Regularization by fractional (in time) noise for the stochastic heat equation with singular drift

Workshop on stochastic analysis

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1. Motivation
2. Setting
   - Regularization by noise for SPDEs
   - Fractional correlation in time
   - Notion of solution
3. Main results
   - Existence and uniqueness of strong solutions
   - Existence of weak solutions
   - Uniqueness of strong solutions
   - Strong existence
4. Further perspectives

Regularization by fractional noise for the SHE

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Let $\mathbb{T}$ be the one dimensional torus and Δ the Laplace operator on $\mathbb{T}$. Fix a time horizon $T \in (0, \infty)$ and an initial condition $u_0 \in L^2(\mathbb{T})$.

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If $b : \mathbb{R} \longrightarrow \mathbb{R}$ is a locally Lipschitz continuous function, then an extension of the **Cauchy-Lipschitz (Picard–Lindelöf) theorem** guarantees that the following parabolic PDE

$$\begin{aligned} \partial_t u(t,x) &= \Delta u(t,x) + b(u(t,x)), \quad \forall (t,x) \in [0,T] \times \mathbb{T}, \\ u(0,x) &= u_0(x), \quad \forall x \in \mathbb{T}, \end{aligned} \tag{1}$$

has local existence and uniqueness of (mild) solutions.

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- $b \in \mathcal{C}^{\alpha}$ for some $\alpha \in (0,1)$,
- Bounded and measurable,
- $b \in \mathcal{C}^{-\alpha}$ for some $\alpha > 0$ (a distribution).

Regularization by fractional noise for the SHE

Motivation

Take $b(u) = 2\operatorname{sign}(u)\sqrt{|u|}$ for example. Then for every $t_0 \in [0, T]$, the function defined as

$$u_{\pm}^{t_0}(t, x) = \begin{cases} 0 & \text{if } t \in [0, t_0), \\ \pm(t - t_0)^2 & \text{if } t \in [t_0, T], \end{cases}$$

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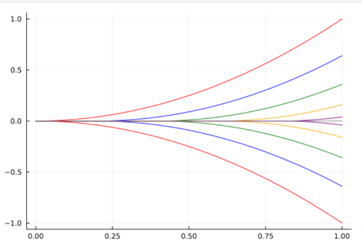

Figure: Peano's brush

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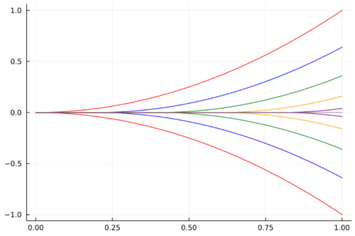

Figure: Peano's brush

How to recover uniqueness?

Through

additive noise! $\partial_t u = \Delta u + b(u) + \xi$.

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Regularization by noise for SPDEs

There are certain choices of b and ξ for which the SPDE

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has uniqueness (and existence) of *strong (mild) solutions*.

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Set $e_k = e^{i2\pi k(\cdot)}$ $\forall k \in \mathbb{Z}$, so that $(e_k)_{k\in\mathbb{Z}}$ is a complete orthonormal basis of $L^2(\mathbb{T})$ with $\Delta e_k = -4\pi^2 k^2 e_k$.

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Fix a complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ over which we define the cylindrical Wiener process

$$W(t,x) = \sum_{k\in\mathbb{Z}} \beta_k(t) e_k(x), \quad (t,x) \in [0,T] \times \mathbb{T}.$$

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Stochastic convolution :

$$\int_0^t \int_{\mathbb{T}} p_{t-r}(x,y) W(dr,dy) := \sum_{k\in\mathbb{Z}} \int_0^t e^{-4\pi^2 k^2 (t-r)} d\beta_k(r) e_k(x), \quad (t,x) \in [0,T]\times\mathbb{T}.$$

For the choice $\xi := \partial_t W$, we say that a random field $u : [0,T] \times \mathbb{T} \times \Omega \longrightarrow \mathbb{R}$ is a solution to (2) if

$$u(t,x) = \int_{\mathbb{T}} p_t(x,y) u_0(y) dy + \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y) b(u(r,y)) dy dr$$
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Known results

- In [PG93], it was shown that (2) is well-posed for b measurable and bounded.
- In [ABLM22], for b a distribution in some Besov space (Hölder space of negative regularity for example).

Regularization by fractional noise for the SHE

Fractional correlation in time

Objective : Consider a family of **fractional Brownian motions** $(\beta_k^H)_{k\in\mathbb{Z}}$ with common Hurst parameter $H \in (0,1)$ in the definition of the noise. This means that for all $k \in \mathbb{Z}$, β_k^H is a Gaussian process verifying

- $\beta_k^H(0) = 0$ and $\mathbb{E}[\beta_k^H(t)] = 0,\ t > 0$,
- $\mathbb{E}[\beta_k^H(t)\beta_k^H(s)] = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}), \quad \forall s,t > 0$.

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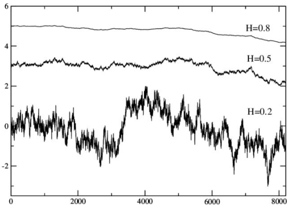

Figure: G.Vasconcelos. A Guided Walk Down Wall Street: An Introduction to Econophysics

Regularization by fractional noise for the SHE

Notion of solution

For the infinite dimensional setting, we consider the fractional cylindrical Wiener process given by

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We consider the case $H \in \left(\frac{1}{2}, \frac{3}{4}\right)$.

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Fractional stochastic convolution (FSC):

$$V_t^H(x) := \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y)W^H(dr,dy) := \sum_{k\in\mathbb{Z}} \int_0^t e^{-4\pi^2k^2(t-r)}d\beta_k^H(r)e_k(x).$$

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Indeed,

$$\begin{aligned}\mathbb{E}\left[\|v_t^H\|_{L^2(\mathbb{T})}^2\right] &\lesssim \sum_{k\in\mathbb{Z}} \lambda_k^{-2H} \\ &\lesssim \sum_{k\in\mathbb{Z}} k^{-4H} < \infty\end{aligned}$$

if and only if $H \in \left(\frac{1}{4}, 1\right)$.

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Existence and uniqueness of strong solutions

Assumption 1

Assume that $\alpha = 0$ if $H \in (\frac{1}{2}, \frac{3}{4})$ and

$$\frac{H - \frac{3}{4}}{H - \frac{1}{4}} < \alpha < 1, \quad \text{if } H \in [\frac{3}{4}, 1).$$

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Theorem (N., Bréhier, Quer-Sardanyons)

Under the conditions of Assumption 1, equation (1) *has a unique strong solution in a suitable class of random fields.*

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Extension of results first shown by Nualart and Ouknine ([NO04]).

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Existence of weak solutions

Weak existence (N., Bréhier, Quer-Sardanyons)

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Strategy of the proof :

For every $n \in \mathbb{N}$ consider the mollification $b^n := G_{1/n}b$, so that equation (1) with $b = b^n$ is clearly well-posed. Denote by $(u^n)_{n\in\mathbb{N}}$ the corresponding sequence of (strong) solutions.

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To identify the limit as a weak solution, one can use *Skorohod's representation theorem* to construct a different probability space and a sequence of random fields $\tilde{u}^n \overset{\text{Law}}{=} u$ such that $(\tilde{u}^n)_{n\in\mathbb{N}}$ converges a.s. to some random fields $\tilde{u}$.

Theorem (N., Bréhier, Quer-Sardanyons)

Let the conditions of Assumption 1 hold and assume that

$$\big(u(t,x)\big)_{(t,x)\in[0,T]\times\mathbb{T}},\ \big(v(t,x)\big)_{(t,x)\in[0,T]\times\mathbb{T}}$$

are two strong solutions of (1) *starting from the same initial condition* u_0. *Then* $u = v$ *a.s.*

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Strategy for the proof : Stochastic Sewing [Lê20].

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Notice that for all $(t,x) \in [0,T] \times \mathbb{T}$,

$$\begin{aligned} |u(t,x) - v(t,x)| &= \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y)(b(u(s,y) - b(v(s,y))dydr \\ &\leq \|b\|_\alpha \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y)|u(s,y) - v(s,y)|^\alpha dydr. \end{aligned}$$

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Write

$$V_t^H(x) := \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y) W^H(dr,dy), \quad (t,x) \in [0,T] \times \mathbb{T},$$

and $\tilde{u} := u - V^H$, so that

$$\int_0^t \int_{\mathbb{T}} p_{t-r}(x,y) b(u(s,y)) dyds = \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y) b(\tilde{u}(s,y) + V_s^H(y)) dyds.$$

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Stochastic Sewing :

For $(t,x) \in [0,T] \times \mathbb{T}$, better to consider the whole mapping

$$z \longrightarrow \mathbb{E} \int_0^t \int_{\mathbb{T}} p_{t-r}(x,y) b(z + V_s^H(y)) dyds.$$

It is more regular!

Regularization by fractional noise for the SHE

Existence of strong solutions

For strong existence : **Yamada-Watanabe**

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Strong existence = weak existence + strong uniqueness

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Proposition (Gyöngi-Krylov [GK21])

Let $(Z_n)_{n\in\mathbb{N}}$ be a sequence of random elements in a Polish space (E,ρ), equipped with the Borel σ-algebra. Assume that for every pair of subsequences (Z_{l_k}) and (Z_{m_k}) there exists a further sub-sequence $(Z_{k_{l_r}}, Z_{m_{l_r}})$ which converges weakly in $E\times E$ to a random element $w=(w',w'')$ such that $w'=w''$ a.s. Then there exists an E-valued random element Z such that (Z_n) converges in probability to Z as $n\longrightarrow\infty$.

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Further perspectives

- Numerical schemes: Given $N \in \mathbb{N}$, let $h = T/N$ and define $\kappa_h(t) = h\lfloor t/h \rfloor$ at every $t \in [0, T]$. For each time step $k \in \{1, \dots, N\}$ consider the numerical scheme given by

$$u_k^h = u_{k-1}^h + h\Delta u_k^h + hb(u_{k-1}^h) + \delta_k W^H, \tag{3}$$

where $\delta_k W^H := W^H(hk, \cdot) - W^H(h(k-1), \cdot)$.

- Cover the remaining case: $H \in \left(\frac{1}{4}, \frac{1}{2}\right)$ both theoretically and numerically.

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