UNOBSERVABILITY OF THE SIGN CHANGE OF SPINORS UNDER A 2π ROTATION IN NEUTRON INTERFEROMETRIC EXPERIMENTS

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Abstract

We show that the neutron interferometric experiments do not imply that the neutron wave function must be described by a Pauli c-spinor wave function that changes sign under a 2π rotation. We argue that the papers supporting the opposite view have jumbled up the time evolution of the Pauli c-spinor wave function with its transformation law under rotations. Even more, we show that the experiment can be well described using a Pauli algebraic spinor wave function that does not change sign under a 2π rotation.

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There are essentially three different definitions of spinors in the literature: (i) the *covariant* definition, where a particular kind of covariant spinor (c-spinor) is a set of complex variables defined by their transformations under a particular kind of spin group; (ii) the *ideal* definition, where a particular kind of *algebraic* spinor (e-spinor) is an element of a lateral ideal (defined by the idempotent e) in an appropriate Clifford algebra (when e is a primitive idempotent we call it an a-spinor, instead of e-spinor); and (iii) the *operator* definition, where a particular kind of operator spinor (o-spinor) is a Clifford number in an appropriate Clifford algebra $\mathbb{R}_{p,q}$ determining a set of tensors by bilinear mappings. In [1,2] we have clarified the relations between and the possible equivalence of all these kinds of spinors and in [3,4] we studied the corresponding spinor fields as sections of appropriate bundles over a manifold modelling spacetime.

Physicists use almost exclusively c-spinor fields (despite the fact that operator spinor fields have been introduced by Ivanenko and Landau [5] already in 1928 and rediscovered by Kähler[6] in 1961) as the representatives of spin 1/2 fermionic matter. As is well known, a c-spinor wave function has the property of changing its sign under an active 2π rotation, which is not the case for algebraic or operator spinor wave functions interpreted as sections of appropriate

Clifford bundles [4]. Which kind of spinor fields, covariant or algebraic/operator gives the best mathematical and physical representation of fermionic matter is a very important problem, since algebraic and operator spinor fields can be written as sums of non-homogeneous differential forms [1,2,4,5,7,8] thus challenging the "majority view" that spinors are objects more fundamental than tensors [9,10,11]. (We emphasize here that when a-spinor fields are interpreted as sections of the so called Spin-Clifford bundle they have the usual transformation law [4].)

Bernstein [12], Aharanov and Susskind [13] and Moore [14] proposed experiments for the verification of the sign change of c-spinors under an active 2π rotation. Hegerfeldt and Krauss [15] put forth a critical remark on the Aharanov and Susskind argument, showing that it is in flaw (a point on which we agree). Also Jordan [16] invoked the spin statistics theorem for spin 1/2 particles to argue that 2π rotations are unobservable.

After the neutron interferometric experiments [17,18,19] the controversy on the interpretation of the sign change of the neutron c-spinor wave function in a magnetic field went out, as it is well illustrated by the many papers that appeared on this subject [20–30]. It seems to be the "majority view" that the neutron interferometric experiments do indeed prove that the neutron wave function must be described by a Pauli c-spinor wave function (on the nonrelativistic limit appropriate for the experiment) that changes sign under an active 2π rotation.

Here we challenge such a viewpoint. Indeed, we are going to show that the neutron interferometric experiment as described e.g. in [30] can be perfectly explained when the spin 1/2 neutron matter is described by a Pauli a-spinor wave function that does *not* change sign under a 2π active rotation. What happens is simply that the unitary evolution operator for such a wave function is an element of Spin(3) $\simeq$SU(2) ! For what follows nonrelativistic (first quantization) quantum mechanics will suffice. We are going to use *elementary* definitions of the c-spinor and a-spinor wave functions, i.e. we are not going to present these objects as sections of some vector bundle. (The interested reader may consult e.g. [4] on that topic.)

We take as arena of physical phenomena the Newtonian spacetime $N = \mathbb{R}^3 \times \mathbb{R}$ and define a Pauli c-spinor wave function as a mapping

$$\Psi : N \to \mathbb{C}^2 \tag{1}$$

where $\mathbb{C}^2$ is a two-dimensional vector space over the complex field $\mathbb{C}$. The space $\mathbb{C}^2$ is equipped with the spinorial metric

$$\beta_p : \mathbb{C}^2 \times \mathbb{C}^2 \to \mathbb{C};\ \beta_p(\Psi, \Phi) = \Psi^\dagger \Phi \tag{2}$$

where $\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$ and † stands for Hermitian conjugation. The spinorial metric is invariant under the action of SU(2)$\simeq$Spin$_+$(3) (in fact it is invariant under the action of U(2) [2]). As it is well known Pauli c-spinors carry the fundamental representation $D^{1/2}$ of SU(2). Under an active rotation R in the Euclidian space $\mathbb{R}^3$ the Pauli c-spinor wave function transforms as

$$\Psi \overset{R}{\mapsto} U(R)\Psi,\ U(R) \in \mathrm{SU}(2) \tag{3}$$

and if R is a 2π rotation around a given axis, then $\Psi \overset{2\pi}{\mapsto} -\Psi$. In a given magnetic field $B : N \to \mathbb{R}^3$ the neutron wave function Ψ satisfies as it is well known [31] Pauli's equation

$$i\frac{\partial \Psi}{\partial t} = H_i\Psi - \frac{\nabla^2\Psi}{2m} \tag{4}$$

where we use units such that $\hbar = 1$, m is the neutron mass and

$$H_i = -\mu \cdot B = -\mu(\sigma_1 B_1 + \sigma_2 B_2 + \sigma_3 B_3) \tag{5}$$

where σ_j, $j = 1,2,3$ are the Pauli spin matrices, B_j, $j = 1,2,3$ are the components of B in a given reference frame of $\mathbb{R}^3$ and μ is the neutron's magnetic moment. In what follows we are interested only in the spin precession motion and so we consider instead of eq.(4) the equation

$$i\frac{\partial \Psi}{\partial t} = H_i\Psi,\ \Psi : t \mapsto \Psi(t) \in \mathbb{C}^2 \tag{6}$$

We choose B in the z-direction and then write $H_i = -\mu B\sigma_3$. We now write $\Psi = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \sum c_j|j>$ and observe that $\sigma_1\sigma_2\sigma_3|1>= i|1>$ and $\sigma_1\sigma_2\sigma_3|2>= -i|2>$. Then eq.(6) can be written

$$\sigma_1\sigma_2\sigma_3\frac{\partial \Psi}{\partial t} = -\mu B\sigma_3\Psi. \tag{7}$$

We now define the Pauli a-spinor wave function and write the (Pauli) equation satisfied by this object for the situation of the neutron interferometric experiment. We first recall [1,2] that the Pauli algebra $\mathbb{R}_3$ is the Clifford algebra generated by 1 and e_j, $j = 1,2,3$ such that $e_ie_j + e_je_i = 2\delta_{ij}$ where $\{e_j; j = 1,2,3\}$ is a basis of the Euclidian vector space $V \simeq \mathbb{R}^3 \hookrightarrow \mathbb{R}_3$. We take $\{\sigma_i; i = 1,2,3\}$ as a basis of V^*, the dual space of $\mathbb{R}^3$, with $\sigma_i(e_j) = \delta_{ij}$ and call $\mathbb{P}(\simeq \mathbb{R}_3)$ the Clifford algebra generated by 1 and the σ_i, $i = 1,2,3$. A Pauli a-spinor wave function in then defined as a mapping

$$\psi : N \to \{\mathbb{P}e\} \tag{8}$$

where $e = \frac{1}{2}(1 + \sigma_3)$ is a primitive idempotent of $\mathbb{P}$ and $\{\mathbb{P}e\}$ is the class of equivalent minimal left ideal of $\mathbb{P}$ generated by e, i.e. ψ is a sum of non-homogeneous differential forms [3,4,7]. Under an active rotation R in $\mathbb{R}^3$ the Pauli a-spinor wave function transforms as

$$\psi \overset{R}{\mapsto} u(R)\,\psi\, u^{-1}(R) \tag{9}$$

where $u \in \mathrm{Spin}_+(3)(\simeq \mathrm{SU}(2)) \subset \mathbb{P}$. (More precisely this is the transformation law when $(x, \psi(x))$ is taken as a section of the Clifford bundle. See [3,4] for details.) This has as a consequence that under a 2π rotation $\psi \overset{2\pi}{\mapsto} \psi$. The spinorial metric defined by eq.(2) can also be defined within the Pauli algebra [1,2] but it is not necessary here.

The spinorial basis generated by $e = \frac{1}{2}(1 + \sigma_3)$ is $\{e, \sigma_1 e\}$ [1,2] and we can write $\psi = c_1 e + c_2 \sigma_1 e$ with $c_1, c_2 \in \mathbb{C}$, generated by $\{1, \mathrm{i}\}$. Also $\mathrm{i} = \sigma_1\sigma_2\sigma_3$ is the volume element of $\mathbb{R}^3$ and $\mathrm{i}\Lambda_p$ is essentially $*\Lambda_p$, where $\Lambda_p \in \bigwedge(\mathrm{T}^*\mathbb{R}^3)$ is a p-form and $*$ is the Hodge dual operator. To write the (Pauli) equation satisfied by ψ for the neutron interferometric experiment we need only to take $\psi : t \mapsto \{\mathbb{P}e\}$ and to make in eq.(7) the substitutions $\Psi \mapsto \psi$, $\sigma_i \mapsto \sigma_i$, $(i = 1, 2, 3)$. We get

$$\frac{\partial \psi}{\partial t} = \mu B(\mathrm{i}\sigma_3)\psi. \tag{10}$$

The solution of this equation is

$$\psi(t) = \exp(\mu B \mathrm{i}\sigma_3 t)\psi(0) \tag{11}$$

where $\mathrm{Spin}_+(3) \ni u(t) = \exp(\mu B \mathrm{i}\sigma_3 t) = \cos(\mu B t) + \sigma_1\sigma_2 \sin(\mu B t)$ [33].

Equation (11) shows that the predictions for the neutron interferometric experiment *when one uses a Pauli a-spinor wave function* are the same as when a Pauli c-spinor wave function is used. Since these two kinds of spinor wave functions have different transformation laws under rotations (eq.(3) and eq.(9)), it follows that the experiment *does not prove* that the fermionic matter of the neutron must be described by a Pauli c-spinor wave function.

Before we end we must add that the notion of algebraic spinor fields leads to a new point of view [4] concerning the spinor structure of spacetime and the relation between bosons and fermions (supersymmetry) [34]. Also our translation of the Pauli equation satisfied by Ψ into the (Pauli) equation satisfied by ψ provides a geometrical meaning for the imaginary unit $i = \sqrt{-1}$, a fact that may have nontrivial consequences as already emphasized by Hestenes [35–38] who has been since long using algebraic and operator spinor wave functions for the interpretation of the relativistic quantum mechanics of the electron.

At least, to those who might not be convinced by our arguments, we recall the fact that there are many two-state quantum systems described by equations identical to eq.(6). Indeed as shown in Chap. 11-3 of [31] this is the case of the amonia molecule (a boson) in an eletric field. In a (possible) interferometric two-slit experiment with amonia molecules, with one of the paths passing through an eletric field $\boldsymbol{E}$, we could see for an appropriate $\boldsymbol{E}$ a phase change $\phi \mapsto -\phi$. Nevertheless we are sure that in such a case nobody would claim that we are observing a 2π rotation of a spinor!

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