ON FACTORIZATION OF HILBERT-SCHMIDT MAPPINGS

CRISTIANE DE ANDRADE MENDES

ABSTRACT. We present some results on factorization of Hilbert-Schmidt multilinear mappings and polynomials through infinite dimensional Banach spaces, $\mathcal{L}_1$ and $\mathcal{L}_\infty$ spaces. We conclude this work with a result on factorization of holomorphic mappings of Hilbert-Schmidt type.

From the linear theory, we know that Hilbert-Schmidt operators factor through an $\mathcal{L}_1$ space and an $\mathcal{L}_\infty$ space (theorem 2.3) and also through infinite dimensional Banach spaces (theorem 2.4). The converse is also true in both cases. Our aim is to study whether it it possible or not to have similar results for non-linear Hilbert-Schmidt mappings.

This paper consists of 3 sections. In the first one, we make some comments on notation and we remind important results and definitions to be used later. In section 2, we present the definitions of Hilbert-Schmidt mappings and we remind the factorization results for Hilbert-Schmidt linear operators. We also examine two sorts of factorizations for the multilinear and polynomial cases. The last part is dedicated to the holomorphic mappings of Hilbert-Schmidt type.

1. NOTATION AND IMPORTANT RESULTS

Throughout this paper, the symbol $\mathbb{K}$ represents the fields of real numbers and complex ones. The set of all positive integers is denoted by $\mathbb{N}$ and $\mathbb{N}_o = \mathbb{N} \cup \{0\}$. $E, E_1, ..., E_n, F$ always represent Banach spaces and $H, H_1, ..., H_n, G$, Hilbert spaces over $\mathbb{K}$. B_E represents the closed unit ball of the space E. $\mathcal{L}(E_1, ..., E_n; F)$ denotes the space of the n-linear continuous mappings from $E_1 \times ... \times E_n$ into F. If $E_1 = ... = E_n = E$, we write $\mathcal{L}(^nE; F)$. The space of the n-homogeneous continuous polynomials from E into F is denoted by $\mathcal{P}(^nE; F)$. If $T \in \mathcal{L}(^nE; F)$, we write $\hat{T} \in \mathcal{P}(^nE; F)$ for the corresponding polynomial. $\check{P} \in \mathcal{L}(^nE; F)$ indicates the symmetric n-linear mapping which corresponds to $P \in \mathcal{P}(^nE; F)$ (see [12], 1.10).

For the holomorphic mappings, $\mathcal{H}(U; F)$ denotes the space of all holomorphic mappings from $U \subset E$ (a non-void open subset of E) into F (E and F are complex spaces). The n-th derivative of f in $x \in U$ is represented by $d^n f(x) \in \mathcal{L}(^nE; F)$ and the corresponding polynomial, by $\hat{d}^n f(x) \in \mathcal{P}(^nE; F)$.

The space of all sequences $(x_n)_n$ in E such that $\| (x_n)_n \|_p := \left(\sum_{n \in \mathbb{N}} \| x_n \|^p \right)^{\frac{1}{p}} < \infty$ is denoted by $l_p(E)$. $l_{p,w}(E)$ denotes the space of all sequences $(x_n)_n$ in E such that $(\langle x', x_n \rangle)_n$ is a sequence in $l_p = l_p(\mathbb{K})$ for all $x' \in E'$. A norm (p-norm if $p < 1$) is defined by $\| (x_n) \|_{p,w} := \sup_{x' \in B_{E'}} \left(\sum_{j=1}^{\infty} | x'(x_j) |^p \right)^{\frac{1}{p}}$.

The definition of absolutely summing n-linear functionals is due to Pietsch [15]. In [1], Alencar and Matos have presented a definition for vector-valued mappings.

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Definition 1.1. *For $r, s_1, ..., s_n \in (0, +\infty]$, with $\frac{1}{r} \leq \frac{1}{s_1} + ... + \frac{1}{s_n}$, a mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ is absolutely $(r; s_1, ..., s_n)$ summing if there is a constant $C \geq 0$ such that*

$$\left(\sum_{i=1}^{m} \| (T(x_i^1, ..., x_i^n))_{i=1}^m \|^r \right)^{\frac{1}{r}} \leq C \prod_{k=1}^{n} \| (x_i^k)_{i=1}^m \|_{s_k, w}$$

for every $m \in \mathbb{N}$, $k = 1, ..., n$ and $i = 1, ..., m$. The vector space of these mappings is indicated by $\mathcal{L}_{as,(r;s_1,...,s_n)}(E_1, ..., E_n; F)$ and the smallest C satisfying the inequality above, by $\| T \|_{as,(r;s_1,...,s_n)}$. This defines a norm (r-norm if $r < 1$) on $\mathcal{L}_{as,(r;s_1,...,s_n)}(E_1, ..., E_n; F)$. If $s_1 = ... = s_n = s$, we indicate $as, (r; s)$ in the place of $as, (r; s_1, ..., s_n)$ and if $r = s$, we just write as, r.

It can be shown that $T \in \mathcal{L}_{as,(r;s_1,...,s_n)}$ if and only if $\left(T(x_j^1, ..., x_j^n)\right)_{j=1}^{\infty} \in l_r(F)$ whenever $(x_j^k)_{j=1}^{\infty} \in l_{r_k,w}(E_k)$, $k = 1, ..., n$.

Pérez-García has proved the following theorem for multilinear mappings [14].

Theorem 1.2. *If E_j is an $\mathcal{L}_{\infty,\lambda_j}$ space, then $\mathcal{L}(E_1, ..., E_n; \mathbb{K}) = \mathcal{L}_{as,(1;2)}(E_1, ..., E_n; \mathbb{K})$ and $\| T \|_{as,(1;2)} \leq K_{G,n} \prod_{j=1}^{n} \lambda_j \| T \|$, for all $T \in \mathcal{L}(E_1, ..., E_n; \mathbb{K})$.*

In [10], Matos has introduced a more restrictive concept for multilinear mappings.

Definition 1.3. *For $r, s_1, ..., s_n \in (0; +\infty]$, with $r \geq s_k$, $k = 1, ..., n$, a mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ is fully absolutely $(r; s_1, ..., s_n)$-summing if there is a constant $C \geq 0$ such that*

$$\left(\sum_{j_1,...,j_n=1}^{m} \| T(x_{j_1}^1, ..., x_{j_n}^n) \|^p \right)^{\frac{1}{p}} \leq C \prod_{k=1}^{n} \| (x_j^k)_{j=1}^m \|_{s_k, w}$$

for every $m \in \mathbb{N}$, $x_j^k \in E_k$, $k = 1, ..., n$ and $j = 1, ..., m$. $\mathcal{L}_{fas,(r;s_1,...,s_n)}(E_1, ..., E_n; F)$ denotes the space of such mappings and $\| T \|_{fas,(r;s_1,...,s_n)}$, the smallest C which satisfies the inequality above. This is a norm (r-norm, if $r < 1$) for the space $\mathcal{L}_{fas,(r;s_1,...,s_n)}(E_1, ..., E_n; F)$. If $s_1 = ... = s_n = s$, we indicate $fas, (r; s)$ in the place of $fas, (r; s_1, ..., s_n)$ and if $r = s$, we just write fas, r.

An important result on fully absolutely summing mappings, due to Bombal, Pérez-García and Villanueva [2], is the following

Theorem 1.4. *If E_j is an $\mathcal{L}_{\infty,\lambda_j}$ space, $1 \leq j \leq n$ and H a Hilbert space, then $\mathcal{L}(E_1, ..., E_n; H) = \mathcal{L}_{fas,2}(E_1, ..., E_n; H)$ and $\| T \|_{fas,2} \leq k_n \prod_{j=1}^{n} \lambda_j \| T \|$ where $k_n = (B_4)^{2n}$ and B_4 is a Khinchin's inequality constant (see [5], 1.10).*

2. Hilbert-Schmidt multilinear mappings and polynomials

The definition of Hilbert-Schmidt m-functionals is due to Dwyer [7]. Those mappings were also studied by Matos in [9] and [10] for the vector-valued case.

Definition 2.1. *A multilinear mapping $T \in \mathcal{L}(H_1, ..., H_n; G)$ is a* Hilbert-Schmidt *mapping if, for each $k = 1, ..., n$, there exist an orthonormal basis $(h_{j_k}^k)_{j_k \in J_k}$ of H_k such that $(\sum_{j_1,...,j_n} \|T(h_{j_1}^1, ..., h_{j_n}^n)\|^2)^{\frac{1}{2}} < +\infty$. We denote by $\mathcal{L}_{HS}(H_1, ..., H_n; G)$ the space of such mappings. This is a Hilbert space with the following inner product $(T \mid S) = \sum_{j_1,...,j_n} (T(h_{j_1}^1, ..., h_{j_n}^n) \mid S(h_{j_1}^1, ..., h_{j_n}^n))$ $(T, S \in \mathcal{L}_{HS}(H_1, ..., H_n; G))$. The corresponding norm is denoted by $\| T \|_{HS}$.*

We can prove that, if $T \in \mathcal{L}_{HS}(H_1, ..., H_n; G)$, then $(\sum_{j_1,...,j_n} \|T(h_{j_1}^1, ..., h_{j_n}^n)\|^2)^{\frac{1}{2}} < +\infty$ for all orthonormal bases of $H_1, ..., H_n$.

Definition 2.2. *A polynomial* $P \in \mathcal{P}(^nH;G)$ *is a Hilbert-Schmidt polynomial if* $\check{P} \in \mathcal{L}_{HS}(^nH;G)$. *The space of such polynomials is indicated by* $\mathcal{P}_{HS}(^nH;G)$ *and a norm is defined by* $\| P \|_{HS} = \| \check{P} \|_{HS}$.

We present now the results on factorization of Hilbert-Schmidt operators.

Theorem 2.3. *(Lindenstrauss-Pelczynski,* [8]*) Let* $u \in \mathcal{L}(H;G)$. *The following are equivalent:*
(i) $u \in \mathcal{L}_{HS}(H;G)$.
(ii) u factors through an $\mathcal{L}_\infty$ *space.*
(iii) u factors through an $\mathcal{L}_1$ *space.*

A more recent result was proved by Diestel, Jarchow and Tonge (see [5], 19.2).

Theorem 2.4. $u \in \mathcal{L}_{HS}(H;G)$ *if and only if, for any infinite dimensional Banach space* Z, *there are operators* $v \in \mathcal{L}(Z;G)$ *and* $w \in \mathcal{L}(H;Z)$ *such that* $u = v \circ w$. *Moreover, we can choose* w *to be compact and* v, *compact and 2-summing.*

There is an important relationship between Hilbert-Schmidt multilinear mappings and fully absolutely summing multilinear mappings.

Proposition 2.5. *(Matos,* [10]*) If* $p \in [2,\infty)$, *then* $\mathcal{L}_{HS}(H_1,...,H_n;G) = \mathcal{L}_{fas,p}(H_1,...,H_n;G)$ *and there are constants* $b_p > 0$ *and* $d_p > 0$ *such that* $(d_p)^n \| T \|_{fas,p} \leq \| T \|_{HS} \leq (b_p)^n \| T \|_{fas,p}$ *for all* $T \in \mathcal{L}_{fas,p}(H_1,...,H_n;G)$.

The first factorization result is the following

Theorem 2.6. *Let* $T \in \mathcal{L}(H_1,...,H_n;G)$. *If there exist an* $\mathcal{L}_\infty$ *space* X_j, $S_j \in \mathcal{L}(H_j;X_j)$, $j = 1,...,n$ *and* $R \in \mathcal{L}(X_1,...,X_n;G)$ *such that* $T = R \circ (S_1,...,S_n)$, *then* $T \in \mathcal{L}_{HS}(H_1,...,H_n;G)$. *The converse is not true in general.*

Proof. We have $R \in \mathcal{L}_{fas,2}(X_1,...,X_n;G)$ (theorem 1.4) and consequently, $T = R \circ (S_1,...,S_n) \in \mathcal{L}_{fas,2}(H_1,...,H_n;G) = \mathcal{L}_{HS}(H_1,...,H_n;G)$ (proposition 2.7).

The converse is not true. The mapping $T \in \mathcal{L}(^2l_2;\mathbb{K})$ given by $T(x^1,x^2) = \sum_{j\in\mathbb{N}} \frac{1}{j} x^1_j x^2_j$ is a Hilbert-Schmidt mapping. If the factorization was possible, say $T = R \circ (S_1,S_2)$, we would have $R \in \mathcal{L}_{as,(1,2)}(X_1,X_2;\mathbb{K})$ (theorem 1.2) and consequently, $T \in \mathcal{L}_{as,(1,2)}(^2l_2;\mathbb{K})$ which is not true, because $\sum_{j\in\mathbb{N}} | T(e_j,e_j) | = \sum_{j\in\mathbb{N}} \frac{1}{j}$. □

Corollary 2.7. *Let* $P \in \mathcal{P}(^nH;G)$. *If there exist an* $\mathcal{L}_\infty$ *space,* $S \in \mathcal{L}(H;X)$ *and* $Q \in \mathcal{P}(^nX;G)$ *such that* $P = Q \circ S$, *then* $P \in \mathcal{P}_{HS}(^nH;G)$. *The converse is not true in general.*

The same is true if we consider the factorization through an $\mathcal{L}_1$ space. The proof of theorem 2.6 is the same, using the analogous of theorem 1.4 for $\mathcal{L}_1$ spaces [2]. In this case, we have no answer about the converse.

There is a class of multilinear mappings and polynomials, formed by the Schatten class type $\mathcal{S}_2$ mappings ([3] and [4]), for which it is possible to prove also the converse of theorem 2.6 and corollary 2.7, not only for the $\mathcal{L}_\infty$ and $\mathcal{L}_1$ spaces, but also for factorization through infinite dimensional Banach spaces (see [11]).

As the first form of factorization did not work as well as we would like it to do, we will study another sort of factorization. Before we announce the next result, we need the following definition and lemma.

Definition 2.8. *Let* $T \in \mathcal{L}(E_1,...,E_n;F)$. *The* adjoint operator $T' \in \mathcal{L}(F';\mathcal{L}(E_1,...,E_n;\mathbb{K}))$ *is defined by* $T'(\varphi)(x_1,...,x_n) = \varphi(T(x_1,...,x_n))$, *for all* $\varphi \in F'$ *and* $x_j \in E_j$, $j = 1,...,n$.

Lemma 2.9. *If $T \in \mathcal{L}_{HS}(H_1, ..., H_n; G)$, then $T'' \in \mathcal{L}_{HS}\left(\mathcal{L}_{HS}(H_1, ..., H_n; \mathbb{K})'; G\right)$.*

Proof. First, observe that if $T \in \mathcal{L}_{HS}(H_1, ..., H_n; G)$, then $T'g' \in \mathcal{L}_{HS}(H_1, ..., H_n; \mathbb{K})$ for all $g' \in G'$.

If $(h^i_{j_i})_{j_i \in J_i}$ is an orthonormal basis for H_i, $i = 1, ..., n$ and if $L \in \mathcal{L}(H_1, ..., H_n; H')$ is given by $L(h_1, ..., h_n)(u) = u(h_1, ..., h_n)$, for all $u \in H = \mathcal{L}(H_1, ..., H_n; \mathbb{K})$, then $\left(L(h^1_{j_1}, ..., h^n_{j_n})\right)_{j_i \in J_i, i=1,...,n}$ is an orthonormal basis for H' and $T''(L(h^1_{j_1}, ..., h^n_{j_n})) = J(T(h^1_{j_1}, ..., h^n_{j_n}))$, where $J : G \hookrightarrow G''$ is the canonical inclusion. Therefore

$$\sum_{j_1,...,j_n} \| T''(\varphi_{j_1,...,j_n}) \|^2 = \sum_{j_1,...,j_n} \| J(T(h^1_{j_1}, ..., h^n_{j_n})) \|^2$$
$$= \sum_{j_1,...,j_n} \| T(h^1_{j_1}, ..., h^n_{j_n}) \|^2 < +\infty$$

□

Theorem 2.10. *Let $T \in \mathcal{L}_{HS}(H_1, ..., H_n; G)$. Then, for all infinite dimensional Banach space Z, we have $T = V \circ S$, where $S \in \mathcal{L}(H_1, ..., H_n; Z)$ and $V \in \mathcal{L}(Z; G)$.*

In addition, we may choose V in such a way that it is compact and 2-summing.

Proof. By the lemma, we have $T'' \in \mathcal{L}_{HS}(H'; G)$, $H = \mathcal{L}_{HS}(H_1, ..., H_n; \mathbb{K})$. Using the Diestel-Jarchow-Tonge result (theorem 2.4), there exist $w \in \mathcal{L}(H; Z)$ and $v \in \mathcal{L}(Z; G)$ such that $T'' = v \circ w$. v and w may be chosen compact and v, 2-summing.

If $L \in \mathcal{L}(H_1, ..., H_n; H)$ is the n-linear mapping defined on the lemma, write $S = w \circ L$ and $V = v$. Then $V \circ S = J \circ T \equiv T$. □

Corollary 2.11. *Let $P \in \mathcal{P}_{HS}(^nH; G)$. Then, for all infinite dimensional Banach space Z, we have $P = V \circ Q$, where $Q \in \mathcal{L}(^nH; Z)$ and $V \in \mathcal{L}(Z; G)$.*

Proof. If $P \in \mathcal{P}_{HS}(^nH; G)$, then $\check{P} = V \circ S$ as in 2.10. Therefore, $P = V \circ \hat{S}$. □

As a consequence, we can say that Hilbert-Schmidt multilinear mappings and polynomials factors through an $\mathcal{L}_1$ or $\mathcal{L}_\infty$ space.

The converse is not true in general as the following example show.

Example 2.12. *$T \in (^2l_2; \mathbb{K})$, $T(x^1, x^2) = \sum_{j=1}^{\infty} \frac{1}{\sqrt{j}} x^1_j x^2_j$. Let Z be a Banach space, $Z \neq \{0\}$, and $b \in Z$, $b \neq 0$. We define $S : l_2 \times l_2 \to Z$, $S(x^1, x^2) = T(x^1, x^2)b$. If $Y = [b]$ and $V : Y \to \mathbb{K}$ is given by $V(\alpha b) = \alpha$, $\alpha \in \mathbb{K}$, using the Hahn-Banach theorem, there exists $\tilde{V} \in Z'$ such that $\tilde{V} \mid Y = V$. It is not difficult to see that $\tilde{V} \circ S = T$. On the other hand, $T \notin \mathcal{L}_{HS}(^2l_2; \mathbb{K})$.*

We can prove that if $T \in \mathcal{L}(H_1, ..., H_n; G)$ factors as in theorem 2.10, then T is strongly 2-summing (Dimant's definition - see [6]). It is an open problem to decide whether strongly 2-summing multilinear mappings defined on Hilbert spaces can be decomposed as in the theorem.

Remark 2.13. *Some comments must be done about the norms of the operators involved on the proof of theorem 2.10 and its corollary.*

Let $u \in \mathcal{L}_{HS}(H; G)$, $u(h) = \sum_{n=1}^{\infty} \tau_n (h \mid h_n) g_n$, where $(\tau_n)_n \in l_2$ and $(h_n)_n$, $(g_n)_n$ are orthonormal sequences in H and G, respectively.

For each $n \in \mathbb{N}$, we can write $\tau_n = \alpha_n \sigma_n \beta_n$, where $\sigma = (\sigma_n)_n \in l_2$, $\alpha = (\alpha_n)_n \in c_o$ and $\beta = (\beta_n)_n \in c_o$. Examining the proof of Diestel-Jarchow-Tonge factorization theorem 2.4 , w and v can be chosen in such a way that

$$\| w \| \leq 8 \| \beta \|^{\frac{1}{4}} \| u \|_{HS}^{\frac{1}{4}}$$

$$\| v \|_{as,2} \leq 12C \| \alpha \|_{\infty} \| \beta \|_{\infty}^{\frac{1}{4}} \| \sigma \|_2 \| u \|_{HS}^{\frac{1}{4}}$$

(see [5], 19.2) where $1 \leq C \leq 4$ *is a constant that depends on* Z.

Let $u = T''$ *(theorem 2.10).* $V \in \mathcal{L}(Z; G)$ *and* $S \in \mathcal{L}(H_1, ..., H_n; Z)$ *can be chosen such as*

$$\| S \| \leq \| w \| \leq 8 \| \beta \|_{\infty}^{\frac{1}{4}} \| T \|_{HS}^{\frac{1}{4}}$$

$$\| V \|_{as,2} \leq 12C \| \alpha \|_{\infty} \| \beta \|^{\frac{1}{4}} \| \sigma \|_2 \| T \|_{HS}^{\frac{1}{4}}.$$

In corollary 2.11, V *and* Q *can be chosen such as*

$$\| Q \| \leq \| S \| \leq 8 \| \beta \|_{\infty}^{\frac{1}{4}} \| P \|_{HS}^{\frac{1}{4}}$$

$$\| V \|_{as,2} \leq 12C \| \alpha \|_{\infty} \| \beta \|^{\frac{1}{4}} \| \sigma \|_2 \| P \|_{HS}^{\frac{1}{4}}.$$

The choice made above will be of great importance for the holomorphic mappings factorization result.

3. Hilbert-Schmidt holomorphic mappings

In this section, we will study the Hilbert-Schmidt holomorphic mappings. Our aim is to prove a factorization result similar to the one we have proved for polynomials (corollary 2.11). All the spaces considered in this section will be complex ones.

The Hilbert-Schmidt holomorphic mappings has been also studied by Dwyer [7].

Definition 3.1. *Let* $U \subset H$ *be a non-void open subset of* H *and* $f \in \mathcal{H}(U; G)$. f *is a Hilbert-Schmidt mapping in* $h \in U$ *if the following conditions hold true*
(1) $\hat{d}^m f(h) \in \mathcal{P}_{HS}(^m H; G)$ *for all* $m \in \mathbb{N}_o$ *and*
(2) there exist real numbers $C \geq 0$ *and* $c \geq 0$ *such that* $\| \frac{1}{m!} \hat{d}^m f(h) \|_{HS} \leq C c^m$ *for all* $m \in \mathbb{N}_o$.

If f *is a Hilbert-Schmidt mapping in all* $h \in U$*, we say that* f *is a Hilbert-Schmidt mapping on* U*. We denote the class of such mappings as* $\mathcal{H}_{HS}(U; G)$.

We say that $f \in \mathcal{H}(H; G)$ *is a Hilbert-Schmidt mapping of bounded type if* f *is a Hilbert-Schmidt mapping in* H *and* $\lim_{m \to \infty} \left(\frac{1}{m!} \| \hat{d}^m f(0) \|_{HS} \right)^{\frac{1}{m}} = 0$*. We indicate the class of such mappings by* $\mathcal{H}_{Hb}(H; G)$.

The factorization result is the following

Theorem 3.2. *Let* $f \in \mathcal{H}(U; G)$*, where* $U \subset H$ *is a non-void open subset of* H*. Suppose that* f *is a Hilbert-Schmidt mapping in* $h_o \in U$*. Then, there exist a neighborhood* U_o *of* h_o *in* U*, an* $\mathcal{L}_\infty$ *space* X*, an holomorphic mapping* $g \in \mathcal{H}(U_o; X)$ *and an operator* $V \in \mathcal{L}(X; G)$ *such that* $f = V \circ g$ *in* U_o.

Before we show the proof of 3.2, we will prove the following lemma

Lemma 3.3. *Given* $n \in \mathbb{N}$, $\delta > 0$ *and* $\tau = (\tau_s)_s \in l_2$*, there exist* $\gamma = (\gamma_s)_s \in c_o$ *and* $\sigma = (\sigma_s)_s \in l_2$ *such that* $\| \gamma \|_\infty = \dfrac{1}{n^{8+\delta}}$, $\| \sigma \|_2 \leq A$, $A > 0$ *does not depend on the choice of* $n \in \mathbb{N}$ *and* $\tau_s = \gamma_s \sigma_s$ *for each* $s \in \mathbb{N}$.

Proof. With no loss of generality, we will suppose that $\| \tau \|_2 = 1$ and $\tau_s \geq 0$ for all $s \in \mathbb{N}$.Write $N_o = 0$. We can inductively define a sequence of positive integers $N_1 < N_2 < ... < N_k < ...$ such that, for each $k \in \mathbb{N}$, $\tau_1^2 + ... + \tau_{N_k}^2 \geq \dfrac{2^{n+k} - 1}{2^{n+k}}$ and N_k is the smallest positive integer with this property. Then, we write

$$\gamma_{N_{k-1}+1} = ... = \gamma_{N_k} = \frac{1}{(n+k-1)^{8+\delta}}$$
$$\sigma_{N_{k-1}+1} = (n+k-1)^{8+\delta}\tau_{N_{k-1}+1} \ , \ ... \ , \sigma_{N_k} = (n+k-1)^{8+\delta}\tau_{N_k}.$$

In this way, we can define the two sequences $\gamma = (\gamma_s)_s \in c_o$ with $\| \gamma \|_\infty = \frac{1}{n^{8+\delta}}$ and $\sigma = (\sigma_s)_s$ such that $\sigma_s\gamma_s = \tau_s$. We have to verify that $\sigma \in l_2$.

$$\sum_{s=1}^{\infty}\sigma_s^2 = \sum_{k=1}^{\infty} \sum_{s=N_{k-1}+1}^{N_k} (n+k-1)^{2(8+\delta)}\tau_s^2 \leq \sum_{k=1}^{\infty}(n+k-1)^{2(8+\delta)} \sum_{s=N_{k-1}+1}^{\infty} \tau_s^2$$

$$\leq \sum_{k=1}^{\infty}(n+k-1)^{2(8+\delta)} \left(1 - \frac{2^{n+k-1}-1}{2^{n+k-1}}\right) = \sum_{k=1}^{\infty} \frac{(n+k-1)^{2(8+\delta)}}{2^{n+k-1}}$$

$$= \sum_{l=n+1}^{\infty} \frac{(l-1)^{2(8+\delta)}}{2^{l-1}} \leq \sum_{l=1}^{\infty} \frac{(l-1)^{2(8+\delta)}}{2^{l-1}}$$

Using the ratio test, we can prove that $\sum_{l=1}^{\infty} \frac{(l-1)^{2(8+\delta)}}{2^{l-1}}$ is a convergent series.

□

Proof. (theorem 3.2) We suppose initially that $f(h_o) = 0$. We write $P_n = \frac{1}{n!}\hat{d}^n f(h_o) \in \mathcal{P}_{HS}(^nH;G)$, $n \in \mathbb{N}$. There exist $C \geq 0$ and $c \geq 0$ such that $\| P_n \|_{HS} \leq Cc^n$ for all $n \in \mathbb{N}$. Using corollary 2.11 with $Z = l_\infty$, we have $P_n = V_n \circ Q_n$, where $Q_n \in \mathcal{P}(^nH;l_\infty)$, $V_n \in \mathcal{L}_{as,2}(l_\infty;G)$. They can be chosen such that (see comments 2.13 and lemma 3.3) $\| Q_n \| \leq 8\frac{1}{n^{1+\frac{\delta}{8}}} \| P_n \|_{HS}^{\frac{1}{4}}$ and $\| V \|_{as,2} \leq 48A\frac{1}{n^{5+\frac{5\delta}{8}}} \| P_n \|_{HS}^{\frac{1}{4}}$, where $A > 0$ does not depend on $n \in \mathbb{N}$ (use $\alpha_s = \beta_s = (\gamma_s)^{\frac{1}{2}}$ for all $s \in \mathbb{N}$ - the same notation of lemma 3.3 and comments 2.13).

Let $\epsilon > 0$ be such that $\epsilon c^{\frac{1}{4}} < 1$. Write $R_n = \epsilon^{-n}Q_n \in \mathcal{P}(^nH;l_\infty)$, $v_n = \epsilon^n V_n \in \mathcal{L}(l_\infty;G)$, $X = l_\infty(l_\infty)$ (an $\mathcal{L}_\infty$ space), $i_n : l_\infty \to X$ the inclusion in the n-th coordinate and $\pi_n : X \to l_\infty$ the projection on the n-th coordinate. We have $v_n \circ R_n = P_n$ for all $n \in \mathbb{N}$. Also $\| R_n \|^{\frac{1}{n}} \leq \epsilon^{-1} \| Q_n \|^{\frac{1}{n}} \leq \epsilon^{-1}\left(\frac{8}{n^{1+\frac{\delta}{8}}}\right)^{\frac{1}{n}} C^{\frac{1}{4n}}c^{\frac{1}{4}}$ for each $n \in \mathbb{N}$ and $\limsup_{n\to\infty} \| R_n \|^{\frac{1}{n}} \leq \epsilon^{-1}c^{\frac{1}{4}}$. Therefore, the mapping $g(h) = \sum_{n=1}^{\infty} i_n \circ R_n(h - h_o)$ is holomorphic in some neighborhood U_o of $h_o \in U$.

Observe that, for each $n \in \mathbb{N}$,

$$\| v_n \circ \pi_n \| \leq \frac{48AC^{\frac{1}{4}}}{n^{5+\frac{5\delta}{8}}}.$$

By the comparison test, we have that $\sum_n \| v_n \circ \pi_n \|$ is convergent. So, we define $v \in \mathcal{L}(X;G)$, $vx = \sum_n (v_n \circ \pi_n)(x)$. For all $h \in U_o$, we have $(v \circ g)(h) = f(h)$.

Now we work with the case $f(h_o) \neq 0$. Define $f_1 \in \mathcal{H}(U;G)$, $f_1(h) = f(h) - f(h_o)$. Using the first part of the proof for f_1, there exist $v^1 \in \mathcal{L}(X_1;G)$ (X_1 is an $\mathcal{L}_\infty$ space) and $g_1 \in \mathcal{H}(U_1;X_1)$ such that $f_1 = v^1 \circ g_1$ in U_1 (a neighborhood of $h_o \in U$). We call $X = \mathbb{C} \times X_1$ and $p_1 : X \to \mathbb{C}$, $p_2 : X \to x_1$ the corresponding projections. Define

$$g_1 : U_1 \to X, \qquad g(h) = (1, g_1(h))$$
$$v : X \to G, \qquad v(x) = f(h_o)p_1(x) + v^1 \circ p_2(x).$$

$g \in \mathcal{H}(U_1;X)$, $v \in \mathcal{L}(X;G)$ and $v \circ g = f$ in U_1. □

The same proof can be done for $\mathcal{L}_1$ spaces. Use l_1 in the place of l_∞ and define $X = l_1(l_1)$.

As a consequence, we have

Corollary 3.4. *If $f \in \mathcal{H}_{Hb}(H; G)$, then there exist an $\mathcal{L}_\infty$ space (or an $\mathcal{L}_1$ space) X, an holomorphic mapping $g \in \mathcal{H}(H; X)$ and an operator $V \in \mathcal{L}(X; G)$ such that $f = V \circ g$.*

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IMECC, UNICAMP, Caixa Postal 6065, Campinas, SP, 13081 970 (Brazil)
E-mail address: camendes@ime.unicamp.br; camendes1@zipmail.com.br