with a center at the origin, after a linear change of variables and a time rescaling, can be represented in one of the following three forms, where $X(x, y)$ and $Y(x, y)$ are real analytic functions without constant and linear terms, defined in a neighborhood of the origin:

$$\dot{x} = -y + X(x, y), \quad \dot{y} = x + Y(x, y),$$

known as a *linear type center*,

$$\dot{x} = y + X(x, y), \quad \dot{y} = Y(x, y), \tag{4}$$

known as a *nilpotent center*, and

$$\dot{x} = X(x, y), \quad \dot{y} = Y(x, y),$$

known as a *degenerate center*.

Consider a planar polynomial differential system of degree three with a nilpotent center at the origin, i.e., of the form (4):

$$\dot{x} = y+a_1x^2y+a_2xy^2+a_3y^3, \qquad \dot{y} = b_0x^3+b_1x^2y+b_2xy^2+b_3y^3, \tag{5}$$

where $a_i, b_i \in \mathbb{R}$. We must assume $b_0 < 0$ for the origin to be a monodromic singular point, meaning that every trajectory in a neighborhood of this singular point is either a spiral or an oval. By imposing the reversible symmetry given by $(x, y, t) \longrightarrow (-x, y, -t)$, system (5)

can be reduced, after some change of variables, to

$$\dot{x} = y(1 + ax^2 + by^2), \qquad \dot{y} = x(cx^2 + dy^2), \tag{6}$$

where $a, b, c, d \in \mathbb{R}$ and $c < 0$. This implies that system (6) is a cubic reversible polynomial differential system with a nilpotent center at the origin and cubic nonlinearities.

While many papers have focused on investigating the phase portraits of various classes of linear type centers, only a few have examined the phase portraits of nilpotent and degenerate centers. In this poster, we present the classification of the phase portraits of system (6) in the Poincaré disc.

This work was conducted during an internship at the Universitat Autònoma de Barcelona under the supervision of Professor Jaume Llibre. The author is a master's student at ICMC-USP, supervised by Professor Regilene Oliveira.

Injetividade global de difeomorfismos locais

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A Conjectura Jacobiana Real no Plano diz que se $F : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ é uma aplicação polinomial com jacobiano não nulo em todo ponto, então F é invertível. No entanto, sabemos que, em geral, essa conjectura é falsa. De fato, Pinchuk, em 1994, exibiu uma aplicação polinomial no plano não injetora com jacobiano não nulo em todo ponto. Todavia, é de grande interesse encontrar condições que garantam a invertibilidade global de difeomorfismos locais, em particular, de aplicações polinomiais no plano. Nesse sentido, existem resultados na literatura que

exploram a conexão entre a injetividade e a conexidade dos conjuntos de nível de uma das funções coordenadas de F, bem como a existência de um centro global de um campo de vetores Hamiltoniano. Neste trabalho, nós estudamos a relação entre a injetividade global de um difeomorfismo local $f : U \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, sendo U um conjunto aberto e simplesmente conexo, e a existência de uma equivalência topológica com campos de vetores planares do tipo anular, radial ou faixa.

Empirical Dynamic Modeling for Powerline Demand Prediction

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Understanding the underlying dynamics of time series to predict future trends is critical in many industrial and power system applications. Empirical dynamic modeling (EDM) as a framework comes in handy in such scenarios as a set of tools capable of reconstructing the state-dependent dynamics of a nonlinear dynamic. For this purpose, Takens' theorem establishes conditions under which one can recover an equivalent dynamic based on observations of the projections of the variables of the system states as time series using techniques such as univariate or multivariate delayed embedding. This work is devoted to using EDM methods to recover the dynamics of the power demand target to make predictions for short-term planning (1 hour, 12 hours, or 36 hours ahead). The chosen application includes real-time electricity demand measured hourly in 2016, 2017, 2018, and 2019. The three older years were used to calibrate the EDMs and the year 2019 was used for forecasting. The predictive skill of the EDM methods resulted

in performance slightly below that of the classical techniques of practical use in electricity demand. However, no feature such as temperature has been used so far, as its inclusion may be promising and will be considered in investigations shortly.

Sparse recovery and nonlinear basis pursuit

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In this presentation, we propose an iterative method to find sparse solutions of nonlinear equations.

Characterization and dynamics of certain classes of polynomial vector fields on the torus

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In this talk, I will classify polynomial vector fields in $\mathbb{R}^3$ of degree up to three such that their flow makes the torus

$$\mathbb{T}^2 = \{(x, y, z) \in \mathbb{R}^3 : (x^2 + y^2 - a^2)^2 + z^2 - 1 = 0\} \text{ with } a \in (1, \infty)$$

invariant. I will also classify cubic Kolmogorov vector fields on $\mathbb{T}^2$ and prove that they exhibit a rational first integral. I will introduce 'pseudo-type-n' vector fields on $\mathbb{T}^2$ and show that any such vector field is completely integrable. The Lie bracket of polynomial vector fields on $\mathbb{T}^2$ of degree up to 2 will be discussed. I will present necessary and sufficient conditions when invariant meridians and parallels of cubic vector fields on $\mathbb{T}^2$ are periodic orbits or limit cycles. Invariant meridians

and parallels of pseudo-type-n vector fields will be discussed as well. Finally, I will talk about the singular points of a class of polynomial vector fields on $\mathbb{T}^2$.

About interactions between information theory and ergodic theory

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Ergodic theory began with the statistical physics work of Boltzmann and Poincaré in Dynamical Systems, soon followed by the work of Kolmogorov, Birkhoff, among others. In 1948, Claude Shannon wrote an article on information theory. In this work I intend to present relations between information theory and ergodic theory through Ornstein's and Marton's theorems, the first on Ergodic Theory, which was reformulated under perspective of Information Theory by the second.

Estimation of Ordinary Differential Equation Parameters in Mass-Spring Systems Modeled by the Duffing Equation

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The observation of physical or biological phenomena leads to studies interested in explaining or predicting their behavior over time or observed variations in the original model. Mathematical models developed from observations are conceived using established physical laws and are made up of parameters, which can be measured initially or not, as well as having fixed or random behaviors. The aim of this work is to estimate the parameters present in a computational mathematical

model, idealized from a translational harmonic oscillator, described by the Duffing Equation, developed using Wolfram Mathematica software, using the NLMEModeling package. The model initially developed was designed without considering the effects of stochasticity in its modeling, which, when analyzed using model validation and analysis tools, such as *Goodness-of-Fit, Visual Predictive Check, Box Plot*, among others, present in the NLMEModeling package, proved to be excessively sensitive to the smallest variations in the values of the errors inherent in the observations and the random effects represented by the Σ and Ω matrices, respectively. Parameter determinations were analyzed numerically considering variations in the initial value of the oscillation period, totaling 51 initial kicks between the interval of 1.500 and 2.250 seconds, for synthetic data generated from a period of 2.0 seconds, with variance equal to 0.1^2 and observation error in the position variable equal to 0.1^2. After obtaining and analyzing these results, the numerical effect of adding stochasticity to the computational model was also studied, resulting in less sensitive behavior and more robust estimation compared to the model without stochasticity, in addition to presenting a lower number of local minima, which is one of the difficulties in estimating parameters in systems of ordinary differential equations that result in oscillatory behavior.

Isochronicity constants for Hopf points in $\mathbb{R}^3$

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The isochronicity problem consists of determining when periodic orbits have the same period. This is equivalent to the condition that the period function is constant for all points around the origin. In this work, we develop a method for calculating the isochronicity constants of three-dimensional differential systems at Hopf point. The method is similar to the well-known approach for planar systems and the significant advantage is that it allows for the calculation of the constants without the need to restrict the three-dimensional system to a center manifold. As a part of our results, we apply the method to the three-dimensional system given by

$$\begin{aligned}
\dot{x} &= y,\\
\dot{y} &= -x + a_1x^2 + a_2xy + a_3xz + a_4y^2 + a_5yz + a_6z^2,\\
\dot{z} &= -z + c_1x^2 + c_2xy + c_3y^2,
\end{aligned}$$

which was presented in Giné, J., Valls, C.. *Center problem in the center manifold for quadratic differential systems in* $\mathbb{R}^3$. J. Symb. Comput. (2016), where the authors provide sufficient conditions for the origin to be a center on the center manifold. Using our method, we obtain necessary and sufficient conditions for isochronicity in almost all cases.

Joint work with **Claudio Pessoa** and **Lucas Queiroz Arakaki**

Estudo dos Retratos de Fase Globais de Sistemas de EDO's polinomiais no plano aplicados à Modelagem da Leucemia Promielocítica Aguda

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Frequentemente, utilizamos modelos matemáticos para entender a dinâmica de alguns fenômenos naturais e também para estudar o comportamento de algumas doenças. Em particular, para entender a dinâmica da Leucemia Promielocítica Aguda (LPA), que é um subtipo de Leucemia Mieloide Aguda (LMA), em que a medula óssea é incapaz de produzir células sanguíneas normais em quantidade suficiente devido a bloqueios na maturação dessas células. Neste trabalho, realiza-se uma análise global da família a dois parâmetros de campo de vetores polinomiais $F : \mathbb{R}^2 \to \mathbb{R}^2$ definido como $F(x, y) = (x', y') = (1 - x - \beta_1 xy, y - y^2 - \beta_2 xy)$ que modela a LPA. São analisadas todas as possíveis Bifurcações Locais e Globais, e, utilizando a Compactificação de Poincaré, são fornecidas as descrições completas das suas dinâmicas no Disco de Poincaré.

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