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EFFICACY AND TEACHING MATHEMATICS BY TELLING: A CHALLENGE FOR REFORM

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This paper analyzes the tension between the traditional foundation of efficacy in teaching mathematics and current reform efforts in mathematics education. Drawing substantially on their experiences in learning mathematics, many teachers are disposed to teach mathematics by "telling": by stating facts and demonstrating procedures to their students. Clear and accurate telling provides a foundation for teachers' sense of efficacy—the belief that they can affect student learning—because the direct demonstration of mathematics is taken to be necessary for student learning. A strong sense of efficacy supports teachers' efforts to face difficult challenges and persist in the face of adversity. But current reforms that de-emphasize telling and focus on enabling students' mathematical activity undermine this basis of efficacy. For the current reform to generate deep and lasting changes, teachers must find new foundations for building durable efficacy beliefs that are consistent with reform-based teaching practices. Although productive new "moorings" for efficacy exist, research has not examined how practicing teachers' sense of efficacy shifts as they attempt to align their practice with reform principles. Suggestions for research to chart the development of, and change in, mathematics teachers' sense of efficacy are presented.

A broad national consensus has emerged among mathematics educators that the policy and practice of precollege mathematics education requires serious reform (National Council of Teachers of Mathematics [NCTM], 1989, 1991; National Research Council [NRC], 1989, 1990). Thus far, the major loci of this wave of reform have been curricular content, teaching practice, and teacher preparation (NCTM, 1989, 1991). This article addresses a different issue raised by the reform: the relationship between teachers' need to feel efficacious in their work and the vision of mathematics teaching sketched by the reform. The core argument is that (a) a mismatch exists between the pedagogy of current reform and the basis on which mathematics teachers have traditionally felt efficacious in directing student learning; (b) new sources of efficacy consistent with the reform are needed to fill that gap; and (c) the failure to explore, identify, and build new foundations of efficacy in teaching mathematics may seriously limit the impact of the reform.

The analysis is framed, in part, by research on mathematics teachers' knowledge, beliefs, and practices, especially those studies that have examined the reactions of prospective and practicing teachers to the reform (e.g., Borko et al., 1992; Putnam, Heaton, Prawat, & Remilliard, 1992). This literature has demonstrated the widespread practice of teaching mathematics by telling, identified the conceptions

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about mathematics and learning that underlie that practice, and underscored the difficulties teachers face in making fundamental changes in their pedagogy. The argument also draws on analyses of efficacy in teaching that relate efficacy beliefs to teachers' commitment, motivation, and teaching practice and to student achievement. This article attempts to synthesize these two lines of research because no currently published studies have linked teachers' sense of efficacy to changes in teaching practice in the context of reform. This focus on efficacy addresses previous calls to examine teachers' feeling about mathematics and teaching (Ball, 1990a) and contributes to the discussion of tensions generated by the reform.[1]

The major claim of this article is that the reform of school mathematics content, learning, and teaching undermines the base for teachers' sense of efficacy that teaching by telling provides. Telling mathematics allows teachers to build a sense of efficacy by (a) defining a manageable mathematical content that they have studied extensively and (b) providing clear prescriptions for what they must do with that content to affect student learning. The current reform removes both supports. The mathematics that teachers know best is reduced in value, substantial emphasis is given to unfamiliar content, and only the most general instructional principles are provided for teaching that content. Teaching by demonstration and practice is no longer acceptable, because students cannot learn mathematics as passive listeners. More deeply, the reform challenges the fundamental assumption that teachers can be direct causal agents in student learning. The new metaphor of teachers as guides or facilitators appears to restrict their agency (see, for example, the presumption not to tell noted in Ball & Chazan, 1994) and suggests that they can act only indirectly, by creating settings in which students learn mathematics through their own activity.

The argument develops in two main parts. The first part sketches the central tension between teaching mathematics by telling; the sense of efficacy it supports; and the reform conceptions of mathematics, teaching, and learning. The main properties and consequences of the sense of efficacy in teaching are discussed; the evidence that documents the practice of and underlying beliefs about telling mathematics is reviewed; and the link between telling and efficacy is drawn. The second part examines the impact of the reform, first by highlighting how deeply the reform departs from the telling model and then by identifying the substantive reasons why such fundamental changes in teaching practice are difficult to achieve. The paper closes with a short list of potential new foundations for a sense of efficacy in teaching mathematics and a call for theoretical and empirical research on the nature and development of efficacy beliefs consistent with reform principles.

TEACHERS' SENSE OF EFFICACY

Ashton (1985) defines teachers' sense of efficacy as "their belief in their ability to have a positive effect on student learning" (p. 142). Teachers with a strong sense

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[1]Critical examinations of the impact of the reform in mathematics curriculum, learning, and teaching that are even-handed in their stance have been few in number. Notable exceptions have been Apple's (1992) critical analysis of the social, economic, and political context and Ball's (1992b) discussion of the inherent tensions in "implementation."

of efficacy attribute a major causal role to their own actions within the constellation of factors that influence their students' learning; those with a weaker sense of efficacy grant a weaker role to their actions relative to other factors. Following Bandura's sociocognitive theory of self-efficacy (1982, 1986, 1992), Ashton & Webb (1986) distinguish general and personal dimensions of teaching efficacy. Teaching efficacy refers to general beliefs about teachers' ability (as a group) to produce student learning in the face of external challenges (e.g., weak student ability or motivation). Personal teaching efficacy is an individual teacher's own sense of his or her ability to take effective action in teaching.[2]

The sense-of-teaching-efficacy construct draws on five basic properties of more general efficacy beliefs. A sense of efficacy is a self-attribution; people must construct their beliefs about the linkage between their actions and the consequences of those actions. This linkage involves beliefs both about themselves and about the world; they must believe in their ability to act effectively and that the world will respond positively.[3] Efficacy beliefs underlie and enable actions in which people expend substantial effort in pursuit of goals, persist in the face of adversity, rebound from temporary setbacks, and in general, attempt to project control over their environment (Bandura, 1992). The source of efficacy beliefs lies in a complex matrix of factors, but a history of perceived past success plays the most important role (Bandura, 1986). Finally, a sense of efficacy is activity and situation specific; typically, people do not feel efficacious in all the situations they encounter. Accordingly, a sense of efficacy is more appropriately understood as a fluid, dynamic set of beliefs than a fixed personality trait (Woolfolk & Hoy, 1990). This general analysis of efficacy beliefs suggests that a strong sense of teaching efficacy requires that teachers (a) conceptualize what is efficacious about their actions and find the positive results of those actions in student learning, (b) maintain and draw upon a personal history of past teaching success, and (c) recognize that their effectiveness will vary across different students and contexts.

Educational research has indicated that teachers' sense of efficacy is an important causal influence on their practice and their students' learning. Teachers to whom a higher sense of efficacy was attributed produced higher measures of student achievement (Ashton & Webb, 1986), maintained learning environments that were more responsive to students (Woolfolk, Rosoff, & Hoy, 1990), persisted longer with struggling students (Ashton & Webb, 1986; Gibson & Dembo, 1984), and orchestrated more productive small-group work (Gibson & Dembo, 1984). They understood their authority to reside in their competence rather than their social position in the classroom (Ashton & Webb, 1986), were more committed to teaching (Coladarci, 1992), and were more willing to try some forms of innovative curricula (Guskey, 1988).

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[2]The conceptual parallel between Bandura's distinction between outcome and efficacy expectancies, on the one hand, and teaching efficacy and personal teaching efficacy beliefs, on the other, is not perfect (see Woolfolk & Hoy, 1990).

[3]Bandura's notion of outcome expectations is couched in terms of relatively specific results and rewards. The responsiveness of the world suggested here is a much more general and wider view of "results."

In contrast, weak efficacy beliefs carry heavy educational costs. Teachers with a low sense of efficacy tended to attribute student failures to factors beyond their control—to students' lack of ability, deficient motivation, character flaws, or poor home environments (Ashton & Webb, 1986); passed over students who answered incorrectly (Gibson & Dembo, 1984); and maintained more rigid and controlling classrooms (Woolfolk et al., 1990). In locating the root problem in poor institutional support, Ashton (1985) underscored the dangers of a weakened sense of efficacy.

> [T]he teacher is ever vulnerable to self-doubt induced by the unpredictability and uncontrollability of human interaction. Given this uncertainty, teachers' sense of efficacy is in continual jeopardy, in danger of attack by resistant or hostile students, angry parents, demanding administrators, and dissatisfied colleagues.... Teachers who succumb to feelings of inefficacy are likely to suffer debilitating stress and a reduction in their effectiveness with students. (p. 166)

Such stress can lead to burnout, shutting down, eventually dropping out of the profession (Dworkin, 1987).

A robust sense of efficacy therefore appears an important, if not necessary, part of teachers' belief systems. To teach to the best of their abilities, teachers need to construct and maintain conscious beliefs that link their teaching actions to their students' learning, that project a measure of control over their difficult and complex work setting, and that allow them to persist in the face of obstacles.

TEACHING MATHEMATICS BY TELLING

A core set of beliefs characterizes how most mathematics teachers (as well as other adults and students) in the United States view the content of school mathematics, how students learn that content, and how teachers should teach it. Studies of the conceptions, beliefs, and practices of prospective teachers (Ball, 1988, 1990a, 1990b, 1990c; Borko et al., 1992; Eisenhart et al., 1993; Wilcox, Schram, Lappan, & Lanier, 1991; Wilson, 1994) and of practicing teachers (Putnam et al., 1992; Thompson, 1984; Wood, Cobb, & Yackel, 1991), as well as observational research on teachers' practice (Schoenfeld, 1988; Stodolsky, 1985), provide consistent support for this claim. The central commitments in this interlocking set are listed below.

- *Mathematical content.* School mathematics is a fixed set of facts and procedures for computing numerical and symbolic expressions to find determinant "answers." Through middle school, the focus is the arithmetic of natural and rational numbers; the high school years shift the focus to computation with algebraic expressions. School mathematics is defined by the content of mathematics textbooks and is largely invariant. Textbooks contain the problems that students must learn to solve, and each problem—whether stated in numerical, symbolic, or verbal terms—is associated with a particular solution procedure.
- *Teaching mathematics.* Given that view of content, teachers' central task is to provide clear, step-by-step demonstrations of each procedure, restate steps in response to student questions, provide adequate opportunities for students to practice the procedures, and offer specific corrective support when necessary. If students

do not master a procedure, teachers should repeat their demonstration. Teachers should also provide recurrent opportunities for students to refresh and strengthen their mastery of previously taught content.

- *Learning mathematics.* Students learn by listening to teachers' demonstrations, attending carefully to their modeling actions, and practicing the steps in the procedures until they can complete them without substantial effort. Solving problems is a matter of recalling and applying the procedure appropriate for the given problem type. Because of the heavy demands on memory, success in mathematics may well depend on students' effort or innate ability.
- *Mathematical authority.* The answers to all mathematical problems are known and found in textbooks. Teachers who control and interpret texts are the intermediate authorities for students on matters of mathematical truth.

These commitments are not independent; collectively, they comprise an internally consistent and mutually reinforcing system of beliefs about the nature of school mathematics. The description used in this article, "teaching mathematics by telling," refers to teaching practices that are grounded in this system of commitments. "Telling" refers directly to the central teaching action of demonstrating the proper sequence of steps in mathematical procedures (Stodolsky, 1985; Putnam, 1992). But the plausibility of telling depends in turn on the underlying conception that mathematics is a fixed and finite collection of procedures for computing answers. Because students do not know these procedures, they must derive their knowledge from teachers. As a result of teachers' reliance on step-by-step procedural display, students are required to listen carefully and practice diligently.

As research on prospective teachers of mathematics has documented (Ball, 1988, 1990a, 1990c; Borko et al., 1992; Holt-Reynolds, 1992; Wilcox et al., 1991), this cluster of commitments is for the most part learned prior to university courses in mathematics and mathematics education. It is the product of teachers' protracted "apprenticeship of observation" as students (Lortie, 1975), complemented by experiences in the everyday culture (Ball, 1990a; Thompson, 1992). As listening students of mathematics for 12 to 16 years, future teachers have extensive participation in the practice of teaching by telling. As a student in Wilcox et al.'s (1991) teacher preparation program candidly stated in distinguishing the program's mathematics class from her previous work in mathematics: "There is a difference between other math classes and this one. In other math classes you don't say anything. You just sit there and watch the professor write problems on the board all hour" (p. 33).

Having seen little variation in their experience as students of mathematics, most prospective teachers come to their professional education with relatively stable commitments to the beliefs listed above. For future elementary school teachers, these commitments are often coupled with feelings of alienation and fear in mathematical settings and anxiety about the prospects of teaching it (Ball, 1990a, 1990c).

If teaching mathematics by telling has strong and deep roots in experience, it also provides a clear-cut basis on which teachers can build a sense of efficacy—more or less independent of their previous success in mathematics. To develop and maintain a sense of their efficacy teachers must project their agency in the classroom and conceptualize and maintain a linkage between their actions and their students' learning. This means acting in ways that generate a sense of effectiveness and avoiding activities that undercut or question it. The conceptions of mathematical content, teaching, and learning summarized under the "telling" model clearly structure and define this process.

- The conception of mathematics as a fixed set of facts and procedures restricts the content that teachers must master to a manageable range. It excludes the more complex and less deterministic components of mathematical argument, discourse, and understanding and excludes consideration of theoretical, historical, or philosophical issues. If there are any gaps in teachers' knowledge of the procedural content, they can refer to their textbooks for review.
- Telling provides a relatively detailed model of what teachers should do in their teaching. Because textbooks provide the proper sequence of procedures and problems for students to work on, the teaching task is narrowed to presenting procedures and assigning practice. When teachers hold classroom activities to the procedural content, they feel effective in their knowledge of the content, how to explain it, and what to say in response to students' questions (Borko et al., 1992; Prawat, 1992; Thompson, 1984; Wilson, 1994). Simply restating rules can count as explanation (Ball, 1990a). When the activity involves nondeterministic mathematics (e.g., real-world problem solving, estimation, alternate methods, or addressing "why?" questions), teachers become confused and feel out of control, are unsure how to answer and lead their students, and make mathematical mistakes (Ball, 1990c; Borko et al., 1992; Heaton, 1992; Putnam, 1992). By holding their teaching to the procedural core content, teachers emphasize their strengths and minimize their weaknesses.
- Telling accentuates the sense of knowledge transfer in teaching. Because students do not know mathematical procedures until teachers tell them, the success that students have in mastering them must be largely due to their teachers' competence. Demonstrations let teachers display their mastery of the content to their students and to themselves, as a reminder that they are competent to teach (Holt-Reynolds, 1992). The basis for feeling like a "knowledge provider" is less clear with a nondeterministic mathematics content where students and teachers collectively share the task of making sense (Schoenfeld, 1992).
- Telling defines what students should do to learn: listen, watch, and practice. Teachers take the corresponding behavioral indicators (attentiveness, time on task during practice) as measures of their daily, minute-to-minute effectiveness with students (Remillard, 1992; Thompson, 1984). In the long term, they judge their effectiveness by the level of computational proficiency their students achieve (Stodolosky, 1985; Thompson, 1984; Wilcox et al., 1991), frequently as measured by standardized tests (Borko et al., 1992; Putnam, 1992).
- The basic structure of demonstrating a procedure, providing practice, correcting individual students' work, and reteaching when necessary provides a rough outline

of a typical day's instruction and simplifies issues of planning and classroom management (Doyle, 1986). Teaching content that is unproblematic allows teachers to focus more attention on establishing a warm, orderly, and respectful classroom, which is also an indicator of success for some teachers (Thompson, 1984).

To sum up, teachers of mathematics, like all teachers, need to believe that their teaching actions have significant causal impact on their students' learning. Telling, irrespective of its pedagogical strengths and weaknesses, provides a clear model for teachers of mathematics to develop a sense of efficacy. Though good telling cannot guarantee that students will learn, it narrows the scope of the content to manageable proportions, clearly defines what the central acts of teaching are and what counts as evidence of student learning, and provides structure for daily classroom life. Teachers can feel efficacious when their students accomplish the reasonable tasks of remembering facts and computing with the standard procedures.

CHANGING THE GAME

The current reform of school mathematics presents conceptions of content, teaching, and learning that are quite different from the telling model (NCTM, 1989, 1991; NRC, 1989, 1990). At the heart of the reform is a very different view of mathematics. Arithmetic and algebra are but two components of mathematics defined broadly as the science of patterns (NRC, 1990; Steen, 1988). School mathematics should also include work in geometry, measurement, statistics, probability, and patterns and functions (NCTM, 1989). Within the number strand, students should learn not only exact computation but number sense, estimation, and the properties of number systems as well. Coupled with this diverse and expanded content is a model of mathematical reasoning derived from analyses of mathematical practice (Restivo, 1993; Tomoczko, 1986). Mathematics is no longer seen as a fixed collection of facts and procedures; it is a dynamic body of knowledge that is continually enriched through conjecture, exploration, analysis, and proof. The fundamental goals of school mathematics are to teach students to understand and reason with mathematical concepts, solve problems arising from new and diverse contexts, and develop a sense of their own mathematical power (NCTM, 1989; NRC, 1989).

These dramatic changes in the conceptions of mathematical content and activity are paralleled by equally radical changes in models of teaching and learning. Students are no longer seen as the recipients of knowledge transmitted directly from the teacher. They possess prior knowledge and intuitions that shape what they see, hear, and understand (NRC, 1989, 1990; Resnick, 1987). In order to make sense of mathematics in their own terms, they must take mathematical actions: represent their ideas, make conjectures, build models, collaborate with other students, and give explanations and arguments (NCTM, 1989; NRC, 1989).

This active view of learning mathematics substantially changes what teachers must do to enable learning. They no longer present the content through clear demonstrations; they must instead create the conditions that will allow students to take their own effective mathematical actions. This shift in agency is characterized dramatically by the authors of *Everybody Counts* (NRC, 1989): "In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students learn mathematics well only when they *construct* their own mathematical understanding" (p. 58, authors' emphasis). To support and enable students' mathematical actions, teachers must play more the role of classroom facilitator than knowledge source. In addition to their central task of selecting problems for students, they must model important mathematical actions, coach student thinking—individually and in groups, pose mathematical questions, and stimulate and moderate classroom discourse (NCTM, 1991; NRC, 1990).

In changing the task of teaching mathematics, the reform has also made it more knowledge intensive and demanding. From a relatively compact collection of computational procedures, the mathematical knowledge base has fairly exploded into a wider set of fundamental domains that only partially subsumes the prior procedural content. Because it emphasizes the importance of conceptual knowledge, reasoning, discourse, and representation and easily outstrips teachers' mathematical experiences (Ball, 1990a, 1990c; Mathematics Association of America, 1991), teachers can find themselves attempting to teach mathematics they have not mastered (Heaton, 1992; Putnam, 1992).

Likewise, the knowledge required to make good instructional decisions has changed and grown dramatically. Where, prior to the reform, textbook designers made many of the important decisions about content, sequence, and teacher actions, teachers are now responsible for allocating time to the various strands of mathematics, selecting or adapting problems, designing and introducing useful representations, and directing discourse, with only the most general principles for guidance. In place of giving clear demonstrations of the content that they know, they must now plan, implement, and revise a program of action to support students' constructive mathematical activity over a much broader range of unfamiliar content—a task that is difficult to manage, even in conceptual terms.

EFFICACY AND REFORM

How then can teachers who have learned by listening to traditional mathematics and have taught by telling achieve a sense of efficacy that is consistent with the reform? If a strong sense of efficacy is necessary to sustain teachers' ongoing professional development and if the tenets of the reform continue to define good mathematics teaching, where can the grounds for linking teaching actions and student learning be found? Answers to these questions will require that two initial conditions are satisfied: that teachers learn about the reforms and attempt to change their practice accordingly, and that their success in making those changes brings about a reconceptualization of their sense of efficacy. Unfortunately, although a growing body of research on teachers' knowledge, beliefs, and practices in the context of reform has addressed the first issue, there has been no explicit focus on the second. The balance of this article is directed toward framing research on teachers' sense of efficacy within the serious obstacles to changing practice, on the one hand, and the potential new moorings for efficacy beliefs, on the other.

Issues in Teachers' Reactions to Reform

Mathematics teachers, both practicing and prospective, respond to the reforms in different ways: from ignoring, downplaying, or openly resisting the changes; to retaining the telling model under the cover of reform; to embracing the heart of the reform, struggling to change, but falling short; to managing deep changes, and achieving new levels of success. A brief review of research on teachers' reception of the reform and their attempts to change their practice illustrates this range of reaction and highlights the problems and issues that underlie the search for a new sense of efficacy.

The potency of prior experience and resulting beliefs. Students' apprenticeship of observation can generate deep commitments about the teaching and learning of mathematics that contradict reform principles. Prospective teachers can be open and vocal about their opposition to the reform pedagogy, asserting that by nature, the subject matter of mathematics requires telling; that learning is primarily a matter of motivation (not activity), and that listening can be an active learning process (Holt-Reynolds, 1992). Moreover, these beliefs are usually linked to what they perceive as their warrant to teach. The strength of these beliefs is problematic for teacher education, because there is much that is valuable in their experience and the wholesale replacement of beliefs is neither desirable nor likely (Ball, 1988; Holt-Reynolds, 1992). The question for teacher educators is both how and what to influence in students' beliefs.

Even when prospective teachers have found course experiences that emphasize their own reasoning and discourse to produce deeper mathematical understandings, they do not always enact those practices in their own classrooms (Borko et al., 1992; Wilcox et al., 1991; Wilson, 1994). Under the pressure of external constraints and the absence of support, they can retreat back to the familiar and safe telling model. Similarly, experienced teachers use their own models of effective mathematics teaching to interpret the reform and incorporate only those elements consistent with their views (Prawat, 1992; Remilliard, 1992). Their positive reception of the reform is more a matter of assimilating surface features to their current teaching practice than giving thoughtful consideration to the basic principles.

The limits of past mathematical experiences. When the reform shifts the emphasis in mathematics content and pedagogy from learning rules and procedures to understanding, explanation, and problem solving, teachers are often at a loss to know what and how to teach (Ball, 1988, 1990a, 1990c; Borko et al., 1992; Eisenhart et al., 1993; Putnam et al., 1992; Simon, 1993; Simon & Schifter, 1991). Reciting rules and procedures correctly does not guarantee knowing where they apply or why they work (Ball, 1990b; Borko et al., 1992), and new content demands can lead to faulty presentations of mathematical concepts (Heaton, 1992; Putnam, 1992). Recognition of their limited mathematical understandings can lead beginning teachers educated in the reform to return to the telling model to avoid situations where those limits are apparent, to themselves and their students (Borko et al., 1992; Eisenhart et al., 1993).

Paste-on adjustments. The reform combines a theory of mathematics content, learning, and teaching with some more specific and explicit prescriptions for teaching (e.g., using manipulative materials or small-group problem solving). In response to strong suggestions that they "implement" the reform, some teachers add these specific elements to their practice without addressing the more fundamental issues that underlie and inspire them (Prawat, 1992; Remillard, 1992; Wilcox et al., 1991; Wilson, 1994). Small-group work, student projects, and manipulatives can be easily assimilated to views of content that emphasize the standard rules and algorithms, the teacher's role of knowledge telling, and students' roles of listening and practicing, leaving the pedagogy of telling fundamentally intact.

Resistance within the school culture. Despite wide acceptance of the reform among mathematics educators and policymakers, teachers can feel pressure from many sources to compromise the reform principles and return to telling (Wilcox et al., 1991). Students can be vocal about their discomfort with, and lack of belief in, new classroom mathematics and openly resist teachers' efforts to experiment and change (Cooney, 1985). Parents, themselves schooled in telling, are often at a loss to understand the goals and rationale of the reform and therefore cannot actively support their children's efforts to learn to think mathematically. Administrators and colleagues pass conflicting messages, emphasizing traditional outcomes (e.g., results on standardized tests [Prawat, 1992]), alongside teaching for understanding (Eisenhart et al., 1993; Wilcox et al., 1991).

Fundamental change is unsettling. Because the reform proposes such deep changes in integrated and well-developed belief systems, because teaching in the spirit of the reform is complex and difficult (Ball, 1992a), and because sources of support are scarce, making fundamental changes in teaching means taking risks. Even when experienced teachers see the reforms as embodying changes that they want to make, the shift away from telling can leave them feeling uncertain, vulnerable, and out of control (Wood, Cobb, & Yackel, 1991). The textbook is no longer a useful reference for many questions, and the key reform documents provide little guidance on specific curricular topics (Heaton, 1994). Teachers know that they should answer students' questions, which now center more on "why" than "how," but they struggle to interpret and know how to address them. When they open up classroom discussion to students' strategies and reasoning they cannot completely anticipate the ideas that will be generated or firmly set the direction of the discussion in advance (Heaton, 1994).

New Moorings for Efficacy

Despite the daunting nature of these problems, the reform does provide fresh opportunities for reconceptualizing efficacy in teaching mathematics. Understanding how individual teachers come to feel efficacious in aligning their practice to the reform is an important and essentially open empirical question (see below), but existing accounts of reform-based practice suggest at least four components of teaching that are promising sites for building and maintaining efficacy beliefs. The first three are emphasized as central components of teaching practices that are consistent with reform principles (NCTM, 1991).

Choosing problems. The expansion of mathematical content proposed by the reform dramatically increases the range of problems and situations teachers can present, and the constructivist emphasis on the role of problems in learning underscores the central power of choosing good ones. Teachers who choose and present tasks and situations that quickly engage students in work with significant mathematical content have already played a crucial role in student learning (see, e.g., the role of problems in Lampert, 1990, and Schoenfeld, 1989). The design and use of instructional representations that support students' work and extend their existing understandings in particular problem situations is a closely related source of efficacy (Ball, 1992a).

Predicting student reasoning. The choice of good problems is particularly empowering when teachers can anticipate how students will understand and work on them. The familiar dictum, "Know the content," expands in the reform to include knowing (a) the standard representations and solution methods in particular topic areas, (b) some of the approaches that students will likely take on problems, and (c) the language they may use to express their thinking. Even partially accurate anticipation of the character of students' reasoning provides a sense of stability and a basis for managing classroom discourse.

Generating and directing discourse. When teachers initiate, manage, and participate in mathematical discourse without becoming the sole authority, they address multiple important issues in teaching and learning mathematics. Creating contexts where students can safely express their own mathematical ideas is a central teaching task and a step toward developing students' mathematical power. Shaping open and fair examinations of those ideas suggests a different view of mathematics to students—a view that mathematics is dynamic, growing, and created by people. Emphasizing that choices between ideas must be justified on mathematical grounds teaches students about the nature of mathematical knowledge. When teachers appreciate the importance of these goals and practices, they can see that their actions play a necessary role in creating and sustaining discourse with these features (Ball & Chazan, 1994). They can also feel confident in conferring increasing responsibility to students for maintaining such discussions (Schoenfeld, 1991).

Judicious telling. Teachers mediate between the accepted knowledge and methods of mathematics and the particular intellectual communities of their classrooms. In this role they must both value their students' constructive activity and introduce elements of accepted mathematics when appropriate. Judicious telling of mathematics supports students' reasoning with particular new additions—such as useful terminology, ways of representing mathematical ideas, and counterexamples to student conjectures—in settings where those additions are necessary (Ball & Chazan, 1994). These additions enable, rather than disable, students to learn to think mathematically. They provide opportunities for teachers to bring their mathematical knowledge to bear and see its effects in support of students' efforts to make mathematical sense. Selective telling of this sort is quite different from the general telling model where the teacher is the sole knowledge provider and telling is the central teaching task (Heaton, 1994).

FUTURE RESEARCH

This list of new sites for establishing a sense of efficacy and the evidence that they are important for some teachers constitute an adequate starting point for research on efficacy development in reform-based practice. They fall well short, however, of locating where and describing how teachers of mathematics actually find the most important connections between changes in their practice and deeper student learning.[4] The question of how representative beginning and experienced teachers come to view themselves as efficacious in new forms of teaching carries great weight for the lasting impact of the reform. Answers to that question will require both richly descriptive empirical analyses and further theoretical development on the efficacy construct.

The few published studies of practicing teachers successfully aligning their teaching with reform principles provide hints about the construction of new efficacy beliefs but do not address the issue directly. In one well-documented case, an experienced second-grade teacher who was participating in a teaching experiment substantially changed her views of her teaching role, her students' roles as learners, and the nature of mathematical activity from a telling model toward the principles of reform (Cobb, Wood, & Yackel, 1990; Wood et al., 1991). Though this teacher eventually adjusted to the fundamental changes in her beliefs and practice, the authors did not explain how she came to reconceptualize the linkage between her revised pedagogy and student learning. Her intriguing claims that she was "not teaching anymore" and "teaching more this year than ever before" (Wood et al., 1991, p. 609) suggest that she revised her model of teaching to include new positive actions, as well as constraints (e.g., do not tell without listening first), and saw the effects on her students. Yet her understanding of how new elements in her pedagogy were effectual was not directly explored or analyzed.

Empirical analyses of efficacy. The central question for empirical studies is how teachers reconceptualize their causal agency in teaching mathematics. These studies should document particular teachers' beliefs and practices about mathematics, teaching, and learning as influenced by the reform and the sense of efficacy that emerges from those beliefs and practices. Participants should include both teachers who are exemplary in their enactment of the reform and have achieved a relatively durable sense of efficacy and those who are working to align their practice with the reform and therefore are experimenting with their practice. In this latter group, efficacy beliefs will likely be fluid and unstable. The analyses should (a) identify the categories of teachers' actions that contribute most significantly to their sense of efficacy; (b) determine how they see the effects of those actions on their students' learning; (c) characterize, in particular, the elements of mathematical knowledge and reasoning they

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[4]The analyses of reform-based teaching cited in the previous section, and the associated "moorings" for efficacy they provide, have all been presented by university-based mathematics educators who also teach in K–16 classrooms. Their education, expertise, and opportunities for analyzing their teaching practice distinguish them as mathematics teachers. We should therefore not presume too close an identity between their experience in developing a sense of efficacy and that of full-time classroom teachers.

show their students and when and how they chose to do so; and (d) assess the durability of their sense of efficacy over time. For those teachers who previously practiced the telling model, contrasts can be drawn between their old and newly emerging sense of efficacy on each of these dimensions.

Empirical analyses of efficacy must be centered in teachers' own perspectives and points of view. They may be situated in case studies of changes in practice (e.g., Wood et al., 1991) but represent a distinct research agenda. They should focus on how teachers themselves *see and understand the effects of their teaching practice on students*, not on how others (usually, reseachers) assess their practice relative to reform principles. The goal of efficacy studies is to characterize teachers' responses to the pedagogical question, "When am I doing a good job?" Understanding their answers will in turn depend on understanding the kind of evidence they identify and take to be centrally relevant to that question.

Prospective teachers, a third population, offer both limitations and opportunities for efficacy research. Because they are not yet engaged in full-time practice, they lack the experiential basis for drawing specific links between their actual teaching to student learning—appealing instead to their expectations (Holt-Reynolds, 1992; Lortie, 1975). On the other hand, they are often just encountering and grappling with the reform principles (Borko et al., 1992); they are protected from some of the demands of full-time practice; and they are still involved in organized activities to learn mathematics (Ball, 1988, 1990c). Research on their efficacy beliefs should examine their reception of the reform, documenting the most common "cracks" in their commitments to the telling model and locating where their understandings of the reform provide potential "niches" for new efficacy beliefs. Classroom activities that provide new experiences and perspectives on doing and learning mathematics are promising contexts for uncovering these niches. For those students who link new mathematical experiences to their future practice, how (if at all) do they anticipate feelings of effectiveness in creating similar experiences for their students?

Support systems. Studies of teachers' attempts to align their practice with the reform, particularly in contexts where support is problematic, consistently suggest the need for local, teacher-directed learning communities that provide mutual support and enable professional growth (Ball, 1993; Cobb et al., 1990; Holt-Reynolds, 1992; Wilcox et al., 1991). These groups allow teachers to raise questions, consider teaching dilemmas, listen to their colleagues' struggles, and weigh pedagogical alternatives in safe settings. These communities may be necessary for current reforms to have a deep and abiding impact, but they are also promising research contexts for studying the changing nature and development of efficacy. Their dialogic character appears to provide richer context and support for feelings of efficacy to surface than more individualized settings, such as one-on-one interview settings.

Clarifying the sense of efficacy construct. Although there is wide agreement on the basic notion of teaching efficacy and on its effects on teachers, more careful, empirically grounded analysis is necessary. It is presently unclear how the personal and general components of teaching efficacy are related (Hoy & Woolfolk, 1990) and how strict and useful that distinction is. Despite the general agreement that efficacy beliefs vary substantially across situations, researchers have often presented teachers' sense of efficacy as relatively stable sets of beliefs (Ashton & Webb, 1986). Perhaps most important, it remains unclear how efficacy beliefs are related to new conceptions of student learning and performance. Progress in clarifying the concept will require new methods of analysis to supplement the questionnaire measures that have provided the empirical basis for most past research (Gibson & Dembo, 1984). It will also require a conception of efficacy beliefs that is nested in the interconnected web of social settings, teaching actions, perceived effects, and local educational values that make up teachers' work environments. Finally, because teaching efficacy is a complex cognitive structure that is related to teachers' views of mathematics, teaching, and learning and the linkages they project onto teaching and learning, conceptual analyses should propose models that are well grounded in detailed case studies of individual teachers.

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