

$$\frac{x-2}{x^2+1}, \frac{x-2}{x^2-1}, \frac{x^2-4x+3}{x^2-x-2} = \frac{(x-1)(x-3)}{(x+1)(x-2)}$$

$$(0, -2) \quad (0, 2)$$

$$(0, -3/2)$$

$$(x_0, y_0)$$

$$1) \text{ NUM} = 0$$

ZEROS DA FUNÇÃO

$$2) \text{ DEN} = 0$$

ASSÍNTOTAS VERTICAIS

$$3) x \rightarrow \pm \infty$$

ASSÍNTOTAS HORIZONTAIS

$$F'(x) = [f'(x)g(x) - f(x)g'(x)] / g^2(x)$$

$$4) \text{ MAX / MIN}$$

$$F'(x) = 0$$

$$5) \text{ TANGENTE EM } (x_0, y_0)$$

$$y = F'(x_0)(x - x_0) + y_0$$

$$\frac{x-2}{x^2+1}$$

$$(x_0, y_0) = (0, -2)$$

$$1) \text{ NUM} = 0$$

$$x = 2$$

ZERO DA FUNÇÃO (2, 0)

$$2) \text{ DEN} \neq 0$$

NÃO TEM ASSÍNTOTAS VERTICAIS

$$3) x \rightarrow +\infty$$

$$\frac{x}{x^2} \sim 0^+$$

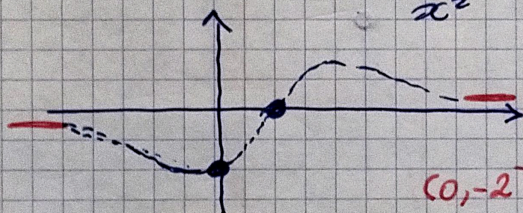
$$(+\infty, 0^+)$$

$$x \rightarrow -\infty$$

$$\frac{x}{x^2} \sim 0^-$$

$$(-\infty, 0^-)$$

GRÁFICO



MAX
MIN

$$2 \pm \sqrt{5}$$

$$(0, -2)$$

$$y = x - 2$$

$$\frac{x-2}{x^2-1}$$

$$(x_0, y_0) = (0, 2)$$

$$1) \text{ NUM} = 0$$

$$x = 2$$

ZERO DA FUNÇÃO (2, 0)

$$2) \text{ DEN} = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$-1.1 \quad (-1^-, \frac{-}{+} \infty)$$

$$(-1^-, -\infty)$$

$$-0.9 \quad (-1^+, \frac{-}{+} \infty)$$

$$(-1^+, +\infty)$$

$$0.9 \quad (1^-, \frac{-}{+} \infty)$$

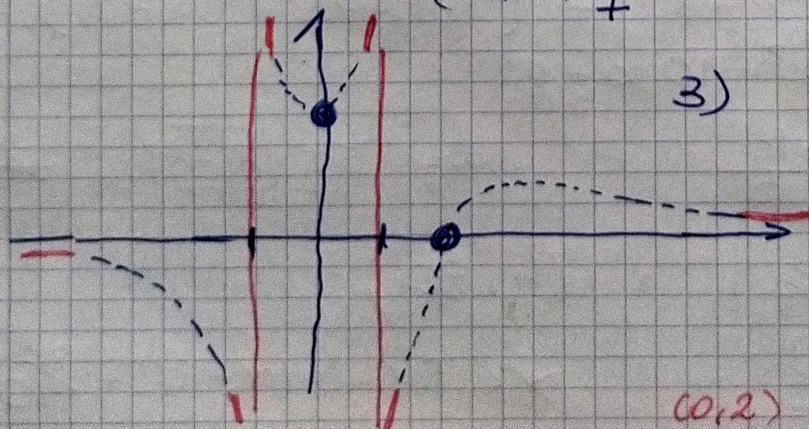
$$(1^-, +\infty)$$

$$1.1 \quad (1^+, \frac{-}{+} \infty)$$

$$(1^+, -\infty)$$

$$3) \quad (+\infty, 0^+)$$

$$(-\infty, 0^-)$$



MAX
MIN

$$2 \pm \sqrt{3}$$

$$(0, 2)$$

$$y = -x + 2$$

$$\frac{x-2}{x^2+1}$$

$$F'(x) = (-x^2 + 4x + 1) / (x^2 + 1)^2$$

$$\frac{x-2}{x^2-1}$$

$$F'(x) = (-x^2 + 4x - 1) / (x^2 - 1)^2$$

$$\frac{x^2 - 4x + 3}{x^2 - x - 2} = \frac{(x-1)(x-3)}{(x+1)(x-2)}$$

$$1) \text{ NUM} = 0$$

$$(2, 0) \quad (3, 0)$$

$$2) \text{ DEN} = 0$$

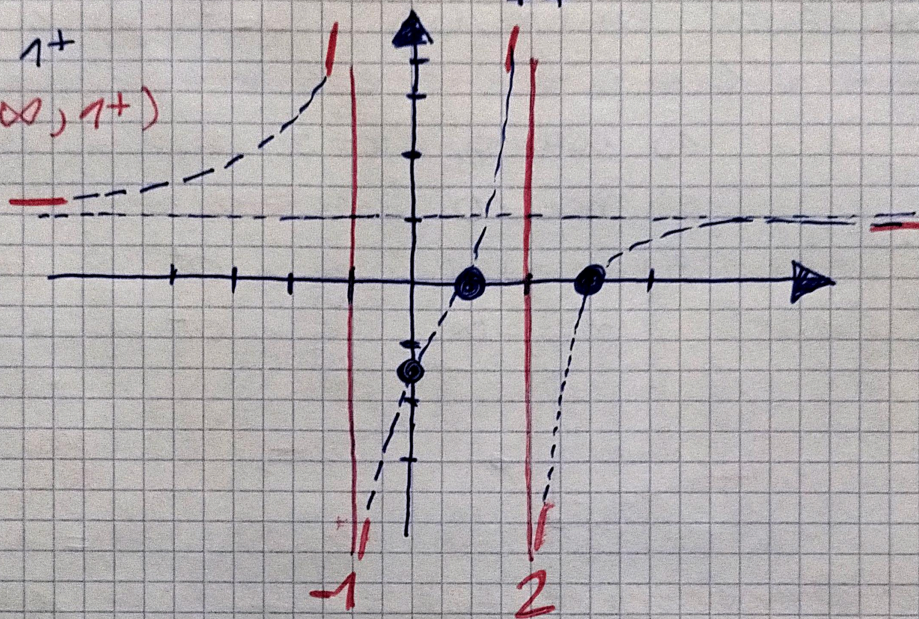
$$x = -1 \quad x = 2$$

$$(x_0, y_0) = \left(0, -\frac{3}{2}\right)$$

$$3) \begin{array}{l} x \rightarrow \pm \infty \quad \frac{x^2 - 4x}{x^2 - x} \\ x \rightarrow +\infty \quad \frac{x^2 - 4x}{x^2 - x} < 1 \quad 1^- \\ x \rightarrow -\infty \quad \frac{x^2 - 4x}{x^2 - x} > 1 \quad 1^+ \end{array} \quad \begin{array}{l} -1.1 \quad (-1^-, \frac{-}{-} \infty) \\ -0.9 \quad (-1^+, \frac{-}{+} \infty) \\ 1.9 \quad (2^-, \frac{+}{-} \infty) \\ 2.1 \quad (2^+, \frac{+}{+} \infty) \end{array} \quad \begin{array}{l} (-1^-, +\infty) \\ (-1^+, -\infty) \\ (2^-, +\infty) \\ (2^+, -\infty) \end{array}$$

$$(+\infty, 1^-)$$

$$(-\infty, 1^+)$$



$$f(x) = x^2 - 4x + 3$$

$$f'(x) = 2x - 4$$

$$g(x) = x^2 - x - 2$$

$$g'(x) = 2x - 1$$

$$F'(x) = \frac{(2x-4)(x^2-x-2) - (2x-1)(x^2-4x+3)}{(x^2-x-2)^2}$$

$$\frac{2x^3 - 2x^2 - 4x - 4x^2 + 4x + 8}{2x^3 - 6x^2 + 8} - \frac{2x^3 - 8x^2 + 6x - x^2 + 4x - 3}{2x^3 - 6x^2 + 8}$$

$$\frac{2x^3 - 6x^2 + 8 - (2x^3 - 9x^2 + 10x - 3)}{3x^2 - 10x + 11}$$

$$3x^2 - 10x + 11$$

$$F'(x) = \frac{3x^2 - 10x + 11}{(x^2 - x - 2)^2}$$

$$x_{1/2} = \frac{5 \pm \sqrt{25 - 33}}{3} \quad *$$

$$\text{tangentes en } \left(0, -\frac{3}{2}\right) \text{ e } \left(4, \frac{3}{10}\right)$$

$$y = F'(0)(x-0) - \frac{3}{2}$$

$$y = F'(4)(x-4) + \frac{3}{10}$$

$$F'(0) = \frac{11}{4}$$

$$F'(4) = \frac{48 - 40 + 11}{(16 - 4 - 2)^2} = \frac{9}{100}$$

$$(0, -\frac{3}{2})$$

$$(4, \frac{3}{10})$$

$$y = \frac{11}{4}x - \frac{3}{2} = \frac{11x - 6}{4}$$

$$y = \frac{9}{100}(x-4) + \frac{3}{10} = \frac{9x - 6}{100}$$