1. Questão 1.

   (a) Temos que

$$\begin{aligned} P(2V) &= P(U1,1B,2V) + P(U1,1V,2V) + P(U2,1B,2V) + P(U2,1V,2V) \\ &= P(U1)P(1B|U1)P(2V|U1,1B) + P(U1)P(1V|U1)P(2V|U1,1V) \\ &+ P(U2)P(1B|U2)P(2V|U2,1B) + P(U2)P(1V|U2)P(2V|U2,1V) \\ &= \frac{1}{3}\frac{1}{2}\frac{2}{3} + \frac{1}{3}\frac{1}{2}\frac{1}{3} + \frac{2}{3}\frac{1}{4}1 + \frac{2}{3}\frac{3}{4}\frac{2}{3} = \frac{1}{9} + \frac{1}{18} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3} \end{aligned}$$

   (b) Temos que

$$P(U2|2V) = \frac{P(U2,2V)}{P(2V)} = \frac{P(U2,1B,2V) + P(U2,1B,2V)}{P(2V)} = \frac{1/6 + 1/3}{2/3} = \frac{3}{4}$$

   (c) Temos que

$$P(1B|2V) = \frac{P(1B,2V)}{P(2V)} = \frac{P(U1,1B,2V) + P(U2,1B,2V)}{P(2V)} = \frac{1/9 + 1/6}{2/3} = \frac{5}{12}$$

2. Questão 2

(a) Temos que ($u = x^2 \to du = 2xdx$)

$$\int_0^\infty cxe^{-x^2}dx = \int_0^\infty \frac{c}{2}e^{-u}du = 2 \to c\Gamma(1) = 2 \to c = 2$$

(b) Temos que ($u = z^2 \to du = 2zdz$)

$$P(X \leq x) = \int_0^x 2ze^{-z^2}dz = \int_0^{x^2} e^{-u}du = (1 - e^{-u})\Big|_0^{x^2} = \left(1 - e^{-x^2}\right)\mathbb{1}_{(0,\infty)}(x)$$

(c) Temos que ($k > 0, u = x^2 \to du = 2xdx$)

$$\begin{aligned} \mathcal{E}(X^k) &= 2\int_0^\infty x^{k+1}e^{-x^2}dx = \int_0^\infty \left(\sqrt{u}\right)^k e^{-u}du = \int_0^\infty u^{k/2+1-1}e^{-u}du \\ &= \Gamma(k/2+1) \end{aligned}$$

Assim

$$\begin{aligned} \mathcal{E}(X) &= \Gamma(3/2) = \frac{1}{2}\Gamma(1/2) = \frac{\sqrt{\pi}}{2} \\ \mathcal{E}(X^2) &= \Gamma(2) = 1 \\ \mathcal{V}(X) &= \mathcal{E}(X^2) - \mathcal{E}^2(X) = \frac{4-\pi}{4} \end{aligned}$$

3. Questão 3

   (a) Temos que

$$\begin{aligned} P(X = x) &= \frac{3}{11}\mathbb{1}_{\{-1\}}(x) + \frac{4}{11}\mathbb{1}_{\{0\}}(x) + \frac{4}{11}\mathbb{1}_{\{1\}}(x) \\ P(Y = y) &= \frac{3}{11}\mathbb{1}_{\{0\}}(y) + \frac{3}{11}\mathbb{1}_{\{1\}}(y) + \frac{5}{11}\mathbb{1}_{\{3\}}(y) \end{aligned}$$

   (b) Temos que:

$$\begin{aligned} P(X \leq 1|Y = 1) &= \frac{P(X = -1, Y = 1) + P(X = 0, Y = 1) + P(X = 1, Y = 1)}{P(Y = 1)} \\ &= \frac{3/11}{3/11} = 1 \\ P(Y = 0|X \leq 0) &= \frac{P(Y = 0, X \leq 0)}{P(X \leq 0)} = \frac{P(Y = 0, X = -1) + P(Y = 0, X = 0)}{P(X = -1) + P(X = 0)} \\ &= \frac{2/11}{7/11} = \frac{2}{7} \end{aligned}$$

   (c) Note que

$$P(X = 0, Y = 0) = 0 \neq P(X = 0)P(Y = 0) = \frac{4}{11}\frac{3}{11} = \frac{12}{121}$$

   Logo X e Y são dependentes.

4. Questão 4.

   (a) Temos que

$$f_X(x) = F'(x) = \frac{1}{b-a}\mathbb{1}_{(a,b)}(x)$$

   (b) Temos que

$$P(X \geq x) = \frac{2}{3} \to P(X < x) = \frac{1}{3} \to \frac{x-a}{b-a} = \frac{1}{3} \to x = \frac{b-a}{3} + a = \frac{b+2a}{3}$$

   (c) Sabemos que:

$$\begin{cases} \mathcal{E}(X) = \dfrac{a+b}{2} = 3 \to a = 6 - b \\ \mathcal{V}(X) = \dfrac{(b-a)^2}{12} = 3 \to (2b-6)^2 = 36 \to 2b - 6 = \pm 6 \end{cases}$$

$$\begin{cases} a = 6 - b \to a' = 3, a'' = 6 \\ b' = 3, b'' = 0 \end{cases}$$

   Como $a < b$ então $a = 0, b = 6$.