A Comment on: 'On Some Contradictory Computations in Multi-Dimensional Mathematics'*

E. Capelas de Oliveira and W. A. Rodrigues, Jr.
Institute of Mathematics, Statistics and Scientific Computation
IMECC, UNICAMP, CP 6065
13083-859 Campinas SP, Brazil.
e-mail: capelas@ime.unicamp.br walrod@ime.unicamp.br

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Abstract

In this paper we analyze the status of some 'unbelievable results' presented in the paper 'On Some Contradictory Computations in Multi-Dimensional Mathematics' [1] published in *Nonlinear Analysis,* a journal indexed in the Science Citation Index. Among some of the unbelievable results 'proved' in the paper we can find statements like that: (i) a rotation $\mathcal{T}_\theta : \mathbb{R}^2 \to \mathbb{R}^2$, $\theta \neq n\pi/2$, is inconsistent with arithmetic, (ii) complex number theory is inconsistent. Besides these 'results' of mathematical nature [1], offers also a 'proof' that Special Relativity is inconsistent. Now, we are left with only two options (a) the results of [1] are correct and in this case we need a revolution in Mathematics (and also in Physics) or (b) the paper is a potpourri of nonsense. We show that option (b) is the correct one. All 'proofs' appearing in [1] are trivially wrong, being based on a poor knowledge of advanced calculus notions. There are many examples (some of them discussed in [2, 3, 4, 5, 6]) of complete wrong papers using nonsequitur Mathematics in the Physics literature. Taking into account also that a paper like [1] appeared in a Mathematics journal we think that it is time for editors and referees of scientific journals to become more careful in order to avoid the dissemination of nonsense.

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1 Introduction

From time to time we give the following exercise to some of our students: find mathematical errors and inconsistencies in articles appearing in scientific journals, books or in the arXiv.[1] Of course a paper with the title 'On Some Contradictory Computations in Multi-Dimensional Mathematics' called immediately our attention as a potential one suggesting a possible exercise. When we read that paper we first thought that the author was joking[2], that he wrote it only to prove that many referees indeed do not understand absolutely nothing about many of papers for which they wrote reports[3]. Indeed, how a red herring was not immediately activated when reviewing a paper that claims among other results that:

(i) "multi-variable mathematics is inconsistent with arithmetic ($1 = 0$) and also auto-contradictory as calculus is part of this theory",
(ii) "A rotation $\mathcal{T}_\theta$, $\theta \neq n\pi/2$, is inconsistent with arithmetic",
(iii) "Complex number theory is inconsistent",
(iv) "Lorentz's transformation is contradictory unless $v = 0$ (in which case the transformation is the identity) i.e., SRT is trivial".

Below we show explicitly that all the above claims are based on a single misconception, which result from the fact that author of [1] forgot some crucial results of advanced calculus.

2 Critical Analysis of [1]

The paper under review is divided in five sections. In the introduction it is said that the permanent requirement of consistence is primordial for exact science. So, if the claims (i-iv) above 'proved' by author of [1] were true it is just the time to stop doing mathematics for a while and certainly stop using it as presently known in any 'exact' science. After some confuse observations (including some ones concerning the theory of relativity) he recalls the merge of algebra and geometry introduced by Descartes and says:

"Thus contemporary mathematics adopts the notion of a change of variables (see following section) as a tool to choose among all the coordinate systems, the one which better simplifies the problem under study. It application began to spread by the middle of the 18th century. What would happen if two of these allowed choices did show up to be contradictory? Clearly, Descartes' idea should be refined, and modern mathematics reformulated. Observe that many new concepts (the negative and the complex numbers) and theories (the infinitesimal calculus, linear algebra, multi-variable calculus, non-Euclidean geometries,

[1]As result of this activity we eventually write some notes which in some specific cases are sent for publication. As example of this activity we quote the following papers: [2, 3, 4, 5, 6].
[2]Unfortunately this was not the case, i.e., author was not joking, as we discover reading some other papers signed by Carvalho which have been quoted in [1].
[3]Something we also said, e.g, in [2, 3, 4, 5, 6].

topology, etc.) were conceived as a by-product of Descartes' idea. The notion of change of variables stands as a fundamental concept to any of those theories."

After that he introduces in Section 2 his arguments for considering the change of variables in $\mathbb{R}^2$ (different form the identity) to be inconsistent.

One of his thesis in that Section is formulate as follows. Consider the standard statement:

"Any linear transformation $\mathcal{T} : \mathbb{R}^2 \to \mathbb{R}^2$, $\mathcal{T}(x, y) = (\xi, \eta)$, given by[4]

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{c2.1}$$

with $\det M = ad - bc \neq 0$, is a valid transformation of variables (coordinates) in $\mathbb{R}^2$".

Well, Carvalho claims that such statement is "obviously meaningless in nature". He endorses his argument with an almost incomprehensible example with barrels of wine, which shows clearly that he does not know how to use mathematical theory in applications to problems of the physical world[5]. Moreover, he continues his discourse saying: "Thus, the only relevant semantic content of the term *valid* used in this statement must be *consistent with mathematics, but not necessarily always consistent with nature*". Next he says that "the assumed validity of the property must be proved before definition is cast" and, of course, all this naive discourse is only a prelude for his 'proof' in Section 3 of his paper of the inconsistencies in the theory of changes of variables.

To 'prove' his main thesis author first says that "in general a transformation from $\mathbb{R}^2$ to $\mathbb{R}$ of the form $u = ax + by$, where a and b are constants that do not depend on parameters is consistent with arithmetic as it represents the sum of two multiplicative tables. In the change of variables Eq.(c2.1) it is required that two independent transformations of this kind, namely

$$\begin{cases} \xi = ax + by \\ \eta = cx + dy \end{cases} \tag{c3.2}$$

be simultaneously satisfied".

From this he says that from Eq.(c3.2) (or what is the same, his Eq.(c2.1)) it follows that recalling that from Eq.(c3.2)

$$\begin{aligned} x &= \eta/c - (d/c)y, \\ y &= \eta/d - (c/d)x \end{aligned} \tag{1}$$

it follows that[6]:

$$\boxed{\xi = \frac{a}{c}\eta + (b - \frac{d}{c})y,} \tag{c3.3}$$

[4]The equations used in [1] will be numbered here by its number there with a prefix c.
[5]More on this below.
[6]Of course, the correct equations are: $\xi = \frac{a}{c}\eta + (b - \frac{ad}{c})y$ and $\xi = \frac{a}{c}\eta + (a - \frac{bc}{d})x$.

and

$$\boxed{\xi = \frac{b}{d}\eta + (a - \frac{c}{d})x.} \tag{c3.4}$$

Then, he says that *synchronization* implies that the values of the variables ξ, η, x and y do not change when we use them in Eq.(c2.1), Eq.(c3.3) or Eq.(c3.4). Next he calculates $\xi_x = a$, $\xi_y = b$, $\eta_x = c$ and $\eta_y = d$ and says that those equations "hold at any point (x, y) in Eq.(c2.1)". Next he says that "the implicit function theorem and the chain rule imply that $\xi_\eta = \xi_x x_\eta$ and $\xi_\eta = \xi_y y_\eta$ also hold everywhere in $\mathbb{R}^2$ (recall that $abcd \neq 0$). Then he says that from Eq.(c2.1) and Eq.(c3.3) it follows that $\xi_\eta = \xi_x x_\eta = a/c$ and $\xi_\eta = \xi_y y_\eta = b/d$ everywhere in $\mathbb{R}^2$. Finally he concludes:

"Hence rationality requires that we must have, say, $\xi_\eta(0,0) = a/c = b/d$ (the origin $(0,0)$ is the same in any system of coordinates) so that arithmetic implies that $ad - bc = 0$, contradicting the standing hypothesis that $ad - bc \neq 0$. Consequently, if we assume both the validity of calculus and of the change of variables Eq.(c2.1), the mathematics that follows from this assumption is contradictory when $abcd \neq 0$."

From this he establishes his theorem:

"**Theorem 3.1**. If $ad - bc \neq 0$ and $abcd \neq 0$ then the adoption of the validity of the concept of change of coordinates Eq.(c2.1) becomes inconsistent with the concept of partial derivative."

From this point Carvalho deduces using arguments similar to the ones he employed in his 'proof' of his Theorem 3.1 that several other theories of mathematics are inconsistents, e.g., he says that d'Alembert solutions of the one-dimensional wave equation is inconsistent, that coordinate transformations which represent rotations in $\mathbb{R}^2$ are inconsistent unless the rotation angle is a multiple of $n\pi/2$. With this last result, he establishes the corollary:

"**Corollary 4.2**. Complex number theory is inconsistent."

And he did not stop here, he also 'shows' in Section 5, that Lorentz transformations are mathematically inconsistent, thus implying that Special Relativity is inconsistent.

Well, my dear reader, at this point we think that if you have had a reasonable course of advanced calculus you already realized the conceptual errors of [1]. Indeed, in any reasonable course of advanced calculus the following exercise is usually given to students to verify if they grasped the main concepts involved in the implicit function theorem (and the chain rule).

Exercise 1 *Let*

$$\xi = f_1(x, y), \qquad \eta = f_2(x, y) \tag{2}$$

with $f_1, f_2 : \mathbb{R}^2 \supset U \to \mathbb{R}$ two C^1 continuous differentiable functions. Prove that the necessary and sufficient condition for the existence of a functional relation between ξ and η of the form

$$\mathbf{F}(\xi, \eta) = 0, \tag{3}$$

is that

$$\frac{\partial(\xi,\eta)}{\partial(x,y)} = 0,$$

where $\frac{\partial(\xi,\eta)}{\partial(x,y)}$ *denotes the Jacobian of the transformation given by* Eq.(2).

If you did not succeed solving the exercise, you may find a detailed solution in Chapter 6 (Exercise 35) of Spiegel's book [7], and in that case we suggest that you take the opportunity to review the basic concepts of calculus.

Here we use the result of the Exercise 1 to analyze Carvalho's Theorem 3.1.

In this case, Eq.(c2.1) or Eq.(c3.1) defines the C^1 differentiable functions f_1 and f_2. However, as the original meaning of Eq.(c2.1) is the one of a transformation of variables in $\mathbb{R}^2$ we need to impose that $\frac{\partial(\xi,\eta)}{\partial(x,y)} = ad - bc \neq 0$. Then the result of Exercise 1 says that it does not exist a functional relation $\mathbf{F}(\xi,\eta) = 0$, i.e., we cannot find a function $\mathbf{f} : \mathbb{R} \to \mathbb{R}$, $\xi \mapsto \eta = \mathbf{f}(\xi)$.

Thus under these conditions the calculations of [1] leading to $\xi_\eta = a/c$ and $\xi_\eta = b/d$ (reproduced above) are *meaningless,* for there is then nonsense in taking the derivative ξ_η.

Note that if $ad - bc = 0$ we can have according to the result of Exercise 1 a functional relation $\mathbf{F}(\xi,\eta) = 0$. Indeed if, e.g., (i) $c = \alpha a$, $d = \alpha b$, $\alpha \neq 0$ we have $\xi = (a/c)\eta$, or (ii) if $b = \beta a$, $d = \beta c$, $\beta \neq 0$ we have again $\xi = (a/c)\eta$, as it may be.

3 Relativity and [1]

The analysis presented above is, of course, enough to convince any reader that [1] is a very bad paper and should never be published in any scientific journal, in particular a Mathematics journal that is indexed in the SCI. However, we shall comment on another of Carvalho's statement. In Section 3 of his paper he recalls that in the Theory of Relativity Lorentz transformations play (as well known) a distinguished role. The nontrivial part of a special Lorentz transformation, also called a boost in the x-direction can obviously be written in the form of a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$,

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \frac{1}{\sqrt{1 - v^2/c^2}} \begin{bmatrix} 1 & -v \\ -v/c^2 & 1 \end{bmatrix} \begin{pmatrix} x \\ t \end{pmatrix}. \tag{4}$$

He says that in Eq.(4) v denotes a 'natural speed' with $0 < v^2 \leq c^2$, c denoting the speed of light. This transformation, as recalled in [1] has the same form of Eq.(2.1) with

$$a = d = \frac{1}{\sqrt{1 - v^2/c^2}},\ b = \frac{-v}{\sqrt{1 - v^2/c^2}},\ c = -\frac{v}{c^2}\frac{1}{\sqrt{1 - v^2/c^2}}. \tag{5}$$

Now, in Section 3 of [1] he calls the function b of the variable v by

$$a =: a(v) = \frac{-v}{\sqrt{1 - v^2/c^2}}. \tag{c3.1}$$

Then he states: “then a is also a speed for each v in its range”. From that statement he thought that it is necessary to impose that $a^2 < c^2$ and then concludes that Eq.(c3.1) is auto-contradictory when $c^2/2 < v^2 < c^2$. He even says that “its numerator and denominator do not agree about their mathematical and natural meanings”.

Of course, every reasonable Physics or Mathematics student that attended lectures on Relativity Theory knows that the imposition that $a^2 < c^2$ is nonsense. Although a has dimension of a velocity it is not a ‘natural speed’, and so we can have $a^2 > c^2$. As well known, what the Theory of Relativity forbids is that $v^2 > c^2$.

So, all criticisms of [1] concerning inconsistencies of Relativity Theory are not valid, for our author besides having forgotten some fundamental results of calculus, also does not know the simple *physical* meaning of the variables used in a theory of Physics[7]. To endorse our statement we ask you to go to Section 3 of [1] where you can read: “The most common idea of change (or transformation) of variables occurs in the real line $\mathbb{R}$ (arithmetic). It is of the form $y = ax$, where $a \neq 0$ is a constant, x is the “old” and y is the “new” variable. Its effect is simply a change of scale: the new unit is a times the old one.”

Well, suppose that x and y denotes the linear measure of a rod in two different units, *meter* and *centimeter*. It is clear that the number y is greater than the number x. This happens because the unit called centimeter is $1/a = 1/100$ times the unit called meter and not that the unit called centimeter is $a = 100$ greater than the unit called meter.

4 Conclusions

In this paper we analyzed the claims of [1] that several theories of Mathematics and also Relativity Theory are inconsistents. We showed that all inferences of [1] are the result of a simple fact: author of [1] does not know the implicit function theorem and, of course, is not able to solve Exercise 1. Also, it must be said that the publication of completely wrong papers containing a potpourri of nonsense Mathematics (and also Physics) is becoming more and more routine in ‘scientific’ journals, books and, of course, also in the arXiv. In our papers [2, 3, 4, 5, 6] we discussed several examples of nonsense Mathematics. Our examples,[8] we believe are enough to claim that it is arrived the time for editors to choose better referees for their journals, which at least must know advanced calculus.

[7] We left as exercise to the reader to find additional errors of Section 5 of [1].

[8] We have, of course, many and many others examples, besides the ones we quoted.

References

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