Workshop on Stochastic Analysis

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A stochastic collisional picture of the Wave Kinetic Equation

Abstract

The wave kinetic equation (WKE) is a fundamental effective equation in wave turbulence theory that describes the evolution of the energy spectrum in a weakly interacting nonlinear wave system, with important applications in fields such as plasma physics, nonlinear optics and oceanography (where it is known as Hasselman's equation). Recent breakthrough results by Deng and Hani established the first full-range mathematical derivation of the 4-wave (or cubic) WKE from the (dispersive, reversible) nonlinear Schrödinger equation under suitable scaling limits.

In this talk, we introduce an alternative, purely stochastic collisional description of the WKE. In this setting, binary collisions between wave-vectors are proposed analogously to particle's in the setting of the spatially homogeneous Boltzmann equation (hBE), but are accepted or rejected depending on the local density at the post-collisional configurations, thus reflecting the additional nonlinearity of the cubic WKE. We formalize this interpretation rigorously by constructing a Poisson-driven nonlinear process in the spirit of Tanaka's process for the hBE, with flow of time-marginal laws equal to a given solution of the WKE.

Furthermore, this stochastic viewpoint allows us to introduce and analyze a regularized mean-field

interacting jump-particle system that extends Kac systems for hBE to the WKE framework. This particle system is expected to satisfy the propagation of chaos property and provide an approximation of the WKE as the number of particles tends to infinity and the regularizing parameter vanishes. We provide partial rigorous mathematical results in these directions and present simple simulations to illustrate the numerical scheme that naturally arises from the particle model.

Last, if time permits, we will discuss potential applications of this stochastic viewpoint on the WKE, to the study of physically relevant questions including non-equilibrium steady states (known as Kolmogorov-Zakharov solutions) and spontaneous stochasticity.

Based on work in progress with Armand Bernou (U. Lyon 1), Roberto Cortez (UNAB, Chile) and G. Krstulovic (U. Cote d'Azur)