BRAZILIAN, CHILEAN AND FRENCH INTERPLAY FOR SYMBOLIC DYNAMICS

DYADISC 7

An attempt at a low-carbon intercontinental conference

# PROGRAM

(In Progess)

#### **Organizing Committee:**

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#### Scientific Committee:

Artur Lopes (Univ. Federal do Rio Grande do Sul, Brazil) Bryna Kra (Northwestern Univ., USA) Maria Isabel Cortez (Pontificia Univ. de Chile, Chile)

Dates: 9, 10 and 11 of December 2024 Site: https://dyadisc7.sciencesconf.org/ Venue: Instituto de Matemática, Estatística e Computação Científica Universidade Estadual de Campinas

## Planning (Brazil)

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# Planning (Brazil)

France (UTC+1)	Chile (UTC−4+DST)	Brazil (UTC−3)	Monday December 9, 2024	Tuesday December 10, 2024	Wednesday December 11, 2024
		7:30 - 8:00	Reception		
		8:00 - 9:00	Christian Rodrigues	Ana Paula Chaves	Benito Pires
		9:00 - 9:30		Break	
13:30 - 14:15	9:30 - 10:15	9:30 - 10:15	Irene Inoquio-Renteria (Online Chile)	Ali Messaoudi	Sébastien Labbé (Online France)
14:30 - 15:15	10:30 - 11:15	10:30 - 11:15	Manuel Stadlbauer	Irene Marcovici (Online France)	Cristóbal Rivas (Online Chile)
15:30 - 16:15	11:30 - 12:15	11:30 - 12:15	David Burguet (Online France)	Julien Boulanger (Online Chile)	Cleber Colle
		12:30 - 14:30	Lunch		
		14:30 - 15:30	Glauco Valle	Régis Varão	Tatiana Rodrigues
		15:30 - 16:30	Marcelo Sobottka	Posters (Virtual Section)	Milton Cobo

Monday - December 9, 2024	Tuesday - December 10, 2024	Wednesday - December 11, 2024
8:00 - 9:00 Geometric properties of disintegration of measures Christian Rodrigues (Univ. de Campinas, Brazil)	8:00 - 9:00 On an upper bound of the degree of polynomial identities regarding linear recurrence sequences Ana Paula Chaves (Univ. Federal de Goiás, Brazil)	8:00 - 9:00 Multi-dimensional piecewise contractions are asymptotically periodic Benito Pires (Univ. de São Paulo, Brazil)
9:30 - 10:15 Statistical properties for expanding circle maps with an indifferent fixed point Irene Inóquio (Univ. de La Serena, Chile)	9:30 - 10:15 TBA Ali Messaoudi (Univ. Estadual Paulista, Brazil)	9:30 - 10:15 A q-analog of Markoff's injectivity conjecture is true Sébastien Labbé (CNRS, Univ. de Bordeaux, France)
11:30 - 11:15 TBA Manuel Stadlbauer (Univ. Federal do Rio de Janeiro, Brazil)	10:30 - 11:15 Frequencies of letters in some random and deterministic self-descriptive sequences Irene Marcovici (Univ. de Rouen Normandie, France)	10:30 - 11:15 TBA Cristóbal Rivas (Univ. de Chile, Chile)
11:30 - 12:15 Multiplicity of topological systems David Burguet (CNRS, Univ. de Picardie Jules Verne, France)	11:30 - 12:15 Hecke continued fractions and connection points on Veech surfaces Julien Boulanger (Univ. de Chile, Chile)	11:30 - 12:15 On periodic decompositions, nonexpansive lines and Nivat's conjecture Cleber Colle (Univ. Federal do ABC, Brazil)
14:30 - 15:30 Invariant measures for substitutions on countable alphabets Glauco Valle (Univ. Federal do Rio de Janeiro, Brazil)	<b>14:30 - 15:30</b> <b>TBA</b> Régis Varão <i>(Univ. de Campinas, Brazil)</i>	<b>14:30 - 15:30</b> <b>TBA</b> Tatiana Rodrigues <i>(Univ. Estadual Paulista, Brazil)</i>
<b>15:30 - 16:30</b> <b>TBA</b> Marcelo Sobottka (Univ. Federal de Santa Catarina, Brazil)	15:30 - 16:30 Posters (Virtual Section)	15:30 - 16:30 Extensions of Pisot substitutions to infinite alphabets Milton Cobo (Univ. Federal do Espírito Santo, Brazil)

#### **Ali Messaoudi** Univ. Estadual Paulista

Tue - I	Dec 10, 2024
Brazil	9:30 - 10:15
Chile	9:30 - 10:15
France	13:30 - 14:15

## On an upper bound of the degree of polynomial identities regarding linear recurrence sequences

#### Ana Paula Chaves

Univ. Federal de Goiás

Let  $(F_n)_{n\geq 0}$  denote the Fibonacci sequence defined by  $F_{n+2} = F_{n+1} + F_n$  for  $n \geq 0$ , with  $F_0 = 0$  and  $F_1 = 1$ . Several interesting identities involve this sequence, such as  $F_n^2 + F_{n+1}^2 = F_{2n+1}$  for all  $n \geq 0$ . Inspired by this identity, in 2012, Chaves, Marques, and Togbé proved that if  $(G_m)_m$  is a linear recurrence sequence (under weak assumptions) and  $G_n^s + \cdots + G_{n+k}^s \in (G_m)_m$  for infinitely many positive integers n, then s is bounded by an effectively computable constant depending only on k and the parameters of  $(G_m)_m$ .

In this talk, we explore a generalization of this result, proving in particular that if  $(G_m)_m$ and  $(H_m)_m$  are linear recurrence sequences (also under weak assumptions), and  $R(z) \in \mathbb{C}[z]$ is a monic polynomial, then if  $R(G_n) + \cdots + R(G_{n+k})$  belongs to  $(H_m)_m$  for infinitely many positive integers n, the degree of R(z) is bounded by an effectively computable constant depending only on the parameters of  $(G_m)_m$  and  $(H_m)_m$  (but, surprisingly, not on k).

 $\frac{\text{Tue - Dec 10, 2024}}{\text{Brazil 8:00 - 9:00}}$ 

# Multi-dimensional piecewise contractions are asymptotically periodic

#### **Benito Pires**

Univ. de São Paulo

Piecewise contractions (PCs) are piecewise smooth maps that decrease distance between pairs of points in the same domain of continuity. The dynamics of a variety of systems is described by PCs. During the last decade, a lot of effort has been devoted to proving that in parametrized families of one-dimensional PCs, the limit set of a typical PC consists of finitely many periodic orbits while there exist atypical PCS with Cantor limit sets. In this talk, we will discuss how to extend the one-dimensional results to the multi-dimensional case. We will also introduce the notion of multiplicity entropy in the framework of piecewise smooth maps. Joint work with José Pedro Gaivão.

> Wed - Dec 11, 2024 Brazil 8:00 - 9:00

## Geometric properties of disintegration of measures

## Christian Rodrigues

Univ. de Campinas

The study of asymptotic behaviour of Dynamical Systems is currently present in almost all fields of science. From a probabilistic point of view, although some of their key statistical properties may depend on the geometry of the space in which it takes place, very little geometric information is taken into account while studying the properties of probability spaces which encodes ergodic properties. In particular, intrinsic geometric properties of the probability spaces are very often neglected. In this talk, we shall address some geometric properties of probability spaces and investigate how they are related to disintegration of measures. This talk is based in joint work with Renata Possobon.

Mon -	Dec 9,	2024
Brazil	8:00 -	9:00

# On periodic decompositions, nonexpansive lines and Nivat's conjecture

#### Cleber Colle

Univ. Federal do ABC

In his Ph.D. thesis, under the guidance of Jarkko Kari, Michal Szabados showed that every low pattern complexity configuration can be decomposed into a finite sum of periodic configurations. For a not fully periodic configuration with a minimal periodic decomposition, that is, a periodic decomposition with the smallest possible number of periodic configurations, any nonexpansive line contains a period of some periodic configuration in the decomposition. Michal Szabados conjectured that the converse also holds. In this talk, we will present recent advances related to Szabados's conjecture as well as some minor progress on Nivat's conjecture, a natural generalization of the Morse-Hedlund Theorem to the two-dimensional case.

Wed -	Dec 11, 2024
Brazil	11:30 - 12:15
Chile	11:30 - 12:15
France	15:30 - 16:15

# TBA Cristóbal Rivas Univ. de Chile

Wed -	Dec 11,	2024
Brazil	10:30 -	11:15
Chile	10:30 -	11:15
France	14:30 -	15:15

# Multiplicity of topological systems

**David Burguet** 

CNRS and Univ. de Picardie Jules Verne

We define the topological multiplicity of an invertible topological system (X, T) as the minimal number k of real continuous functions  $f_1, \ldots, f_k$  such that the functions  $f_i \circ T^n$ ,  $n \in \mathbb{Z}, 1 \leq i \leq k$ , span a dense linear vector space in the space of real continuous functions on X endowed with the supremum norm. We study some properties of topological systems with finite multiplicity. After giving some examples, we investigate the multiplicity of subshifts with linear growth complexity.

Mon -	Dec 9, 2024
Brazil	11:30 - 12:15
Chile	11:30 - 12:15
France	15:30 - 16:15

#### Invariant measures for substitutions on countable alphabets

Glauco Valle

Univ. Federal do Rio de Janeiro

We will discuss ergodic and dynamical properties of symbolic dynamical systems associated to substitutions on infinite countable alphabets. Specifically, we consider shift dynamical systems associated to irreducible substitutions which have well-established properties in the case of finite alphabets. Based on dynamical properties of a countable integer matrix related to the substitution, we obtain results on existence and uniqueness of shift invariant measures. This is a joint work with A. Messaoudi (UNESP-São José do Rio Preto), W. Domingos (UNESP-São José do Rio Preto), S. Ferenczi (Aix Marseille Université).

Mon -	Dec 9,	2024
Brazil	14:30 -	15:30

## Statistical properties for expanding circle maps with an indifferent fixed point

#### Irene Inoquio-Renteria

Univ. de La Serena

In the setting of expanding maps of the circle with an indifferent fixed point, we study the joint behavior of the dynamics and pairs of moduli of continuity  $(\omega, \Omega)$ , which  $\omega$  is related to the regularity classes of potentials and  $\Omega$  the regularity of observables. This interaction plays a central role in the development of equilibrium theory. In this talk, we remark on a specific property of the modulus  $\Omega$  that guarantees favorable statistical properties for the associated equilibrium states, such as exponential decay of correlations and the Central Limit Theorem. Additionally, we present some preliminary elements that suggest the formulation of a Large Deviation Principle for equilibrium states linked to moduli of continuity. This is joint work with E. Garibaldi.

Mon -	Dec 9, 2024
Brazil	9:30 - 10:15
Chile	9:30 - 10:15
France	13:30 - 14:15

## Frequencies of letters in some random and deterministic self-descriptive sequences

#### Iréne Marcovici

CNRS and Univ. de Rouen Normandie

The Oldenburger-Kolakoski sequence is the only infinite sequence over the alphabet 1,2 that starts with 1 and is its own run-length encoding. We take a step back from this largely known and studied sequence by introducing a larger family of self-descriptive sequences, for which the choice of the letters written is determined not only by their index, but also by another sequence, namely the directing sequence. This enables us to provide some results on the convergence of the density of 1's in the resulting sequence. When the directing sequence is an infinite sequence of i.i.d. random variables or a Markov chain, the average densities of letters converge. Moreover, in the case of i.i.d. random variables, we are able to prove that the densities even almost surely converge. We also treat the case of deterministic self-descriptive sequences directed by two periodic sequences.

The presentation will be based on two joint works with Damien Jamet, in collaboration with Chloé Boisson for the first one, and with Shigeki Akiyama and Mai-Linh Trân Công for the second one.

Tue - I	Dec 10, 2024
Brazil	10:30 - 11:15
Chile	10:30 - 11:15
France	14:30 - 15:15

# Hecke continued fractions and connection points on Veech surfaces

Julien Boulanger CNRS and Univ. de Chile

Tue - 1	Dec 10, 2024
Brazil	11:30 - 12:15
Chile	11:30 - 12:15
France	15:30 - 16:15

#### Manuel Stadlbauer Univ. Federal do Rio de Janeiro

Mon -	Dec 9, 2024
Brazil	10:30 - 11:15
Chile	10:30 - 11:15
France	14:30 - 15:15

#### Marcelo Sobottka Univ. Federal de Santa Catarina

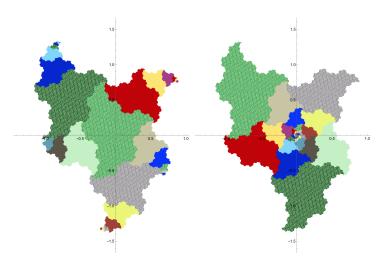
Mon -	Dec 9,	2024
Brazil	15:30 -	16:30

## Extensions of Pisot substitutions to infinite alphabets

Milton Cobo

Univ. Federal do Espírito Santo

We will consider a very broad family of Pisot Substitutions and define its extensions of the infinite alphabet  $\mathbb{N}$ . We will show the same classical results holds for this family of extended substitutions, this is, there is a geometric representation in some  $\mathbb{R}^n$  that corresponds to an exchange of infinitely many domains in the classical Rauzy fractal of the substitution and that projects to the Torus. See for example the figure for the extension to  $\mathcal{A} = \mathbb{N}$  of the Hokkaido substitution  $\sigma(1) = 12$ ;  $\sigma(2) = 3$ ;  $\sigma(3) = 4$ ;  $\sigma(4) = 5$ ;  $\sigma(5) = 1$ :



This is a joint work with A. Messaoudi.

Wed -	Dec 11,	2024
Brazil	15:30 -	16:30

#### **Régis Varão** Univ. de Campinas

Tue -	Dec 10,	2024
Brazil	14:30 -	15:30

## A q-analog of Markoff's injectivity conjecture is true

Sébatien Labbé CNRS and Univ. de Bordeaux

The elements of Markoff triplets are given by the coefficients of certain matrix products defined by Christoffel words, and Markoff's injectivity conjecture, a long- standing open problem also known as the uniqueness conjecture, is then equivalent to injectivity on Christoffel words. A q-analog of these matrix products has been proposed recently by Morier-Genoud and Ovsienko, and we prove that injectivity on Christoffel words is valid for this q-analog. We also extend the problem to arbitrary words and provide a large family of word pairs where the injectivity does not hold. This talk is based on work done in collaboration with Mélodie Lapointe and Wolfgang Steiner available at https://doi.org/10.5802/alco.322

Wed - Dec 11, 2024
Brazil 9:30 - 10:15
Chile 9:30 - 10:15
France 13:30 - 14:15

#### **Tatiana Rodrigues** Univ. Estadual Paulista

Wed -	Dec	11,	2024
Brazil	14:3	0 -	15:30