MÉDIA $\{y_1, y_2, \dots y_N\}$

CALCULAREMOS O VALOR "b" QUE MINIMIZA A SOMA DOS QUADRADOS

$$(y_1-b)^2+(y_2-b)^2+\dots+(y_N-b)^2$$

ABRINDO OS QUADRADOS

$$y_1^2+b^2-2y_1b+y_2^2+b^2-2y_2b+\dots+y_N^2+b^2-2y_Nb$$

$$N\,b^2-2(y_1+y_2+\dots+y_N)\,N+y_1^2+y_2^2+\dots+y_N^2$$

PARA ENCONTRAR O MÍNIMO DERIVAMOS

$$2Nb-2(y_1+y_2+\dots+y_N)=0$$

$$b=\frac{y_1+y_2+\dots+y_N}{N}=\langle y\rangle$$

MEDIA

DESVIO PADRÃO $\sqrt{\dfrac{(y_1-\langle y\rangle)^2+(y_2-\langle y\rangle)^2+\dots+(y_N-\langle y\rangle)^2}{N-1}}$

CONSIDERAMOS 2 GRUPOS DE VALORES

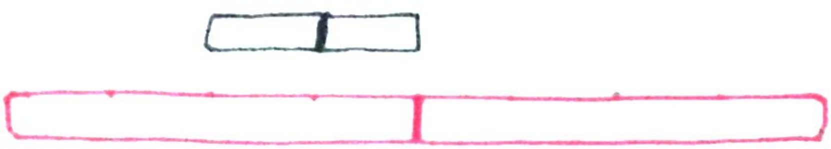
$A=\{40,40,50,60,60\}$ $B=\{20,20,60,100,100\}$

média 50 média 60

desvio padrão $\sqrt{\dfrac{10^2+10^2+0^2+10^2+10^2}{5-1}}$ desvio padrão $\sqrt{\dfrac{(40)^2+(40)^2+0^2+(40)^2+(40)^2}{5-1}}$

10 40

$\{50-10, 50+10\}$ $\{60-20, 60+40\}$



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IMAGINAMOS AGORA DE TER UM CONJUTO DE PONTOS NUM PLANO E QUERER ENCONTRAR A RETA QUE MELHOR REPRESENTA (APPROXIMA) ESTES PONTOS

$$\{(x_1,y_1),(x_2,y_2),\dots,(x_N,y_N)\}$$

$$y = ax + b$$

QUEREMOS OBTER OS COEFICIENTES "a" E "b"

MINIMIZAREMOS OS QUADRADOS

$$(y_1 - a x_1 - b)^2 + \dots + (y_N - a x_N - b)^2$$

$$y_1^2 + a^2 x_1^2 + b^2 - 2a x_1 y_1 - 2b y_1 + 2ab x_1 + \dots$$
$$+ y_N^2 + a^2 x_N^2 + b^2 - 2a x_N y_N - 2b y_N + 2ab x_N$$

$$a^2(x_1^2 + \dots + x_N^2) + N b^2$$
$$-2a(x_1 y_1 + \dots + x_N y_N) - 2b(y_1 + \dots + y_N)$$
$$+2ab(x_1 + \dots + x_N)$$
$$+y_1^2 + \dots + y_N^2$$

DERIVANDO COM RESPEITO AO VALOR "a"

$$2a(x_1^2 + \dots + x_N^2) - 2(x_1 y_1 + \dots + x_N y_N) + 2b(x_1 + \dots + x_N) = 0$$

DERIVANDO COM RESPEITO AO VALOR "b"

$$2Nb - 2(y_1 + \dots + y_N) + 2a(x_1 + \dots + x_N) = 0$$

DIVIDINDO POR N

$$a\langle x^2\rangle + b\langle x\rangle = \langle xy\rangle \qquad (1)$$
$$a\langle x\rangle + b = \langle y\rangle$$
$$a\langle x\rangle^2 + b\langle x\rangle = \langle x\rangle\langle y\rangle \qquad (2)$$

$$(1)-(2) \qquad (\langle x^2\rangle - \langle x\rangle^2)\, a = \langle xy\rangle - \langle x\rangle\langle y\rangle$$

$$a = \frac{\langle xy\rangle - \langle x\rangle\langle y\rangle}{\langle x^2\rangle - \langle x\rangle^2} \qquad b = \langle y\rangle - a\langle x\rangle$$

EXEMPLO $(1,1)$ $(2,3)$ $(3,2)$

$$\langle x\rangle = \frac{1+2+3}{3} = 2 \qquad \langle y\rangle = \frac{1+3+2}{3} = 2$$

$$\langle xy\rangle = \frac{1\cdot 1+2\cdot 3+3\cdot 2}{3} = \frac{1+6+6}{3} = \frac{13}{3}$$

$$\langle x\rangle^2 = \frac{1^2+2^2+3^2}{3} = \frac{1+4+9}{3} = \frac{14}{3}$$

$$a = \frac{\langle xy\rangle - \langle x\rangle\langle y\rangle}{\langle x^2\rangle - \langle x\rangle^2} = \frac{\frac{13}{3} - 2\cdot 2}{\frac{14}{3} - 2^2} = \frac{\frac{13}{3} - 4}{\frac{14}{3} - 4}$$

$$= \frac{1/3}{2/3} = \frac{1}{2}$$

$$b = \langle y\rangle - a\langle x\rangle = 2 - \frac{1}{2}2 = 1$$

$$y = \frac{x}{2} + 1$$

$$\left(1, \frac{3}{2}\right) \quad (2,2) \quad \left(3, \frac{5}{2}\right)$$

$$(1,1) \quad (2,3) \quad (3,2)$$

$$\left(\frac{1}{2}\right)^2 + (-1)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + 1 + \frac{1}{4}$$

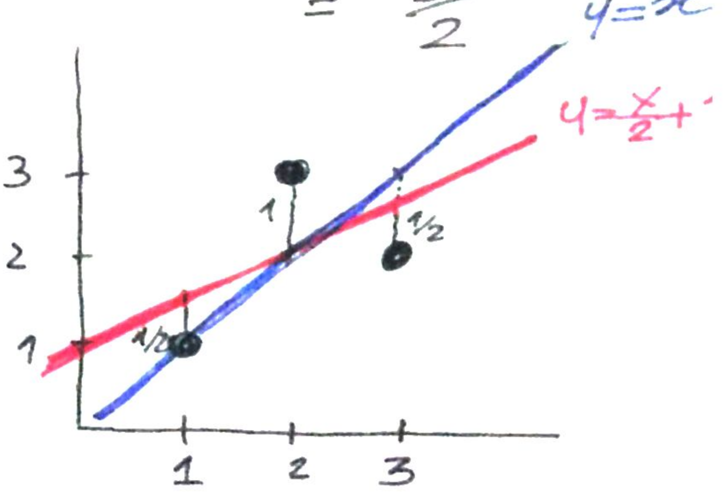
$$= \frac{3}{2}$$

SE PEGAMOS COMO PROVA UMA DIFERENTE RETA, POR EXEMPLO $y = x$

$(1,1)$ $(2,2)$ $(3,3)$

$(1,1)$ $(2,3)$ $(3,2)$

$$0^2 + (-1)^2 + 1^2 = 2$$



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