Workshop in Stochastic Analysis and Applications

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Stochastic continuity equation with non-smooth velocity

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Abstract

In this work we are interested in studying the stochastic continuity equation given by

$$\partial_t u(t,x) + Div\big((b(t,x) + \frac{dB_t}{dt}) \cdot u(t,x)\big) = 0$$

$$u|_{t=0} = u_0.$$

The goal of this work is to show uniqueness of L^2-weak solutions for one-dimensional stochastic continuity equation above with unbounded measurable field vector (drift) without assumptions on the divergence. More precisely, we assume that vector field b (drift) satisfies

$$|b(x)| \leq k(1 + |x|).$$

Thus, to get uniqueness the essential idea is to prove that one primitive V of the solution u is regular and that it verifies the transport equation

$$\partial_t V(t,x) + (b(t,x) + \frac{dB_t}{dt}) \cdot \nabla V(t,x) = 0.$$

In this way, using a modified version of the "commutator Lemma" and the characteristic systems associated to the stochastic partial differential equation above, we shall show that $V = 0$ with initial condition equal to zero, which implies that $u = 0$. This is a joint work with Christian Olivera (Universidade Estadual de Campinas).