The non sequitur mathematics and physics of the "New Electrodynamics" proposed by the *AIAS* group*

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Abstract

We show that the *AIAS* group collection of papers on a "new electrodynamics" recently published in the *Journal of New Energy*, as well as other papers signed by that group (and also other authors) appearing in other established physical journals and in many books published by leading international publishers (see references) are full of misconceptions and misunderstandings concerning the theory of the electromagnetic field and contain fatal mathematical flaws, which invalidates almost all claims done by the authors. We prove our statement by employing a modern presentation of Maxwell Theory using Clifford bundles and also develop the basic ideas of gauge theories using principal and associated vector bundles

1 Introduction

A group of 15 physicists (see footnote 64), hereafter called the *AIAS* group, signed a series of 60 papers published in a special issue of the journal, *J. New Energy*[0] (*JNE*) with the title: "The New Maxwell Electrodynamic Equations" and subtitle: "New Tools for New Technologies". Here we mainly review the first paper of the series, named "On the Representation of the Electromagnetic Field in Terms of Two Whittaker Scalar Potentials", hereafter called $AIAS\mathbf{1}$, but we also present comments on other papers of the series that pretends to have created a new electrodynamics which is a gauge theory based on the $O(3)$ group.

Before presenting the main claims of the *AIAS* group which we will criticize it is important to know the following. If the material concerning the "new electrodynamics" were published only in the *JNE* we probably would never have had contact with it. However, almost all the material of that papers appeared in one form or another in established and traditional physical journals[13−17,34] and in several books[4,66−70] published by leading international publishing houses.

It happens that on May, 1999, one of the present authors (W.A.R.) was asked by the editor of the journal *Found. of Physics* to referee the first three papers published in[0]. Of course, the papers were rejected, the reason being that these publications can be categorized as a collection of mathematical sophisms[71], i.e., are full of *nonsense* mathematics.

We felt that something must start to be doing in order to denounce this state of affairs to the public[1] and to stop the proliferation of mathematical nonsense in scientific journals.

The first version of *AIAS papers* was signed by 19 people and Professor J. P. Vigier was not one of the authors. The other people that 'signed' the first version of the manuscripts (*MSs*) and did not signed the version of that papers published in[0] are: D. Leporini, J. K. Moscicki, H. Munera, E. Recami and D. Roscoe. These names are explicitly quoted here because we are not sure that they knew or even agreed with Evans (the leader of the *AIAS* group) in participating as authors of that papers[2], although the situation is very confused. Indeed, some of the people mentioned above signed other papers as members the *AIAS* group which have been published in several different journals[13−17][3].

All these facts show that there are ethical problems at issue in this whole affair and they are in our opinion more serious than it appears at a first sight, deserving by themselves a whole discussion. However we will not consider this enterprise here, and simply concentrate ourselves in analyzing the mathematics

[1] The present paper is based on a referee's report written for *Found. Physics*, under request of Professor A. van der Merwe, the editor of that journal. We emphasize here that Professor van der Merwe has been authorized to inform the *AIAS* group who wrote the report, but according to him he didn't. Also, the contents of the present paper has been presented in an invited lecture given by W.A.R. at the meeting of the Natural Philosophy Alliance entitled: *An Introduction to 21st Century Physics and Cosmology*, hold at the University of Connecticut, Storrs, CT, June 5-9, 2000. Dr. Hal Fox, the editor of the *JNE* announced by June,1999 in the internet site of his journal that he intended to publish a series of papers siged by the *AIAS* group. He has been discretly advised by W.A.R. that publication of that material could damage for ever the reputation of the *JNE*. Dr. Fox did not follow the advice and published that papers. After attending W.A.R. presentation at Storrs, he invited us to publish our criticisms in his journal, but we decline to to that, since we do not want our names to be in any way associated with that periodic. However, since all this affair is an important one, from several points of view, we decide to publish our criticisms in *ROSE*, with the hope that it will be read by physicists and other scientists interested in mantaining science in the highest possible level.

[2] At the meeting of the Natural Philosophy Alliance quoted in footnote 2, Dr. Munera was present. He confirmed to the public attending W.A.R. lecture at that meeting that his name has been used withouth his consent in some publications of the *AIAS* group.

[3] These papers—despite appearing in established physical journals and in books published by traditional publishing houses—are like the ones published in[0], i.e., are full of mathematical sophisms. This is an indication of the *low* level of significant part of the present scientific literature. We will elaborate more on this issue on another paper. We quote also here that while preparing the review for the *Found. Phys.*, W.A.R. received a new "improved version" of the *MSs*. There, some (but not all) of the absurdities of the papers published in[0](that indeed correspond to the first version of the *MSs* received for review) have been deleted, but unfortunately the papers continued a *pot-pourri* of nonsense. More important is to register here that three authors 'decided' not to sign the 'improved' version of the *MSs*. Eventually they realized in due time that they would compromise their careers if the physics or mathematics community would know about their participation in that papers.

behind some of the main claims of $AIAS\mathbf{1}$[4]. These are

(i) "The contemporary view that classical electromagnetism is a $U(1)$ gauge theory relies on the restricted received view of transverse plane waves, $U(1)$ being isomorphic with $O(2)$ (sic)[5], the group of rotations in a plane."

(ii) "If there are longitudinal components available from the Heaviside-Maxwell equations (ME)[6], then, these cannot be represented by a $U(1)$ gauge theory."

(iii) That Whittaker [1,2] proved nearly a hundred years ago "that longitudinal standing waves exist in the vacuum from the most general possible solutions of the D'Alembert wave equation."

(iv) That "Jackson's well known demonstration of longitudinal waves also illustrates that the group $O(2)$ in gauge theory must be replaced by the group $O(3)$, that of rotations in three space dimensions, the covering group[7] of $SU(2)$ (sic)."

(v) That (i)-(iii) "leads in turn to the fact that classical electromagnetism is according to gauge theory a Yangs-Mills theory, with an internal gauge space that is a vector space, rather than a scalar space as in the received ME."

(vi) *AIAS* authors quote that recently a theory of electrodynamics[3−14] (see also[15−17]) has been proposed based on a physical $O(3)$ gauge space, which reduces to the $U(1)$ counterpart under certain circumstances, together with a novel phase free field $\vec{B}^{(3)}$. The quotation of so many papers has as obvious purpose to suggest to the reader that such a new theory is well founded. Unfortunately, this is not the case, as we show in detail below.

To show their claims, *AIAS* authors review in section 2 the work of Whittaker[1,2] and say that they reviewed the work[8] of Jackson[18]. In section 3 they claim to have developed a theory where a "symmetry breaking of $SU(2)$ to $O(3)$ with the Higgs field gives a view of electromagnetism similar to Whittakers' in terms of two scalar potentials."

In what follows we show[9]:

(a) That claims (i, ii, v) are *wrong*.

(b_1) We are not going to comment on (iv) because everybody can easily realize that this quotation is simply misleading concerning the problem at issue.

Concerned (iii) we make some comments (for future reference) on Whittaker's proof in[1] that from D'Alembert equation it follows that there are lon-

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[4] For other important criticisms concerning $\vec{B}^{(3)}$ theory as originally formulated by Evans, see[58,59,81−83] and references therein.

[5] Of course, as it is well known by any competent physicist $U(1)$ is isomorphic with $SO(2)$, not $O(2)$. In fact, any student with a middle level of knowledge in topology knows that $SO(2)$ is connected as a topological space while $O(2)$ is not. The determinant function $\det : O(2) \to R$ is continuous and send $O(2)$ to the set $\{-1, 1\} \subset R$ which is not connected in R.

[6] Hereafter denoted ME.

[7] Of course, any competent physicist knows that $SU(2)$ is the covering group of $SO(3)$ and not that $O(3)$ is the covering group of $SU(2)$ as stated in the *AIAS* papers. This is only a small example of the fact that *AIAS* authors do not know elementary mathematics.

[8] They did not.

[9] There are also some errors in $AIAS\mathbf{1}$, as e.g., the approximations given eq.(9) that in general are not done by freshman calculus students, at least at our universities.

gitudinal standing waves in the vacuum.

(b_2) That there is a proof that *ME* possess exact solutions corresponding to electromagnetic fields configurations (*EFC*) in vacuum that can move with arbitrary speeds[10] $0 \leq v < \infty$ and that this fact is well known as documented in[19−22] (see also [23,25]). These papers have not been quoted by the *AIAS* authors, although there is proof[11] that at least 2 of the 16 authors of the first version of *AIAS***1** knew references[19,21] very well! This point is *important,* because in [19,21] it is shown that, in general, an *EFC* moving with speeds $0 \leq v < 1$ or $v > 1$ have *longitudinal components* of the electric and/or of the magnetic fields.

(b_3) That Whittakers' presentation of electromagnetism in terms of two purely "longitudinal" potentials, $\vec{f}$ and $\vec{g}$ is not as general as claimed by the *AIAS* authors. Indeed, Whittaker's presentation is a particular case of *Hertz's vector potentials theory,* known since 1888, a fact that is clearly quoted in Stratton's book[30]. The appropriate use of Hertz theory allowed the authors of [19−22,35] to easily prove the existence in vacuum of the arbitrary velocities solutions ($0 \leq v < \infty$) of *ME.*

The claim that Whittaker's approach shows that there are scalar waves in the vacuum and that these scalar waves are more fundamental than the potentials and electromagnetic fields is simply one more of the many unproven claims resulting from wishful thinking. Our statement will become clear in what follows.

Concerning (v) we have the following to say[12]:

(c_1) References[3−10]13 do not endorse the view that the $U(1)$ gauge theory of electromagnetism is incorrect. This claim has been made on many occasions by Evans while defending (as, e.g., in[28,29]) his $\vec{B}^{(3)}$ theory from his critics (which are many competent physicists, see[58,59,81−83] and the references in that papers). Our main purpose in this paper is not to discuss if the concept of the $\vec{B}^{(3)}$ field is of some utility to physical science. However we will introduce in section 2 the main ideas that probably lead Evans[3] to this concept. It will become clear that it is completely superfluous and irrelevant. Indeed, the original definition given by Evans of $\vec{B}^{(3)}$ makes his theory a non sequitur[14]. Trying to save his "theory" he and colleagues (the *AIAS* group) decide to promote $\vec{B}^{(3)}$ as a gauge theory with $O(3)$ as gauge group.These new develpoments show very clearly that the members of that group *never* understood until now what a gauge theory is. This is particularly clear in his paper[34], entitled "Non-

[10]These solutions, as is the case of the plane wave solutions of *ME* have infinite energy and so cannot be produced by any physical device. However, finite aperture approximations to that solutions can be produced. These waves have extraordinary properties which have been studied in details in [37].Among the extraordinary new solutions of *ME* there are, as particular cases, standing *EFC* in vacuum, as proved in[19−21,37]. Moreover, standing *EFC* with $\vec{E} \parallel \vec{B}$ have been produced in the laboratory[33]. See [19−21,37], and also [32,33].

[11]Indeed, [25] quotes [21] and [26] quotes [19,21].

[12]Our statements are proved below.

[13][11,12] need a separated comment, it is also, according to our view, a *pot-pourri* of misconceptions.

[14]On this issue, see also[58,59,64].

Abelian Electrodynamics and the Vacuum $\vec{B}^{(3)}$**"** which is the starting point for the theory of section 3 of *AIAS***1** and also of many other odd papers published in [0].

(c$_2$) The statement that Barrett developed a consistent $SU(2)$ non abelian electrodynamics is non sequitur. Indeed, at least the Barrett's papers[5–8] which we had the opportunity to examine are also a *pot-pourri* of inconsistent mathematics. We will point out some of them, in what follows.

(c$_3$) Quotation of Harmuth's papers[35] by the *AIAS* authors is completely out of context. At the *Discussion* section of *AIAS***1** it is concluded:

> "On the $U(1)$ level there are longitudinal propagating solutions of the potentials $\vec{f}$ and $\vec{g}$, of the vector potential $\vec{A}$ and the Stratton potential $\vec{S}$, but not longitudinal propagating components of the $\vec{E}$ and $\vec{B}$ fields. So, on the $U(1)$ level, any physical effects of longitudinal origin in free space depend on whether or not $\vec{f}$, $\vec{g}$, $\vec{A}$ and $\vec{S}$, are regarded as physical or unphysical"

We explicitly show that:

(d) this conclusion is wrong and results from the fact that the *AIAS* authors could not grasp the elementary mathematics used in Whittaker's paper[2]. Moreover, it is important to quote here that recently[15] finite aperture approximations to *SEXWs*[19–22] (i.e., superluminal electromagnetic X- waves) have been produced in the laboratory[36] and that these waves, differently from the fictitious $\vec{B}^{(3)}$ field of Evans and Vigier, possesses real *longitudinal* electric and/or magnetic components.

2 On scalar and longitudinal waves and $\vec{B}^{(3)}$

As one can learn from Chapter 5 of Whittaker's book[65], the idea that both electromagnetic *transverse* and *longitudinal* waves[16] exists in the aether was a very common one for the physicists of the XIX century.

As it is well known, in 1905 the concept of photons as the carriers of the electromagnetic interaction between charged particles has been introduced. Soon, with the invention of quantum electrodynamics, the photons have been associated to the quanta of the electromagentic field and described by *transverse* solutions of Maxwell equations, interpreted as an equation for a quantum field[72,73]. Longitudinal photons appears in quantum electrodynamics once we want to

[15] In the first version of the *AIAS***1** manuscript received by W.A.R. from *Found. Phys.*, E. Recami, one of member of the group (at that time) certainly knew about the results concerning the X-waves quoted above. Indeed, [26] quotes [19,21]. To avoid any misunderstanding let us emphasize here that the finite aperture approximations to *SEXWs* are such that their peaks can travel (for some time) at superluminal speeds. However since these waves have compact support in the space domain, they have fronts that travel at the speed of light. Thus no violation of the principle of relativity occurs. More details can be found in [37].

[16] A transverse wave has non zero components only in directions orthogonal to the propagation direction, whereas a longitudinal wave has always a non null component in the propagation direction.

mantain *relativistic covariance* in the quantization of the electromagnetic field. Indeed, as it is well known[72,73] the Gupta- Bleuler formalism introduces besides the transverse photons also longitudinal and timelike photons. These, however appears in an equal special "mixture" and cancel out at the end of electrodynamics calculations. These longitudianl and timelike photons did not seem to have a physical status. Their introduction in the theory seems to be only a mathematical necessity[17]. Some authors, like de Broglie[74] thinks that a photon has a very small mass. In this case, photons must be described by Proca's equation (free of sources) and this equation possess fidedigne longitudinal solutions, besides the transverse ones. There is a wrong opinion among physicists that only Proca's equation have longitudinal solutions, but the fact is that the free *ME*[18] possess infinite families of solutions (in vacuum) that have longitudinal electric and/or magnetic components. The existence of these solutions has been shown in[19−22,37] and can be seem to exist in a quite easy way from the Hertz potential theory developed in section 4 below. We emphasize here that Hertz potential theory was known (in a particular case) by Whittaker. He produced formulas (see eqs.(68) below) which clearly show the possiblity of obtaining exact solutions of the free *ME* with longitudinal electric and/or magnetic fields.

Let $\mathcal{M} = (M, g, D)$ be Minkowski spacetime[79]. (M, g) is a four dimensional time oriented and space oriented Lorentzian manifold, with $M \simeq \mathcal{R}^4$ and $g \in \sec(T^*M \times T^*M)$ being a Lorentzian metric of signature (1,3), and D is the Levi-Civita connection. Let $I \in \sec TM$ be an inertial reference frame[79] and let $\langle x^\mu \rangle$ be Lorentz- Einstein coordinates naturally adapted to I.

In the coordinates $\langle x^\mu \rangle$ the free *ME* for the electric field $\vec{E} : M \to R^3$ and magnetic field $\vec{B} : M \to R^3$ satisfy:

$$\begin{cases} \nabla \cdot \vec{E} = 0, & \nabla \times \vec{B} - \partial_t \vec{E} = 0, \\ \nabla \cdot \vec{B} = 0, & \nabla \times \vec{E} + \partial_t \vec{B} = 0. \end{cases} \tag{1}$$

$\vec{E}$ and $\vec{B}$ are derivable from the potentials $\phi : M \to R$ and $\vec{A} : M \to R^3$ by

$$\vec{E} = -\nabla\phi - \partial_t \vec{A}, \vec{B} = \nabla \times \vec{A} \tag{2}$$

Substituting $\vec{E}$ and $\vec{B}$ as giving by eqs.(2) in eqs.(2) gives

$$\Box\phi - \partial_t(\partial_t\phi + \nabla \cdot \vec{A}) = 0, \Box A_i - \partial_i(\partial_t\phi + \nabla \cdot \vec{A}) = 0 \tag{3}$$

Plane wave transverse solutions of eqs.(1) are obtained from eqs.(3) once we impose the so called radiation gauge, i.e., we put

$$\phi = 0, \quad \nabla \cdot \vec{A} = 0. \tag{4}$$

[17] We will discusss this issue in another publication[72].

[18] *ME*, according to the wisdom of quantum field theoy describes a zero mass particle.

When looking for such solutions of ME it is sometimes convenient to regard the fields $\vec{E}$, $\vec{B}$, ϕ and $\vec{A}$ as complex fields[19], i.e., we consider $\phi : M \to \mathcal{C}$; $\vec{E}$, $\vec{B}$; $\vec{A} : M \to \mathcal{C} \otimes R^3$. Defining the complex vector basis for the complexified euclidian vector space as

$$\vec{e}^{(1)} = \frac{1}{\sqrt{2}}(\hat{\imath} - i\hat{\jmath}), \vec{e}^{(2)} = \frac{1}{\sqrt{2}}(\hat{\imath} + i\hat{\jmath}), \vec{e}^{(3)} = \hat{k},\ i = \sqrt{-1}, \tag{5}$$

we can write (in a system of units where $c = 1$ and also $\hbar = 1$ and where q is the value of the electron charge) two linearly independent solutions of eqs.(3) (and of (1)) subject to the restriction (4) as:

$$\begin{aligned}
\vec{A}^{(1)} &= \frac{A^{(0)}}{\sqrt{2}}(i\vec{e}^{(1)} + \vec{e}^{(2)})e^{i(\omega t - kz)},\ \vec{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}}(-i\vec{e}^{(1)} + \vec{e}^{(2)})e^{-i(\omega t - kz)},\\
\vec{B}^{(1)} &= \frac{B^{(0)}}{\sqrt{2}}(i\vec{e}^{(1)} + \vec{e}^{(2)})e^{i(\omega t - kz)},\ \vec{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}(-i\vec{e}^{(1)} + \vec{e}^{(2)})e^{-i(\omega t - kz)},\\
\vec{E}^{(1)} &= -iq\vec{B}^{(1)} = \frac{E^{(0)}}{\sqrt{2}}(\vec{e}^{(1)} - i\vec{e}^{(2)})e^{i(\omega t - kz)}, \qquad (6)\\
\vec{E}^{(2)} &= iq\vec{B}^{(2)} = \frac{E^{(0)}}{\sqrt{2}}(\vec{e}^{(1)} + i\vec{e}^{(2)})e^{-i(\omega t - kz)},\ B^{(0)} = E^{(0)} = \omega A^{(0)} = \frac{1}{q}\omega^2.
\end{aligned}$$

Evans[3] defined the $\vec{B}^{(3)}$ field by

$$\vec{B}^{(3)} = \frac{-i}{B^{(0)}}\vec{B}^{(1)} \times \vec{B}^{(2)} = \frac{-i}{E^{(0)}}\vec{E}^{(1)} \times \vec{E}^{(2)} = -iq\vec{A}^{(1)} \times \vec{A}^{(2)} \tag{7}$$

It is clear from eq.(7) that $\vec{B}^{(3)}$ as normalized has the dimension of a magnetic field, is phase free and longitudinal. If, instead of the normalization in (7) we define the adimensional polarization vector

$$\vec{P} = \frac{-i}{(B^{(0)})^2}\vec{B}^{(1)} \times \vec{B}^{(2)} = \frac{-i}{(E^{(0)})^2}\vec{E}^{(1)} \times \vec{E}^{(2)} \tag{8}$$

we immediately regonize that this object is related to the second Stokes parameters (see [58,59,64]). More precisely, writing

$$\vec{E}^{(1)} = a_1\hat{\imath} + a_2\hat{\jmath}, \tag{9}$$

and defining the Stokes parameters $\rho_i, i = 0, 1, 2, 3$ by

$$\rho_0 = \frac{|a_1|^2 + |a_2|^2}{2},\ \rho_1 = \mathrm{Re}(a_1^+ a_2),\ \rho_2 = \mathrm{Im}(a_1^+ a_2),\ \rho_3 = \frac{|a_1|^2 - |a_2|^2}{2}, \tag{10}$$

we recall that the ratios

$$\tau_L = \sqrt{\frac{{\rho_1}^2 + {\rho_3}^2}{{\rho_0}^2}},\ \tau_C = \frac{\rho_2}{\rho_0} \tag{11}$$

[19] See our comments on the use of complex fields in section 5 and in[72].

are called respectively the degree of linear polarization and the degree of circular polarization. We have,

$$\tau_C = i\vec{k} \cdot \frac{\vec{E}^{(1)} \times \vec{E}^{(2)}}{(E^{(0)})^2} = \frac{i}{E^{(0)}} \vec{k} \cdot \vec{B}^{(3)}, \tag{12}$$

From this coincidence and the fact that the combination $\vec{E}^{(1)} \times \vec{E}^{(2)}$ appears also as an "effective" magnetic field in a term of the phenomenological Hamiltonian formulated by Pershan et al. in 1966[75] in their theory of the *inverse Faraday effect* Evans claimed first in [3] and then in a series of papers and books[20] that the field $\vec{B}^{(3)}$ is a *fundamental* longitudinal magnetic field wich is an integral part of any plane wave field configuration. Obviously, this is sheer nonse, and Silverman's in his wonderful book[64] wrote in this respect:

"Expression (12)[21] is specially interesting, for it is not, in my experience, a particularly well-known relation. Indeed, it is sufficiently obscure that in recent years an extensive scientific literature has developed examining in minute detail the far reaching electrodynamic, quantum, and cosmological implications of a "new" nonlinear light interaction proportional to $\vec{E}^{(1)} \times \vec{E}^{(2)}$ (deduced by analogy to the Poynting vector $\vec{S} \propto \vec{E}^{(1)} \times \vec{B}^{(2)}$) and intrpreted as a "longitudinal magnetic field" carried by the photon. Several books have been written on the subject. Were any of this true, such a radical revision of Maxwellian electrodynamics would of course be highly exciting, but it is regrettably the chimerical product of self-delusion—just like the "discovery" of N-Rays in the early 1900s. (During the period 1903-1906 some 120 trained scientists published almost 300 papers on the origins and characteristics of a tottaly spurious radiation first purpoted by a french scientist, René Blondlot[22])."[23]

Of course, the real meaning of the right hand side of eq.(12) is that it is a generalization of the concept of *helicity* which is defined for a single photon in quantum theory[73]. Here we only quote[.63] that, e.g., for a a right circularly polarized plane wave (helicity -1), $\tau_C = -1$.

According to Hunter[58,59], experiments[76,77] have been done in order to verify Evans' claims and they showed without doubts (despite Evans' claims on the contrary) that the conception of the $\vec{B}^{(3)}$ field is a non sequitur[24].

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[20] Some of these papers are in the list of references of the present paper. See also the references in[58,59,81−83].

[21] In Silverman's book his eq.(34), pp.167 is the one that corresponds to our eq.(12).

[22] The amazing history of the N-rays affair is presented in[80].

[23] Of course, Silverman is refering to Evans, which togheter with some collegues (the *AIAS* group) succeded in publishing several books edited by leading publishing houses and also so many papers even in respectable physical journals. The fact is that Evans and collaborators produced a vast amount of sheer non sense mathematics and physics, some of then discused in other sections of the present paper. Production of mathematical nonsense is not a peculiarity of the *AIAS* group. Indeed, there are inumerous examples of mathematical sophisms published in the recent Physics literature.This fact reflects the low level of university education in the last decades. Hundreeds of people call themselves mathematical physicists, write and succeed in publishing many papers (and books) and the truth is that they probably would not be approved in a freshman calculus examination in any serious university.

[24] Despite these facts, Evans succeded in publishing rebutals to the interpretation and results of these experiments[78], but as clearly showed in[58,59] the rebultats are not valid.

The above discussion shows in our opinion very clearly that Evans' $\vec{B}^{(3)}$ theory is simply wrong. Despite this fact, Evans and collaborators (the *AIAS* group) taking into account the last equality in eq.(7) decided to promote $\vec{B}^{(3)}$ theory to a gauge theory with gauge group $O(3)$. In the development of that idea the *AIAS* group produced a veritable compendium of mathematical sophisms. In their enterprise, the *AIAS* authors used both very good and interesting material from old papers from Whittaker, as well as some non sequitur proposals done by other authors concerning reformulation of Maxwell electrodynamics. In what follows we discuss the main mathematical flaws of these proposals.

3 Comments on Whittaker's 1903 paper of Mathematische Annalen

Whittaker's paper[1] is a classic, however we have some reservations concerning its section 5.5, entitled: *Gravitation and Electrostatic Attraction explained as modes of Wave-disturbance.* There, Whittaker observed that as a result of section 5.1 of his paper, it follows that any solution of the wave equation[25]

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \mathfrak{K}^2 \frac{\partial^2 V}{\partial t^2} \tag{13}$$

can be analyzed in terms of simple plane waves and that this fact throws a new light on the nature of forces, such as gravitation and electrostatic attraction, which vary as the inverse square of the distance. Whittaker's argument is that for a system of forces of this character, their potential (or their component in any given direction) satisfies the Laplace equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{14}$$

and therefore *à fortiori* also satisfies eq.(13), where $\mathfrak{K}$ is *any* constant. Then, Whittaker said that it follows that this potential V (or any force component, e.g., $F_x = -\partial V/\partial x$) can be analyzed into simple plane waves, in various directions, each wave being propagated with constant velocity, and that these waves *interfere* with each other in such a way that, when the action has once set up, the disturbance at any point does not vary with time, and depends only on the coordinates (x, y, z) of the point. To prove his statement, Whittaker constructs the electrostatic or Newton gravitational potential as follows:

(i) Suppose that a particle is emitting spherical waves, such that the disturbance at a distance r from the origin, at time t, due to those waves whose wave length lies between $2\pi/k$ and $2\pi/(k + dk)$ is represented by

$$\frac{2}{\pi}\frac{dk}{k}\frac{\sin(k\mathfrak{v}t - kr)}{r} \tag{15}$$

[25] V is a scalar valued function in Minkowski spacetime.

where $\mathfrak{v}$ is the phase velocity of propagation of the waves. Then after the waves have reached the point r, so that $(\mathfrak{v}t - r)$ is positive, the total disturbance at the point (due to the sum of all the waves) is

$$\int_0^\infty \frac{2}{\pi}\frac{dk}{k}\frac{\sin(k\mathfrak{v}t - kr)}{r} \tag{16}$$

(ii) Next, Whittaker makes the change of variables $k(\mathfrak{v}t - r) = Y$ and write eq.(16) as

$$\frac{2}{\pi r}\int_0^\infty dY\frac{\sin Y}{Y} = \frac{1}{r}. \tag{17}$$

(iii) Whittaker concludes that:

> "The total disturbance at any point, due to this system of waves, is therefore independent of the time, and is everywhere proportional to the gravitational potential due to the particle at that point."

(iv) That in each one of the constituent terms $\sin(k\mathfrak{v}t - kr)/r$ the potential will be constant along each wave-front, and "consequently the gravitational force in each constitutient field will be perpendicular to the wave-front, i.e., *the waves will be longitudinal*."

Now, we can present our comments.

(a) As it is well known, when we have a particle at the origin, the potential satisfies Poisson equation with a delta function source term, and not Laplace equation as stated by Whittaker, and indeed (17) satisfies Poisson equation. It is an interesting fact that a sum of waves which are non singular at the origin produces a "static wave" with a singularity at that point.

(b) Whittaker's hypothesis (i) is *ad hoc*, he did not present any single argument to justify why a charged particle, at rest at the origin must be emitting spherical waves of all frequencies, with the frequency spectrum implicit in eq.(15).

(c) A way to improve Whittaker's model should be to *represent* the electric charge as a particular electromagnetic field configuration modeled by a *UPW*[26] solution of eq.(13) (with[27] $v = c$, the velocity of light in vacuum) non singular at $r = 0$. The simplest stationaries solutions are[19]:

$$\frac{\sin(kr)}{kr}\sin\omega t, \quad \frac{\sin(kr)}{kr}\cos\omega t, \quad k = \omega \tag{18}$$

[26] *UPWs* means *Undistorted Progressive Waves*. In fact, *UPWs* of finite energy do not exist according to Maxwell linear theory, but *quasi-UPWs* with a very long 'lifetime' can eventually be constructed by appropriate superpositions of *UPWs* solutions. Of course, the non existence of finite energy *UPWs* solutions of *ME* shows clearly the limits we can arrive when pursuing such kind of ideas inside the frame of a linear theory.

[27] We use a system of units such that $c = 1$.

From, these solutions it is easy to build a new one with a frequency distribution such that it is possible to recover the Coulomb potential under the same conditions as the ones used by Whittaker. This is a contribution for the idea of modeling particles as *PEPs*, i.e., pure electromagnetic particles[19−21].

We call the readers attention that the idea of longitudinal waves (in the aether) was a very common one for the physicists of the XIX century. In this respect the reader should consult Chapter 5 of Whittaker's book[65].

Moreover, we quote that Landau and Lifshitz in their classical book[60] (section 52) after making the Fourier resolution of the Coulomb electrostatic field, got that

$$\vec{E} = \int_{-\infty}^{\infty} \frac{d^3\vec{k}}{(2\pi)^3} \vec{B}_k \exp(i\vec{k}\cdot\vec{r}) \quad \text{and} \quad \vec{B}_k = -i\frac{4\pi e\vec{k}}{k^2} \tag{19}$$

and concludes:

> "From this we see that the field of the waves, into which we have resolved the Coulomb field, is directed along the wave vector. Therefore these waves can be say to be longitudinal."

Well, we must comment here that since each $\vec{B}_k$ is an imaginary vector it cannot represent any realized electric field in nature. This show the danger that exist when working with complex numbers in the analysis of physical problems.

4 Clifford bundles[61]

Let $\mathcal{M} = (M, g, D)$ be Minkowski spacetime. (M, g) is a four dimensional time oriented and space oriented Lorentzian manifold, with $M \simeq R^4$ and $g \in \sec(T^*M \times T^*M)$ being a Lorentzian metric of signature (1,3). T^*M $[TM]$ is the cotangent [tangent] bundle. $T^*M = \cup_{x\in M} T^*_x M$, $TM = \cup_{x\in M} T_x M$, and $T_x M \simeq T^*_x M \simeq R^{1,3}$, where $R^{1,3}$ is the Minkowski vector space[79]. D is the Levi-Civita connetion of g, *i.e.*, $Dg = 0$, $\boldsymbol{T}(D) = 0$. Also $\boldsymbol{R}(D) = 0$, $\boldsymbol{T}$ and $\boldsymbol{R}$ being respectively the torsion and curvature tensors. Now, the Clifford bundle of differential forms $\mathcal{C}\ell(M)$ is the bundle of algebras[28] $\mathcal{C}\ell(M) = \cup_{x\in M} \mathcal{C}\ell(T^*_x M)$, where $\forall x \in M, \mathcal{C}\ell(T^*_x M) = Cl_{1,3}$, the so called *spacetime algebra*[61]. Locally as a linear space over the real field R, $\mathcal{C}\ell(T^*_x M)$ is isomorphic to the Cartan algebra $\Lambda(T^*_x M)$ of the cotangent space and $\Lambda(T^*_x M) = \sum_{k=0}^{4} \Lambda^k(T^*_x M)$, where $\Lambda^k(T^*_x M)$ is the $\binom{4}{k}$-dimensional space of k-forms. The Cartan bundle $\Lambda(M) = \cup_{x\in M} \Lambda(T^*_x M)$ can then be thought as "imbedded" in $\mathcal{C}\ell(M)$. In this way sections of $\mathcal{C}\ell(M)$ can be represented as a sum of inhomogeneous differential forms. Let $\{e_\mu = \frac{\partial}{\partial x^\mu}\} \in \sec TM, (\mu = 0, 1, 2, 3)$ be an orthonormal basis $g(e_\mu, e_\nu) = \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and let $\{\gamma^\nu = dx^\nu\} \in \sec \Lambda^1(M) \subset \sec \mathcal{C}\ell(M)$ be the dual basis. Moreover, we denote by g^{-1} the metric in the cotangent bundle.

[28] We can show using the definitions of section 5 that $\mathcal{C}\ell(M)$ is a vector bundle associated to the *orthonormal frame bundle*, i.e., $\mathcal{C}\ell(M) = P_{SO_{+(1,3)}} \times_{ad} Cl_{1,3}$ Details about this construction can be found in [61].

4.1 Clifford product, scalar contraction and exterior products

The fundamental *Clifford product* (in what follows to be denoted by juxtaposition of symbols) is generated by $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$ and if $\mathcal{C} \in \sec \mathcal{C}\ell(M)$ we have

$$\mathcal{C} = s + v_\mu\gamma^\mu + \frac{1}{2!}b_{\mu\nu}\gamma^\mu\gamma^\nu + \frac{1}{3!}a_{\mu\nu\rho}\gamma^\mu\gamma^\nu\gamma^\rho + p\gamma^5 \ , \tag{20}$$

where $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = dx^0dx^1dx^2dx^3$ is the volume element and s, v_μ, $b_{\mu v}$, $a_{\mu\nu\rho}$, $p \in \sec \Lambda^0(M) \subset \sec \mathcal{C}\ell(M)$.

Let $A_r, \in \sec \Lambda^r(M), B_s \in \sec \Lambda^s(M)$. For $r = s = 1$, we define the *scalar product* as follows:

For $a, b \in \sec \Lambda^1(M) \subset \sec \mathcal{C}\ell(M)$.,

$$a \cdot b = \frac{1}{2}(ab + ba) = g^{-1}(a, b). \tag{21}$$

We define also the *exterior product* ($\forall r, s = 0, 1, 2, 3$) by

$$\begin{aligned} A_r \wedge B_s &= \langle A_r B_s \rangle_{r+s}, \\ A_r \wedge B_s &= (-1)^{rs} B_s \wedge A_r \end{aligned} \tag{22}$$

where $\langle\rangle_k$ is the component in $\Lambda^k(M)$ of the Clifford field. The exterior product is extended by linearity to all sections of $\mathcal{C}\ell(M)$.

For $A_r = a_1 \wedge ... \wedge a_r, B_r = b_1 \wedge ... \wedge b_r$, the scalar product is defined *here* as follows,

$$\begin{aligned} A_r \cdot B_r &= (a_1 \wedge ... \wedge a_r) \cdot (b_1 \wedge ... \wedge b_r) \\ &= \sum (-1)^{\frac{r(r-1)}{2}} \epsilon^{i_1 ... i_r}_{1 ... r} (a_1 \cdot b_{i_1}) .... (a_r \cdot b_{i_r}) \end{aligned} \tag{23}$$

We agree that if $r = s = 0$, the scalar product is simple the ordinary product in the real field.

Also, if $r, s \neq 0$ and $A_r \cdot B_s = 0$ if r or s is zero.

For $r \leq s, A_r = a_1 \wedge ... \wedge a_r, B_s = b_1 \wedge ... \wedge b_s$ we define the left contraction by

$$\lrcorner : (A_r, B_s) \mapsto A_r \lrcorner B_s = \sum_{i_1 < ... < i_r} \epsilon^{i_1 .... . i_s}_{1 ...... s} (a_1 \wedge ... \wedge a_r) \cdot (b_{i_r} \wedge ... \wedge b_{i_1})^\sim b_{i_r+1} \wedge ... \wedge b_{i_s}, \tag{24}$$

where $\sim$ denotes the reverse mapping (*reversion*)

$$\sim : \sec \Lambda^p(M) \ni a_1 \wedge ... \wedge a_p \mapsto a_p \wedge ... \wedge a_1, \tag{25}$$

and extended by linearity to all sections of $\mathcal{C}\ell(M)$. We agree that for $\alpha, \beta \in \sec \Lambda^0(M)$ the contraction is the ordinary (pointwise) product in the real field and that if $\alpha \in \sec \Lambda^0(M)$, $A_r, \in \sec \Lambda^r(M), B_s \in \sec \Lambda^s(M)$ then $(\alpha A_r) \lrcorner B_s =$

$A_r \lrcorner (\alpha B_s)$. Left contraction is extended by linearity to all pairs of elements of sections of $\mathcal{C}\ell(M)$, i.e., for $A, B \in \sec \mathcal{C}\ell(M)$

$$A \lrcorner B = \sum_{r,s} \langle A \rangle_r \lrcorner \langle B \rangle_s, r \leq s \tag{26}$$

It is also necessary to introduce in $\mathcal{C}\ell(M)$ the operator of *right contraction* denoted by $\llcorner$. The definition is obtained from the one presenting the left contraction with the imposition that $r \geq s$ and taking into account that now if $A_r, \in \sec \Lambda^r(M), B_s \in \sec \Lambda^s(M)$ then $A_r \llcorner (\alpha B_s) = (\alpha A_r) \llcorner B_s$.

4.2 Some useful formulas

The main formulas used in the Clifford calculus can be obtained from the following ones (where $a \in \sec \Lambda^1(M)$):

$$\begin{aligned}
aB_s &= a \lrcorner B_s + a \wedge B_s, B_s a = B_s \llcorner a + B_s \wedge a, \\
a \lrcorner B_s &= \frac{1}{2}(aB_s - (-)^s B_s a), \\
A_r \lrcorner B_s &= (-)^{r(s-1)} B_r \lrcorner A_s, \\
a \wedge B_s &= \frac{1}{2}(aB_s + (-)^s B_s a), \\
A_r B_s &= \langle A_r B_s \rangle_{|r-s|} + \langle A_r \lrcorner B_s \rangle_{|r-s-2|} + \ldots + \langle A_r B_s \rangle_{|r+s|} \\
&= \sum_{k=0}^{m} \langle A_r B_s \rangle_{|r-s|+2k}, \; m = \frac{1}{2}(r + s - |r-s|).
\end{aligned} \tag{27}$$

4.3 Hodge star operator

Let $\star$ be the Hodge star operator $\star : \Lambda^k(M) \to \Lambda^{4-k}(M)$. Then we can show that if $A_p \in \sec \Lambda^p(M) \subset \sec \mathcal{C}\ell(M)$ we have $\star A = \widetilde{A}\gamma^5$. Let d and δ be respectively the differential and Hodge codifferential operators acting on sections of $\Lambda(M)$. If $\omega_p \in \sec \Lambda^p(M) \subset \sec \mathcal{C}\ell(M)$, then $\delta \omega_p = (-)^p \star^{-1} d \star \omega_p$, with $\star^{-1}\star =$ identity.

The Dirac operator acting on sections of $\mathcal{C}\ell(M)$ is the invariant first order differential operator

$$\boldsymbol{\partial} = \gamma^\mu D_{e_\mu}, \tag{28}$$

and we can show the very important result:

$$\boldsymbol{\partial} = \boldsymbol{\partial} \wedge + \boldsymbol{\partial} \lrcorner = d - \delta. \tag{29}$$

5 Maxwell equation and the consistent Hertz potential theory

In this formalism, Maxwell equations for the electromagnetic field $F \in \sec \Lambda^2(T^\star M) \subset \sec \mathcal{C}\ell(M)$ and current $J_e \in \sec \Lambda^1(T^\star M) \subset \sec \mathcal{C}\ell(M)$ are resumed in a single equation (justifying the singular used in the title of the section)

$$\boldsymbol{\partial} F = J_e. \tag{30}$$

Of course, eq.(30) can be written in the usual way, i.e.,

$$\begin{cases} dF & = & 0 \\ \delta F & = & -J_e. \end{cases} \tag{31}$$

We write[29]

$$F = \frac{1}{2} F^{\mu\nu} \gamma_\mu \gamma_\nu = \vec{E} + \boldsymbol{i}\vec{B}, \tag{32}$$

where the real functions $F_{\mu\nu}$ are given by the entries of the following matrix

$$[F^{\mu\nu}] = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{bmatrix}. \tag{33}$$

Moreover,

$$J_e \gamma^0 = \rho_e + \vec{J}_e, \tag{34}$$

and

$$\begin{gathered} \vec{E} = E^i \vec{\sigma}_i, \quad \vec{B} = B^i \vec{\sigma}_i, \quad \vec{J}_e = J^i \vec{\sigma}_i, \\ \vec{\sigma}_i = \gamma_i \gamma_0, \\ \vec{\sigma}_i \vec{\sigma}_j + \vec{\sigma}_j \vec{\sigma}_i = 2\delta_{ij}, \\ \boldsymbol{i} = \vec{\sigma}_1 \vec{\sigma}_2 \vec{\sigma}_3 = \gamma_0 \gamma_1 \gamma_2 \gamma_3 = \gamma_5. \end{gathered} \tag{35}$$

Then, even if the element $\boldsymbol{i} = -\gamma^5 = \gamma_5$ (the pseudoscalar unity of the $Cl_{1,3}$) is such that $\boldsymbol{i}^2 = -1$ it has not the *algebraic* meaning of $\sqrt{-1}$, but has the *geometrical* meaning of an oriented volume in M. In addition, in this formalism, it is possible to identify when convenient the elements $\vec{\sigma}_i$ with the Pauli matrices. We have,

$$\partial \gamma_0 \gamma_0 \Pi \gamma_0 = (\partial_t - \nabla)(-\vec{E} + \boldsymbol{i}\vec{B}) = J_e \gamma_0 = \rho_e + \vec{J}_e \tag{36}$$

For an arbitrary vector field $\vec{C} = C^i \vec{\sigma}_i$, where $C^i : M \to R$, we have

$$\nabla \vec{C} = \nabla \cdot \vec{C} + \nabla \wedge \vec{C} \tag{37}$$

where $\nabla \cdot \vec{C}$ is the (Euclidean) divergence of $\vec{C}$. We define the (Euclidean) rotational of $\vec{C}$

$$\nabla \times \vec{C} = -\boldsymbol{i} \nabla \wedge \vec{C}. \tag{38}$$

By using definitions (37) and (38) we obtain from eq.(36) the vector form of *ME*, i.e.,

$$\begin{cases} \nabla \cdot \vec{E} = \rho_e, & \nabla \times \vec{B} - \partial_t \vec{E} = \vec{J}_e, \\ \nabla \cdot \vec{B} = 0, & \nabla \times \vec{E} + \partial_t \vec{B} = 0. \end{cases} \tag{39}$$

Now we are in position to provide a *modern* presentation of *Hertz theory.*

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[29] We observe that the quantity that really describes the properties of the magnetic field is the bivector field $\boldsymbol{i}\vec{B}$.

5.1 Hertz theory on vacuum

Let $\Pi = \frac{1}{2}\Pi^{\mu\nu}\gamma_\mu\gamma_\nu = \vec{\Pi}_e + \boldsymbol{i}\vec{\Pi}_m \in \sec\Lambda^2(T^\star M) \subset \sec\mathcal{C}\ell(M)$ be the so called *Hertz potential*[19−22,30]. We write

$$[\Pi^{\mu\nu}] = \begin{bmatrix} 0 & -\Pi_e^1 & -\Pi_e^2 & -\Pi_e^3 \\ \Pi_e^1 & 0 & -\Pi_m^3 & \Pi_m^2 \\ \Pi_e^2 & \Pi_m^3 & 0 & -\Pi_m^1 \\ \Pi_e^3 & -\Pi_m^2 & \Pi_m^1 & 0 \end{bmatrix}. \tag{40}$$

Let

$$A = -\delta\Pi \in \sec\Lambda^1(T^\star M) \subset \sec\mathcal{C}\ell(M), \tag{41}$$

and call it the *electromagnetic potential*.

Since $\delta^2 = 0$ it is clear that A satisfies the Lorentz gauge condition, i.e.,

$$\delta A = 0. \tag{42}$$

Also, let

$$\gamma^5 S = d\Pi \in \sec\Lambda^3(T^\star M) \subset \sec\mathcal{C}\ell(M), \tag{43}$$

and call S, the *Stratton potential*. It follows also that

$$d\left(\gamma^5 S\right) = d^2\Pi = 0. \tag{44}$$

But $d(\gamma^5 S) = \gamma^5\delta S$ from which we get, taking into account eq.(40),

$$\delta S = 0 \tag{45}$$

We can put eqs.(41) and (42) into a single Maxwell like equation, i.e.,

$$\boldsymbol{\partial}\Pi = (d - \delta)\Pi = A + \gamma^5 S = \mathcal{A}. \tag{46}$$

>From eq.(46) (using the same developments as in eq.(36)) we get

$$\begin{cases} A^0 = \nabla\cdot\vec{\Pi}_e, & \vec{A} = -\partial_t\vec{\Pi}_e + \nabla\times\vec{\Pi}_m, \\ S^0 = \nabla\cdot\vec{\Pi}_m, & \vec{S} = -\partial_t\vec{\Pi}_m - \nabla\times\vec{\Pi}_e. \end{cases} \tag{47}$$

We also have,

$$\Box\Pi = (d-\delta)^2\Pi = dA + \gamma_5 dS. \tag{48}$$

Next, we define the electromagnetic field by

$$F = \boldsymbol{\partial}\mathcal{A} = \Box\Pi = dA + \gamma_5 dS = F_e + \gamma_5 F_m. \tag{49}$$

We observe that,

$$\Box\Pi = 0 \text{ which leads to } F_e = -\gamma_5 F_m. \tag{50}$$

Now, let us calculate $\boldsymbol{\partial}F$. We have,

$$\begin{aligned}\boldsymbol{\partial}F &= (d-\delta)F \\ &= d^2A + d(\gamma^5 dS) - \delta(dA) - \delta(\gamma^5 dS).\end{aligned}$$

The first and last terms in the second line of eq.(51) are obviously null. Writing,

$$J_e = -\delta dA, \text{ and } \gamma^5 J_m = -d(\gamma^5 dS), \tag{51}$$

we get *ME*

$$\boldsymbol{\partial}F = (d-\delta)F = J_e, \tag{52}$$

if and only if the magnetic current $\gamma^5 J_m = 0$, i.e.,

$$\delta dS = 0. \tag{53}$$

a condition that we suppose to be satisfied in what follows. Then,

$$\begin{aligned}&\Box A = J_e = -\delta dA, \\ &\Box S = 0.\end{aligned} \tag{54}$$

Now, we define,

$$F_e = dA = \vec{E}_e + \boldsymbol{i}\vec{B}_e, \tag{55}$$

$$F_m = dS = \vec{B}_m + \boldsymbol{i}\vec{E}_m. \tag{56}$$

and also

$$F = F_e + \gamma_5 F_m = \vec{E} + \boldsymbol{i}\vec{B} = (\vec{E}_e - \vec{E}_m) + \boldsymbol{i}(\vec{B}_e + \vec{B}_m). \tag{57}$$

Then, eq.(49) gives ,

$$\Box\vec{\Pi}_e = \vec{E}, \qquad \Box\vec{\Pi}_m = \vec{B} \tag{58}$$

Eqs.(58) agree with eqs.(52) and (53) of Stratton's book since we are working in the vacuum.

It is important to keep in mind that:

$$\Box\Pi = 0 \text{ leads to } \vec{E} = 0 \text{ and } \vec{B} = 0. \tag{59}$$

Nevertheless, despite this result we have,

Hertz theorem.[30]

$$\Box\Pi = 0 \text{ leads to } \partial F_e = 0. \tag{60}$$

Proof: From eq.(49) and eq.(53) we have

$$\partial F_e = -\partial(\gamma_5 F_m) = -d(\gamma_5 dS) + \delta(\gamma_5 dS) = \gamma_5 d^2 S - \gamma_5 \delta dS = 0. \tag{61}$$

[30] Eq. (60) has been called the *Hertz theorem* in[19−22].

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Another way to prove this theorem is taking into account that $\delta A = 0$ is: $\partial F_e = (d-\delta)F_e = (d-\delta)(d-\delta)A = \delta d\delta\Pi = -\delta^2 d\Pi = 0$.

From eq.(57) we easily obtain the following formulas:

$$\left\{\begin{array}{ll} \vec{E}_e = -\nabla A^0 - \partial_t \vec{A} & \vec{B}_m = -\nabla S^0 - \partial t\vec{S}, \\ \vec{B}_e = \nabla \times \vec{A} & \vec{E}_m = \nabla \times \vec{S} \end{array}\right. \tag{62}$$

or

$$\left\{\begin{array}{l} \vec{E}_e = -\nabla(\nabla\cdot\vec{\Pi}_e) - \partial_t(\nabla\times\vec{\Pi}_m) + \partial_t^2\vec{\Pi}_e, \\ \vec{B}_e = \nabla\times(\nabla\times\vec{\Pi}_m) - \partial_t(\nabla\times\vec{\Pi}_e), \end{array}\right. \tag{63}$$

and

$$\left\{\begin{array}{l} \vec{B}_m = -\nabla(\nabla\cdot\vec{\Pi}_m) + \partial_t(\nabla\times\vec{\Pi}_e) + \partial_t^2\vec{\Pi}_m, \\ \vec{E}_m = -\nabla\times(\nabla\times\vec{\Pi}_e) - \partial_t(\nabla\times\vec{\Pi}_m). \end{array}\right. \tag{64}$$

We observe that, when $\Box\Pi = 0$, then $\Box\vec{\Pi}_e = 0$ and $\Box\vec{\Pi}_m = 0$. In this case eq.(63) can be written as

$$\left\{\begin{array}{l} \vec{E}_e = -\nabla(\nabla\cdot\vec{\Pi}_e) - \partial_t(\nabla\times\vec{\Pi}_m) + \partial_t^2\vec{\Pi}_e, \\ \vec{B}_e = \nabla(\nabla\cdot\vec{\Pi}_m) - \partial_t(\nabla\times\vec{\Pi}_e) - \partial_t^2\vec{\Pi}_m. \end{array}\right. \tag{65}$$

The first of eqs.(65) is identical to eq.(56) and the second of eqs.(65) is identical to eq.(57) of Stratton's book (p. 31), with the obvious substitution $\vec{\Pi}_e \to -\vec{\Pi}$ and $\vec{\Pi}_m \to \vec{\Pi}^*$, which is exactly the notation used by Stratton.

5.2 Comments on section 2 of *AIAS*1

(i) It is obvious that Whittaker's theory, as presented by the *AIAS* authors, is simply a particular case of Hertz theory and so it does not deserve to be called Whittaker's theory. Indeed, what Whittaker did, in the notation of *AIAS* authors, was to put

$$\vec{f} = -\vec{\Pi}_e = \mathrm{F}\hat{k}, \qquad \vec{g} = \vec{\Pi}_m = \mathrm{G}\hat{k}, \tag{66}$$

$$\Box\vec{\Pi}_e = 0, \qquad \Box\vec{\Pi}_m = 0. \tag{67}$$

i.e., he used only two degrees of freedom of the six possible ones.

From eqs.(66) and (67) we compute the cartesian components of $\vec{E}_e, \vec{B}_e$ appearing in the decomposition of $F_e = \vec{E}_e + \boldsymbol{i}\vec{B}_e$ (see eq.(65)). We have, as first derived by Whittaker,

$$\left\{\begin{array}{ll} E_e^1 = E_x = \dfrac{\partial^2\mathrm{F}}{\partial x\partial z} + \dfrac{\partial^2\mathrm{G}}{\partial y\partial t}, & B_e^1 = B_x = \dfrac{\partial^2\mathrm{F}}{\partial y\partial t} - \dfrac{\partial^2\mathrm{G}}{\partial x\partial z}, \\ E_e^2 = E_y = \dfrac{\partial^2\mathrm{F}}{\partial y\partial z} - \dfrac{\partial^2\mathrm{G}}{\partial x\partial t}, & B_e^2 = B_y = -\dfrac{\partial^2\mathrm{F}}{\partial x\partial t} - \dfrac{\partial^2\mathrm{G}}{\partial y\partial z}, \\ E_e^3 = E_z = \dfrac{\partial^2\mathrm{F}}{\partial z^2} - \dfrac{\partial^2\mathrm{F}}{\partial t^2}, & B_e^3 = B_z = \dfrac{\partial^2\mathrm{G}}{\partial x^2} + \dfrac{\partial^2\mathrm{G}}{\partial y^2}. \end{array}\right. \tag{68}$$

To simplify calculations it is in general useful to introduce the complexified Clifford bundle $\mathcal{C}\ell_{\mathcal{C}}(M) = \mathcal{C} \otimes \mathcal{C}\ell(M)$, where $\mathcal{C}$ is the complex field. We use, $i = \sqrt{-1}$. This does not mean that complex fields have any meaning in classical electromagnetism. Bad use of complex fields produces a lot of nonsense.

(ii) Eqs.(68) makes clear the fact that it is possible to have *exact* solutions of *ME* in vacuum that are not transverse waves, in the sense that there may be components of the electric and/or magnetic field parallel to the direction of propagation of the wave. Indeed, e.g., [17−20] exhibit several solutions of this kind which have been obtained with the Hertz potential method. In these papers it was found that, in general, these exactly solutions of *ME* correspond to theoretical waves traveling with speeds[31] $0 \leq v < 1$ or $v > 1$. Moreover, these waves are *UPWs*, i.e., undistorted progressive waves[32]! More important is the fact that, recently, finite apperture approximations for optical *SEXWs* (i.e., superluminal electromagnetic X-waves) which have longitudinal electric and/or magnetic fields have been produced in the laboratory by Saari and Reivelt[36].

It is clear also from this approach that theoretically there exists transverse electromagnetic waves such that their fields can be derived from the potential 1-forms with longitudinal components, but this fact did not give any *ontology* to the potential vector field.

(iii) From (ii) it follows that the *AIAS* group conclusions, in the discussion section of *AIAS***1**, namely:

> "On the $U(1)$ level there are longitudinal propagating solutions of the potentials $\vec{f}$ and $\vec{g}$, of the vector potential $\vec{A}$ and the Stratton potential $\vec{S}$, but not longitudinal propagating components of the $\vec{E}$ and $\vec{B}$ fields. So, on the $U(1)$ level, any physical effects of longitudinal origin in free space depend on whether or not $\vec{f}$, $\vec{g}$, $\vec{A}$ and $\vec{S}$, are regarded as physical or unphysical."

is completely *wrong*[33], because all results described in (iii) have been obtained from classical electromagnetism which is a $U(1)$ gauge theory. We will discuss more about this issue later because, as already stated, it is clear that *AIAS* authors have not a single idea of what a gauge theory is.

(iv) After these comments, we must say that we are *perplexed* not only with the very bad mathematics of the *AIAS* group, but also with the ethical status of some of its present and/or past members. Indeed, the fact is that the papers mentioned in (ii) above have not been quoted by the *AIAS* group. This is

[31] These are not phase velocities, of course, but genuine propagation velocities. The interpretation of the superluminarity observed in the experiment [36] is presented in [37].

[32] To avoid any misunderstanding here, we recall again that exact superluminal *UPWs* solutions of *ME* cannot be realized in the physical world. The reason is that, like the monochromatic plane waves they have infinity energy. However finite aperture approximations to superluminal waves can be produced. They have very interesting properties (see [37]).

[33] This statement came from the fact that in the example studied by Whittaker and copied by the *AIAS* authors the functions F and G used are linear in the variables x and y.

ethically unacceptable[34], since Evans is one the members of the group, and he knew all the points mentioned above, since he quoted[19−21] in some of his papers, as pointed out in footnote 12.

6 Gauge Theories

We already saw that *ME* possess exact solutions that are *EFC* with longitudinal electric and/or magnetic components. In order,

(a) to understand why the existence of this kind of solutions did not imply that we must consider electromagnetism as a gauge theory with gauge group different from $U(1)$ and,

(b) to understand that the section on "Non-Abelian Electrodynamics" of $AIAS\mathbf{1}$ and also a number of other papers in [0] and also the "Non-Abelian Electrodynamics of Barret" are *non sequitur* and a *pot-pourri* of *inconsistent* mathematics, it is necessary to know exactly what a gauge theory is. The only coherent presentation of such a theory is through the use of rigorous mathematics. We need to know at least very well the notions of:

(*i*) *Principal Bundles*

(*ii*) *Associated Vector bundles to a given principal bundle*

(*iii*) *Connections on Principal Bundles*

(*iv*) *Covariant derivatives of sections of a Vector Bundle*

(*v*) *Exterior Covariant derivatives*

(*vi*) *Curvature of a Connection*

After these notions are known we can introduce concepts used by physicists as gauge potentials, gauge fields, and matter fields.

Of course, we do not have any intention to present in what follows a monograph on the subject[35]. However, to grasp what a gauge theory is, we will recall in the next subsection the main definitions and results of the general theory adapted for the case where the base manifold of the bundles used is Minkowski spacetime. Our presentation clarifies some issues which according to our view are obscure in many physics textbooks.

[34] We take the opportunity to say that paper[26] which deals with "superluminal solutions" of *ME* has *good* and new *things*. However the good things are not *new* and can be found in[21] (and in reference 5 of that paper, which has not been published). The new things are not *good*. Contrary to what is stated there, there is no *UPW* *X*-wave like solution of Schrödinger equation (for a proof see [51]). Moreover, the claim done by the author of[26] (followed with a "proof") that he predicted the existence of superluminal *X*-waves from tachyon kinematics is obviously non sequitur and must be considered as a joke. In time, we are quoting these *facts*, because the author of[26] signed several papers as member of the *AIAS* group, and as we already said, appears as one of the authors of the first version of the *MSs* (now published in[0]) sent to W. A. R. by the editor of *Found. Phys.*, which asked for a review of that papers.

[35] There are now excellent texts and monographies on the subject. We recommend here the following [38−42].

6.1 Some definitions and theorems

As in sections 2 and 3, let (M, g) be Minkowski spacetime manifold[36].

1. A *fiber bundle* over M with Lie group G will be denoted by $(E, M, \boldsymbol{\pi}, G, F)$. E is a topological space called the total space of the bundle, $\boldsymbol{\pi} : E \to M$ is a continuous surjective map, called the *canonical projection* and F is the typical fiber. The following conditions must be satisfied:

a) $\boldsymbol{\pi}^{-1}(x)$, the fiber over x is homeomorphic to F.

b) Let[37] $\{U_i,\, i \in \mathfrak{I}\}$ be a covering of M, such that:

- Locally a fiber bundle E is *trivial*, i.e., it is *difeomorphic* to a product bundle, i.e., $\boldsymbol{\pi}^{-1}(U_i) \simeq U_i \times F$ for all $i \in \mathfrak{I}$.

- The difeomorphism, $\Phi_i : \boldsymbol{\pi}^{-1}(U_i) \to U_i \times F$ has the form

$$\Phi_i(p) = (\boldsymbol{\pi}(p), \phi_i(p)) \tag{69}$$

$$\phi_i|_{\boldsymbol{\pi}^{-1}(x)} \equiv \phi_{i,x} : \boldsymbol{\pi}^{-1}(x) \to F \text{ is onto.} \tag{70}$$

  The collection $\{U_i, \Phi_i\}$, $i \in \mathfrak{I}$, are said to be a family of *local trivializations* for E.

- Let $x \in U_i \cap U_j$. Then,

$$\phi_{j,x} \circ \phi_{i,x}^{-1} : F \to F \tag{71}$$

  must coincide with the action of an element of G for all $x \in U_i \cap U_j$ and $i, j \in \mathfrak{I}$.

- We call *transition functions* of the bundle the continuous *induced mappings*

$$g_{ij} : U_i \cap U_j \to G, \text{ where } g_{ij}(x) = \phi_{j,x} \circ \phi_{i,x}^{-1}. \tag{72}$$

For consistence of the theory the transition functions must satisfy the cocycle condition

$$g_{ij}(x) g_{jk}(x) = g_{ik}(x). \tag{73}$$

Observation 1: To complete the definition of a fiber bundle it is necessary to define the concept of *equivalent* fiber bundles[39]. We do not need to use this concept in what follows and so are not going to introduce it here.

2. $(P, M, \boldsymbol{\pi}, G, F \equiv G) \equiv (P, M, \boldsymbol{\pi}, G)$ is called a *principal fiber bundle (PFB)* if all conditions in **1** are fulfilled and moreover, there is a *right action* of G on elements $p \in P$, such that:

a) the mapping (defining the right action) $P \times G \ni (p, g) \mapsto pg \in P$ is continuous.

[36] Minkowski spacetime is the Minkowski manifold equipped with the Levi-Civita connection of g.

[37] $\mathfrak{I}$ is an index set.

b) given $g, g' \in G$ and $\forall p \in P$, $(pg)g' = p(gg')$.

c) $\forall x \in M, \boldsymbol{\pi}^{-1}(x)$ is invariant under the action of G, i.e., each element of $p \in \boldsymbol{\pi}^{-1}(x)$ is mapped into $pg \in \boldsymbol{\pi}^{-1}(x)$, i.e., it is mapped into an element of the same fiber.

d) G acts transitively on each fiber $\boldsymbol{\pi}^{-1}(x)$, which means that all elements within $\boldsymbol{\pi}^{-1}(x)$ are obtained by the action of all the elements of G on any given element of the fiber $\boldsymbol{\pi}^{-1}(x)$. This condition is, of course necessary for the identification of the typical fiber with G.[38]

3. A bundle $(E, M, \boldsymbol{\pi}_1, G = Gl(m, \mathcal{F}), F = \boldsymbol{V})$, where $\mathcal{F} = R$ or C (respectively the real and complex fields), $Gl(m, \mathcal{F})$, is the linear group, and $\boldsymbol{V}$ is an m-dimensional vector space over , is called a *vector bundle*.

4. A vector bundle $(E, M, \boldsymbol{\pi}, G, F)$ denoted $E = P \times_\rho F$ is said to be *associated* to a *PFB* bundle $(P, M, \boldsymbol{\pi}, G)$ by the linear representation ρ of G in $F = \boldsymbol{V}$ (a linear space of finite dimension over an appropriate field , which is called the *carrier space* of the representation) if its transition functions are the images under ρ of the corresponding transition functions of the *PFB* $(P, M, \boldsymbol{\pi}, G)$. This means the following: consider the local trivializations

$$\Phi_i : \boldsymbol{\pi}^{-1}(U_i) \to U_i \times G \quad \text{ of } (P, M, \boldsymbol{\pi}, G), \tag{74}$$

$$\Xi_i : \boldsymbol{\pi}_1^{-1}(U_i) \to U_i \times F \quad \text{ of } E = P \times_\rho F, \tag{75}$$

$$\Xi_i(q) = (\boldsymbol{\pi}_1(q) = x, \chi_i(q)), \tag{76}$$

$$\chi_i|_{\boldsymbol{\pi}_1^{-1}(x)} \equiv \chi_{i,x} : \boldsymbol{\pi}_1^{-1}(x) \to F, \tag{77}$$

where $\boldsymbol{\pi}_1 : P \times_\rho F \to M$ is projection of the bundle associated to $(P, M, \boldsymbol{\pi}, G)$.

Then, for all $x \in U_i \cap U_j$, $i, j \in \mathfrak{I}$, we have

$$\chi_{j,x} \circ \chi_{i,x}^{-1} = \rho(\phi_{j,x}\phi_{i,x}^{-1}). \tag{78}$$

In addition, the fibers $\boldsymbol{\pi}^{-1}(x)$ are vector spaces isomorphic to the representation space V.[39]

5. Let $(E, M, \boldsymbol{\pi}, G, F)$ be a fiber bundle and $U \subset M$ an open set. A *local cross section*[40] of the fiber bundle $(E, M, \boldsymbol{\pi}, G, F)$ on U is a mapping

$$s : U \to E \quad \text{ such that } \quad \pi \circ s = Id_U. \tag{79}$$

If $U = M$ we say that s is a *global section*. There is a relation between cross-sections and local trivializations. In fact, the existence of a global cross section on a principal bundle implies that this bundle is equivalent to the trivial one.

[38] Any principal bundle P over M (the Minkowski spacetime) is equivalent to a trivial bundle, i.e., it can be shown that $P \simeq M \times G$.

[39] Given a principal budle with structure group G, when we take a representation of G in some vector space we are specifying which kind of particles we want to study.

[40] Definition of a cross section justifies the definitions of multiforms fields (see eq.(20) as sections of the Clifford bundle.

6. To define the concept of a *connection* on a *PFB* $(P, M, \boldsymbol{\pi}, G)$, we recall that since $\dim(M) = 4$, if $\dim(G) = n$, then $\dim(P) = n + 4$. Obviously, for all $x \in M$, $\boldsymbol{\pi}^{-1}(x)$ is an n-dimensional submanifold of P difeomorphic to the structure group G and $\boldsymbol{\pi}$ is a submersion $\boldsymbol{\pi}^{-1}(x)$ is a closed submanifold of P for all $x \in M$.

The tangent space T_pP, $p \in \boldsymbol{\pi}^{-1}(x)$, is an $(n+4)$-dimensional vector space and the tangent space $V_pP \equiv T_p(\boldsymbol{\pi}^{-1}(x))$ to the fiber over x at the same point $p \in \boldsymbol{\pi}^{-1}(x)$ is an n-dimensional linear subspace of T_pP called the *vertical subspace* of T_pP[41].

Now, roughly speaking a connection on P is a rule that makes possible a *correspondence* between any two fibers along a curve $\sigma : I \supseteq \ \to M, t \mapsto \sigma(t)$. If p_0 belongs to the fiber over the point $\sigma(t_0) \in \sigma$, we say that p_0 is parallel translated along σ by means of this *correspondence*.

A *horizontal lift* of σ is a curve $\hat{\sigma} : I \supseteq \ \to P$ (described by the parallel transport of p). It is intuitive that such a transport takes place in P along directions specified by vectors in T_pP, which do not lie within the vertical space V_pP. Since the tangent vectors to the paths of the basic manifold passing through a given $x \in M$ span the entire tangent space T_xM, the corresponding vectors $\mathbf{X}_p \in T_pP$ (in whose direction parallel transport can generally take place in P) span a four-dimensional linear subspace of T_pP called the *horizontal space* of T_pP and denoted by H_pP. Now, the mathematical concept of a connection can be presented. This is done through three equivalent definitions ($\mathbf{c_1}$, $\mathbf{c_2}$, $\mathbf{c_3}$) given below which encode rigorously the intuitive discussion given above. We have,

Definition $\mathbf{c_1}$. A connection on a *PFB* $(P, M, \boldsymbol{\pi}, G)$ is an assignment to each $p \in P$ of a subspace $H_pP \subset T_pP$, called the horizontal subspace for that connection, such that H_pP depends smoothly on p and the following conditions hold:

(i) $\boldsymbol{\pi}_* : H_pP \to T_xM$, $x = \boldsymbol{\pi}(p)$, is an isomorphism.

(ii) H_pP depends smoothly on p.

(iii) $(R_g)_* H_pP = H_{pg}P, \forall g \in G, \forall p \in P$.

Here we denote by $\boldsymbol{\pi}_*$ the *differential*[39] of the mapping $\boldsymbol{\pi}$ and by $(R_g)_*$ the differential[42] of the mapping $R_g : P \to P$ (the right action) defined by $R_g(p) = pg$.

Since $x = \boldsymbol{\pi}(\hat{\sigma}(t))$ for any curve in P such that $\hat{\sigma}(t) \in \boldsymbol{\pi}^{-1}(x)$ and $\hat{\sigma}(0) = p_0$, we conclude that $\boldsymbol{\pi}_*$ maps all vertical vectors in the zero vector in T_xM, i.e., $\boldsymbol{\pi}_*(V_pP) = 0$ and we have,

$$T_pP = H_pP \oplus V_pP. \tag{80}$$

[41] Here we may be tempted to realize that as it is possible to construct the vertical space for all $p \in P$ then we can define a horizontal space as the complement of this space in respect to T_pP. Unfortunately this is not so, because we need a smoothly association of a horizontal space in every point. This is possible only by means of a connection.

[42] Sometimes called push-forward.

Then every $\boldsymbol{X}_p \in T_pP$ can be written as

$$\boldsymbol{X} = \boldsymbol{X}_p^h + \boldsymbol{X}_p^v, \qquad \boldsymbol{X}_p^h \in H_pP, \qquad \boldsymbol{X}_p^v \in V_pP. \tag{81}$$

Therefore, given a vector field X over M it is possible to lift it to a horizontal vector field over P, i.e., $\boldsymbol{\pi}_*(\boldsymbol{X}_p) = \boldsymbol{\pi}_*(\boldsymbol{X}_p^h) = X_x \in T_xM$ for all $p \in P$ with $\boldsymbol{\pi}(p) = x$. In this case, we call $\boldsymbol{X}_p^h$ *horizontal lift* of X_x. We say moreover that $\boldsymbol{X}$ is a horizontal vector field over P if $\boldsymbol{X}^h = \boldsymbol{X}$.
Definition $\mathbf{c_2}$. A *connection* on a *PFB* $(P, M, \boldsymbol{\pi}, G)$ is a mapping $\Gamma_p : T_xM \to T_pP$, such that $\forall p \in P$ and $x = \boldsymbol{\pi}(p)$ the following conditions hold:
(i) Γ_p is linear.
(ii) $\boldsymbol{\pi}_* \circ \Gamma_p = Id_{T_xM}$.
(iii) the mapping $p \mapsto \Gamma_p$ is differentiable.
(iv) $\Gamma_{R_gp} = (R_g)_*\Gamma_p$, for all $g \in G$.
We need also the concept of parallel transport. It is given by,

Definition. Let $\sigma : \ \ni I \to M$, $t \mapsto \sigma(t)$ with $x_0 = \sigma(0) \in M$, be a curve in M and let $p_0 \in P$ such that $\boldsymbol{\pi}(p_0) = x_0$. The *parallel transport* of p_0 along σ is given by the curve $\hat{\sigma} : \ \ni I \to P, t \mapsto \hat{\sigma}(t)$ defined by

$$\frac{d}{dt}\hat{\sigma}(t) = \Gamma_p(\frac{d}{dt}\sigma(t)), \tag{82}$$

with $p_0 = \hat{\sigma}(0)$ and $\hat{\sigma}(t) = p_\parallel$, $\boldsymbol{\pi}(p_\parallel) = x$.

In order to present **definition $\mathbf{c}_3$** of a connection we need to know more about the nature of the vertical space V_pP. For this, let $\hat{\boldsymbol{X}} \in T_eG = \mathfrak{G}$ be an element of the Lie algebra $\mathfrak{G}$ of G. The vector $\hat{\boldsymbol{X}}$ is the tangent to the curve produced by the exponential map

$$\hat{\boldsymbol{X}} = \frac{d}{dt}\left(\exp(t\hat{\boldsymbol{X}})\right)\Big|_{t=0}. \tag{83}$$

Then, for every $p \in P$ we can attach to each $\hat{\boldsymbol{X}} \in T_eG =$ a unique element $\hat{\boldsymbol{X}}_p^v \in V_pP$ as follows: let $\mathfrak{F} : P \to$ be given by $\mathfrak{F}(t) = f(p \exp t\hat{\boldsymbol{X}})$, where $f : (-\varepsilon, \varepsilon) \to P$ is a curve in P. Then we have

$$\hat{\boldsymbol{X}}_p^v(\mathfrak{F}) = \frac{d}{dt}\mathfrak{F}\left(f\left(p\exp(t\hat{\boldsymbol{X}})\right)\right)\Big|_{t=0}. \tag{84}$$

By this construction we attach to each $\hat{\boldsymbol{X}} \in T_eG = \mathfrak{G}$ a unique vector field over P, called the *fundamental field* corresponding to this element. We then have the canonical isomorphism

$$\hat{\boldsymbol{X}}_p^v \longleftrightarrow \hat{\boldsymbol{X}}, \quad \hat{\boldsymbol{X}}_p^v \in V_pP, \quad \hat{\boldsymbol{X}} \in T_eG = \mathfrak{G} \tag{85}$$

from which we get

$$V_pP \simeq \mathfrak{G}. \tag{86}$$

Definition $\mathbf{c_3}$. A connection on a *PFB* $(P, M, \boldsymbol{\pi}, G)$ is a 1-form field $\boldsymbol{\omega}$ on P with values in the Lie algebra $\mathfrak{G} = T_eG$ such that $\forall p \in P$ we have,

(i) $\boldsymbol{\omega}_p(\hat{\mathbf{X}}^v_p) = \hat{\mathbf{X}}$ and $\hat{\mathbf{X}}^v_p \longleftrightarrow \hat{\mathbf{X}}$, where $\hat{\mathbf{X}}^v_p \in V_pP$ and $\hat{\mathbf{X}} \in T_eG = \mathfrak{G}$.

(ii) $\boldsymbol{\omega}_p$ depends smoothly on p.

(iii) $\boldsymbol{\omega}_p[(R_g)_*\mathbf{X}_p] = (Ad_{g^{-1}}\boldsymbol{\omega}_p)(\mathbf{X}_p)$, where $Ad_{g^{-1}}\boldsymbol{\omega}_p = g^{-1}\boldsymbol{\omega}_p g$.

It follows that if $\{\mathcal{G}_a\}$ is a basis of $\mathfrak{G}$ and $\{\theta^i\}$ is a basis for T^*P then

$$\boldsymbol{\omega}_p = \omega^a_p \otimes \mathcal{G}_a = \omega^a_i(p)\theta^i_p \otimes \mathcal{G}_a, \tag{87}$$

where ω^a are 1-forms on P.

Then the horizontal spaces can be defined by defined by

$$H_pP = \ker(\boldsymbol{\omega}_p), \tag{88}$$

which shows the equivalence between the definitions.

7. *Connections on M*. Let $U \subset M$ and

$$s : U \to \boldsymbol{\pi}^{-1}(U) \subset P, \qquad \boldsymbol{\pi} \circ s = Id_U, \tag{89}$$

be a local section of the *PFB* $(P, M, \boldsymbol{\pi}, G)$. Given a connection $\boldsymbol{\omega}$ on P, we define the 1-form $s^*\boldsymbol{\omega}$ (the pullback of $\boldsymbol{\omega}$ under s) by

$$(s^*\boldsymbol{\omega})_x(X_x) = \boldsymbol{\omega}_{s(x)}(s_*X_x), \qquad X_x \in T_xM, \quad s_*X_x \in T_pP, \quad p = s(x). \tag{90}$$

It is quite clear that $s^*\boldsymbol{\omega} \in T^*U \otimes \mathfrak{G}$. It will be called *local gauge potential*. This object differs from the *gauge field* used by physicists by numerical constants (with units). Conversely we have the following

Proposition. Given $\bar{\boldsymbol{\omega}} \in T^*U \otimes \mathfrak{G}$ and a differentiable section of $\boldsymbol{\pi}^{-1}(U) \subset P$, $U \subset M$, there exists one and only one connection $\boldsymbol{\omega}$ on $\boldsymbol{\pi}^{-1}(U)$ such that $s^*\boldsymbol{\omega} = \bar{\boldsymbol{\omega}}$.

We recall that each local section s determines a local trivialization $\Phi : \boldsymbol{\pi}^{-1}(U) \to U \times G$ of P by setting

$$\Phi^{-1}(x, g) = s(x)g = pg = R_g p. \tag{91}$$

Conversely, Φ determines s since

$$s(x) = \Phi^{-1}(x, e). \tag{92}$$

Consider now

$$\begin{aligned}
&\bar{\boldsymbol{\omega}} \in T^*U \otimes \mathfrak{G}, \quad \bar{\boldsymbol{\omega}} = (\Phi^{-1}(x, e))^* \, \boldsymbol{\omega} = s^*\boldsymbol{\omega}, \quad s(x) = \Phi^{-1}(x, e), \\
&\bar{\boldsymbol{\omega}}' \in T^*U' \otimes \mathfrak{G}, \quad \bar{\boldsymbol{\omega}}' = (\Phi'^{-1}(x, e))^* \, \boldsymbol{\omega} = s'^*\boldsymbol{\omega}, \quad s'(x) = \Phi'^{-1}(x, e).
\end{aligned} \tag{93}$$

Then we can write, for each $p \in P$ $(\pi(p) = x)$, parameterized by the local trivializations Φ and Φ' respectively as (x, g) and (x, g') with $x \in U \cap U'$, that

$$\boldsymbol{\omega}_p = g^{-1}dg + g^{-1}\bar{\boldsymbol{\omega}}_x g = g'^{-1}dg' + g'^{-1}\bar{\boldsymbol{\omega}}'_x g'. \tag{94}$$

Now, if

$$g' = hg, \tag{95}$$

we immediately get from eq.(94) that

$$\bar{\boldsymbol{\omega}}'_x = hdh^{-1} + h\bar{\boldsymbol{\omega}}_x h^{-1}, \tag{96}$$

which can be called the *transformation law* for the gauge fields.

8. *Exterior Covariant derivatives.* Let $\Lambda^k(P, \mathfrak{G}), 0 \leq k \leq n$, be the set of all k-form fields over P with values in the Lie algebra $\mathfrak{G}$ of the gauge group G (and, of course, the connection $\boldsymbol{\omega} \in \Lambda^1(P, \mathfrak{G})$).. For each $\boldsymbol{\varphi} \in \Lambda^k(P, \mathfrak{G})$ we define the so called *horizontal form* $\boldsymbol{\varphi}^h \in \Lambda^k(P, \mathfrak{G})$ by

$$\boldsymbol{\varphi}^h_p(\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_k) = \boldsymbol{\varphi}(\boldsymbol{X}^h_1, \boldsymbol{X}^h_2, ..., \boldsymbol{X}^h_k), \tag{97}$$

where $\boldsymbol{X}_i \in T_pP$, $i = 1, 2, .., k$.

Notice that $\boldsymbol{\varphi}^h_p(\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_k) = 0$ if one (or more) of the $\boldsymbol{X}_i \in T_pP$ are vertical.

We define the *exterior covariant derivative* of $\boldsymbol{\varphi} \in \Lambda^k(P, \mathfrak{G})$ in relation to the connection $\boldsymbol{\omega}$ by

$$D^{\boldsymbol{\omega}}\boldsymbol{\varphi} = (d\boldsymbol{\varphi})^h \in \Lambda^{k+1}(P, \mathfrak{G}), \tag{98}$$

where $D^{\boldsymbol{\omega}}\boldsymbol{\varphi}_p(\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_k, \boldsymbol{X}_{k+1}) = d\boldsymbol{\varphi}_p(\boldsymbol{X}^h_1, \boldsymbol{X}^h_2, ..., \boldsymbol{X}^h_k, \boldsymbol{X}^h_{k+1})$. Notice that $d\boldsymbol{\varphi} = d\boldsymbol{\varphi}^a \otimes \mathcal{G}_a$ where $\boldsymbol{\varphi}^a \in \Lambda^k(P)$, $a = 1, 2, ..., n$.

9. We define the *commutator* of $\boldsymbol{\varphi} \in \Lambda^i(P, \mathfrak{G})$ and $\boldsymbol{\psi} \in \Lambda^j(P, \mathfrak{G})$, $0 \leq i, j \leq n$ by $[\boldsymbol{\varphi}, \boldsymbol{\psi}] \in \Lambda^{i+j}(P, \mathfrak{G})$ such that if $\boldsymbol{X}_1, ..., \boldsymbol{X}_{i+j} \in \sec TP$, then

$$[\boldsymbol{\varphi}, \boldsymbol{\psi}](\boldsymbol{X}_1, ..., \boldsymbol{X}_{i+j}) = \frac{1}{i!j!}\sum_{\sigma \in \mathcal{S}_n}(-1)^\sigma[\boldsymbol{\varphi}(\boldsymbol{X}_{\iota(1)}, ..., \boldsymbol{X}_{\iota(i)}), \boldsymbol{\psi}(\boldsymbol{X}_{\iota(i+1)}, ..., \boldsymbol{X}_{\iota(i+j)})], \tag{99}$$

where $\mathcal{S}_n$ is the permutation group of n elements and $(-1)^\sigma = \pm 1$ is the sign of the permutation. The brackets $[,]$ in the second member of eq.(99) are the Lie brackets in $\mathfrak{G}$.

By writing

$$\boldsymbol{\varphi} = \varphi^a \otimes \mathcal{G}_a, \quad \boldsymbol{\psi} = \psi^a \otimes \mathcal{G}_a, \quad \varphi^a \in \Lambda^i(P), \quad \psi^a \in \Lambda^j(P), \tag{100}$$

we can write

$$\begin{aligned} [\boldsymbol{\varphi}, \boldsymbol{\psi}] &= \varphi^a \wedge \psi^b \otimes [\mathcal{G}_a, \mathcal{G}_b] \\ &= f^c_{ab}(\varphi^a \wedge \psi^b) \otimes \mathcal{G}_c \end{aligned} \tag{101}$$

where f^c_{ab} are the structure constants of the Lie algebra .

With eq.(101) we can prove *easily* the following important properties involving commutators:

$$[\boldsymbol{\varphi}, \boldsymbol{\psi}] = (-)^{1+ij}[\boldsymbol{\psi}, \boldsymbol{\varphi}], \tag{102}$$

$$(-1)^{ik}[[\boldsymbol{\varphi}, \boldsymbol{\psi}], \boldsymbol{\tau}] + (-1)^{ji}[[\boldsymbol{\psi}, \boldsymbol{\tau}], \boldsymbol{\varphi}] + (-1)^{kj}[[\boldsymbol{\tau}, \boldsymbol{\varphi}], \boldsymbol{\psi}] = 0, \tag{103}$$

$$d[\boldsymbol{\varphi}, \boldsymbol{\psi}] = [d\boldsymbol{\varphi}, \boldsymbol{\psi}] + (-1)^i [\boldsymbol{\varphi}, d\boldsymbol{\psi}]. \tag{104}$$

for $\boldsymbol{\varphi} \in \Lambda^i(P, \mathfrak{G})$, $\boldsymbol{\psi} \in \Lambda^j(P, \mathfrak{G})$, $\boldsymbol{\tau} \in \Lambda^k(P, \mathfrak{G})$.

We shall also need the following identity

$$[\boldsymbol{\omega}, \boldsymbol{\omega}](\boldsymbol{X_1}, \boldsymbol{X_2}) = 2[\boldsymbol{\omega}(\boldsymbol{X}_1), \boldsymbol{\omega}(\boldsymbol{X}_2)]. \tag{105}$$

The proof of eq.(105) is as follows:
(i) Recall that

$$[\boldsymbol{\omega}, \boldsymbol{\omega}] = (\omega^a \wedge \omega^b) \otimes [\mathcal{G}_a, \mathcal{G}_b]. \tag{106}$$

(ii) Let $\boldsymbol{X}_1, \boldsymbol{X}_2 \in \sec TP$ (i.e., $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are vector fields on P). Then,

$$\begin{aligned} [\boldsymbol{\omega}, \boldsymbol{\omega}](\boldsymbol{X_1}, \boldsymbol{X_2}) &= (\omega^a(\boldsymbol{X}_1) \wedge \omega^b(\boldsymbol{X_2}) - \omega^a(\boldsymbol{X_2}) \wedge \omega^b(\boldsymbol{X_1}))[\mathcal{G}_a, \mathcal{G}_b] \\ &= 2[\boldsymbol{\omega}(\boldsymbol{X}_1), \boldsymbol{\omega}(\boldsymbol{X}_2)]. \end{aligned} \tag{107}$$

10. The *curvature form* of the connection $\boldsymbol{\omega} \in \Lambda^1(P, \mathfrak{G})$ is $\boldsymbol{\Omega}^{\omega} \in \Lambda^2(P, \mathfrak{G})$ defined by

$$\boldsymbol{\Omega}^{\omega} = D^{\omega}\boldsymbol{\omega}. \tag{108}$$

Proposition.

$$D^{\omega}\ \boldsymbol{\omega}(\boldsymbol{X_1}, \boldsymbol{X_2}) = d\boldsymbol{\omega}(\boldsymbol{X}_1, \boldsymbol{X}_2) + [\boldsymbol{\omega}(\boldsymbol{X}_1), \boldsymbol{\omega}(\boldsymbol{X}_2)]. \tag{109}$$

Eq.(109) can be written using eq.(107) (and recalling that $\boldsymbol{\omega}(\boldsymbol{X}) = \omega^a(\boldsymbol{X})\mathcal{G}_a$). Thus we have

$$\boldsymbol{\Omega}^{\omega} = D^{\omega}\ \boldsymbol{\omega} = d\boldsymbol{\omega} + \frac{1}{2}[\boldsymbol{\omega}, \boldsymbol{\omega}]. \tag{110}$$

11. **Proposition** (*Bianchi identity):*

$$D\boldsymbol{\Omega}^{\omega} = 0. \tag{111}$$

Proof: (i) Let us calculate $d\boldsymbol{\Omega}^{\omega}$. We have,

$$d\boldsymbol{\Omega}^{\omega} = d\left(d\boldsymbol{\omega} + \frac{1}{2}[\boldsymbol{\omega}, \boldsymbol{\omega}]\right). \tag{112}$$

We now take into account that $d^2\boldsymbol{\omega} = 0$ and that from the properties of the commutators given by eqs.(102), (103), (104) above, we have

$$\begin{aligned} d[\boldsymbol{\omega}, \boldsymbol{\omega})] &= [d\boldsymbol{\omega}, \boldsymbol{\omega}] - [\boldsymbol{\omega}, d\boldsymbol{\omega}], \\ [d\boldsymbol{\omega}, \boldsymbol{\omega}] &= -[\boldsymbol{\omega}, d\boldsymbol{\omega}], \\ [[\boldsymbol{\omega}, \boldsymbol{\omega}], \boldsymbol{\omega}] &= 0. \end{aligned} \tag{113}$$

By using eq.(113) in eq.(112) gives

$$d\boldsymbol{\Omega}^{\omega} = [d\boldsymbol{\omega}, \boldsymbol{\omega}]. \tag{114}$$

(ii) In eq.(114) use eq.(110) and the last equation in (113) to obtain

$$d\boldsymbol{\Omega}^{\omega} = [\boldsymbol{\Omega}^{\omega}, \boldsymbol{\omega}]. \tag{115}$$

(iii) Use now the definition of the exterior covariant derivative [eq.(99)] together with the fact that $\boldsymbol{\omega}(\boldsymbol{X}^h) = 0$, for all $\boldsymbol{X} \in T_pP$ to obtain

$$D^\omega \boldsymbol{\Omega}^\omega = 0.$$

We can then write the very important formula (known as the Bianchi identity),

$$D^\omega \boldsymbol{\Omega}^\omega = d\boldsymbol{\Omega}^\omega + [\boldsymbol{\omega}, \boldsymbol{\Omega}^\omega] = 0. \tag{116}$$

12. *Local curvature in the base manifold.* Let (U, Φ) be a local trivialization of $\pi^{-1}(x)$ and s the associated cross section as defined in **6**. Then, $s^*\boldsymbol{\Omega}^\omega \equiv \bar{\boldsymbol{\Omega}}^\omega$ (the pull back of $\boldsymbol{\Omega}^\omega$) is a well defined 2-form field on U which takes values in the Lie algebra $\mathfrak{G}$. Let $\bar{\boldsymbol{\omega}} = s^*\boldsymbol{\omega}$ (see eq.(93)). If we recall that the differential operator d commutes with the pull back, we immediately get

$$\bar{\boldsymbol{\Omega}}^\omega \equiv D^\omega \, \bar{\boldsymbol{\omega}} = d\,\bar{\boldsymbol{\omega}} + \frac{1}{2}\left[\bar{\boldsymbol{\omega}}, \bar{\boldsymbol{\omega}}\right]. \tag{117}$$

and

$$\begin{aligned} D^\omega \bar{\boldsymbol{\Omega}}^\omega &= 0, \\ D^\omega \bar{\boldsymbol{\Omega}}^\omega &= d\bar{\boldsymbol{\Omega}}^\omega + \left[\bar{\boldsymbol{\omega}}, \bar{\boldsymbol{\Omega}}^\omega\right] = 0. \end{aligned} \tag{118}$$

Eq.(118) is also known as Bianchi identity.

Observation 2. In gauge theories (Yang-Mills theories) $\bar{\boldsymbol{\Omega}}^\omega$ is (except for numerical factors with physical units) called a *field strength in the gauge* Φ.

Observation 3. When G is a matrix group, as is the case in the presentation of gauge theories by physicists, definition (99) of the commutator $[\boldsymbol{\varphi}, \boldsymbol{\psi}] \in \Lambda^{i+j}(P, \mathfrak{G})$ ($\boldsymbol{\varphi} \in \Lambda^i(P, \mathfrak{G})$, $\boldsymbol{\psi} \in \Lambda^j(P, \mathfrak{G})$) gives

$$[\boldsymbol{\varphi}, \boldsymbol{\psi}] = \boldsymbol{\varphi} \wedge \boldsymbol{\psi} - (-1)^{ij} \boldsymbol{\psi} \wedge \boldsymbol{\varphi}, \tag{119}$$

where $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$ are considered as matrices of forms with values in and $\boldsymbol{\varphi} \wedge \boldsymbol{\psi}$ stands for the usual matrix multiplication. Then, when G is a matrix group, we can write eqs.(110) and (117) as

$$\boldsymbol{\Omega}^\omega = D^\omega \, \boldsymbol{\omega} = d\boldsymbol{\omega} + \boldsymbol{\omega} \wedge \boldsymbol{\omega}, \tag{120}$$

$$\bar{\boldsymbol{\Omega}}^\omega = D^\omega \, \bar{\boldsymbol{\omega}} = d\bar{\boldsymbol{\omega}} + \bar{\boldsymbol{\omega}} \wedge \bar{\boldsymbol{\omega}}. \tag{121}$$

13. *Transformation of the field strengths under a change of gauge.* Consider two local trivializations (U, Φ) and (U', Φ') of P such that $p \in \pi^{-1}(U \cap U')$ has (x, g) and (x, g') as images in $(U \cap U') \times G$, where $x \in U \cap U'$. Let s, s' be the associated cross sections to Φ and Φ' respectively. By writing $s'^*\boldsymbol{\Omega}^\omega = \bar{\boldsymbol{\Omega}}^{\omega\prime}$, we have the following relation for the local curvature in the two different gauges such that $g' = hg$

$$\bar{\boldsymbol{\Omega}}^{\omega\prime} = h\bar{\boldsymbol{\Omega}}^\omega h^{-1}, \quad \text{for all } x \in U \cap U'. \tag{122}$$

14. We now give the *coordinate expressions* for the potential and field strengths in the trivialization Φ. Let $\langle x^\mu \rangle$ be a local chart for $U \subset M$ and let $\left\{ \partial_\mu = \dfrac{\partial}{\partial x^\mu} \right\}$ and $\{dx^\mu\}$, $\mu = 0, 1, 2, 3$, be (dual) bases of TU and T^*U respectively. Then,

$$\bar{\boldsymbol{\omega}} = \bar{\omega}^a \otimes \mathcal{G}_a = \bar{\omega}^a_\mu dx^\mu \otimes \mathcal{G}_a, \tag{123}$$

$$\bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}} = (\bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}})^a \otimes \mathcal{G}_a = \frac{1}{2} \bar{\Omega}^a_{\mu\nu} dx^\mu \wedge dx^\nu \otimes \mathcal{G}_a. \tag{124}$$

where $\bar{\omega}^a_\mu$, $\bar{\Omega}^a_{\mu\nu} : M \supset U \to R$ (or $\mathcal{C}$) and we get

$$\bar{\Omega}^a_{\mu\nu} = \partial_\mu \bar{\omega}^a_\nu - \partial_\nu \bar{\omega}^a_\mu + f^a_{bc} \bar{\omega}^b_\mu \bar{\omega}^c_\nu. \tag{125}$$

The following objects appear frequently in the presentation of gauge theories by physicists[43].

$$(\bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}})^a = \frac{1}{2} \bar{\Omega}^a_{\mu\nu} dx^\mu \wedge dx^\nu = d\bar{\omega}^a + \frac{1}{2} f^a_{bc} \bar{\omega}^b \wedge \bar{\omega}^c, \tag{126}$$

$$\bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\mu\nu} = \bar{\Omega}^a_{\mu\nu} \mathcal{G}_a = \partial_\mu \bar{\boldsymbol{\omega}}_\nu - \partial_\nu \bar{\boldsymbol{\omega}}_\mu + [\bar{\boldsymbol{\omega}}_\mu, \bar{\boldsymbol{\omega}}_\nu], \tag{127}$$

$$\bar{\boldsymbol{\omega}}_\mu = \bar{\omega}^a_\mu \mathcal{G}_a. \tag{128}$$

We now give the local expression of Bianchi identity. Using eqs.(118) and (126) we have

$$D^{\boldsymbol{\omega}} \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}} = (D^{\boldsymbol{\omega}} \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}})_{\rho\mu\nu} dx^\rho \wedge dx^\mu \wedge dx^\nu = 0. \tag{129}$$

By putting

$$(D^{\boldsymbol{\omega}} \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}})_{\rho\mu\nu} \doteq D_\rho \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\mu\nu} \tag{130}$$

we have

$$D_\rho \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\mu\nu} = \partial_\rho \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\mu\nu} + [\bar{\boldsymbol{\omega}}_\rho, \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\mu\nu}], \tag{131}$$

and

$$D_\rho \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\mu\nu} + D_\mu \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\nu\rho} + D_\nu \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}_{\rho\mu} = 0. \tag{132}$$

Physicists call the operator

$$D_\rho = \partial_\rho + [\boldsymbol{\omega}_\rho,]. \tag{133}$$

the *covariant derivative.* The reason for this name will be given now.

15. *Covariant derivatives of sections of associated vector bundles to a given PFB.*

Consider again, like in **4,** a vector bundle $(E, M, \boldsymbol{\pi}_1, G, F)$ denoted $E = P \times_\rho F$ *associated* to a *PFB* bundle $(P, M, \boldsymbol{\pi}, G)$ by the linear representation

[43] It is important to keep eqs.(126), (127), (128) in mind in order to understand the mathematical absurdities of the *AIAS* papers.

ρ of G in $F = \boldsymbol{V}$. Consider again the trivializations of P and E given by eqs.(75)-(77). Then, we have the

Definition. The *parallel transport* of $\boldsymbol{\Psi_0} \in E$, $\boldsymbol{\pi}_1(\boldsymbol{\Psi_0}) = x_0$, along the curve $\sigma : \ni I \to M$, $t \mapsto \sigma(t)$ from $x_0 = \sigma(0) \in M$ to $x = \sigma(t)$ is the element $\boldsymbol{\Psi}_\parallel \in E$ such that:

(i) $\boldsymbol{\pi}_1(\boldsymbol{\Psi}_\parallel) = x$,

(ii) $\chi_i(\boldsymbol{\Psi}_\parallel) = \rho(\varphi_i(p_\parallel) \circ \varphi_i^{-1}(p_0))\chi_i(\boldsymbol{\Psi_0})$.

(iii) $p_\parallel \in P$ is the parallel transport of $p_0 \in P$ along σ from x_0 to x as defined in eq.(82) above.

Definition. Let X be a vector at x_0 tangent to the curve σ (as defined above). The covariant derivative of $\boldsymbol{\Psi} \in \sec E$ in the direction of X is $(D_X^E \boldsymbol{\Psi}\ )_{x_0} \in \sec E$ such that

$$(D_X^E \boldsymbol{\Psi})(x_0) \equiv (D_X^E \boldsymbol{\Psi})_{x_0} = \lim_{t\to 0} \frac{1}{t}(\boldsymbol{\Psi}^0_{\parallel,t} - \boldsymbol{\Psi_0}), \tag{134}$$

where $\boldsymbol{\Psi}^0_{\parallel,t}$ is the "vector" $\boldsymbol{\Psi}_t \equiv \boldsymbol{\Psi(\sigma(t))}$ of a section $\boldsymbol{\Psi} \in \sec E$ parallel transported along σ from $\sigma(t)$ to x_0, the only requirement on σ being

$$\left.\frac{d}{dt}\sigma(t)\right|_{t=0} = X. \tag{135}$$

In the local trivialization (U_i, Ξ_i) of E (see eqs.(75)-(77)) if Ψ_t is the element in $\boldsymbol{V}$ representing $\boldsymbol{\Psi}_t$,

$$\chi_i(\Psi^0_{\parallel,t}) = \rho(g_0 g_t^{-1})\chi_{i|\sigma(t)}(\Psi_t). \tag{136}$$

By choosing p_0 such that $g_0 = e$ we can compute eq(134):

$$\begin{aligned} (D_X^E \boldsymbol{\Psi})_{x_0} &= \left.\frac{d}{dt}\rho(g^{-1}(t)\Psi_t)\right|_{t=0} \\ &= \left.\frac{d\Psi_t}{dt}\right|_{t=0} - \left(\rho'(e)\left.\frac{dg(t)}{dt}\right|_{t=0}\right)(\Psi_0). \end{aligned} \tag{137}$$

This formula is trivially generalized for the covariant derivative in the direction of an arbitrary vector field $Y \in \sec TM$.

With the aid of eq.(137) we can calculate, e.g., the covariant derivative of $\boldsymbol{\Psi} \in \sec E$ in the direction of the vector field $Y = \dfrac{\partial}{\partial x^\mu} \equiv \partial_\mu$. This covariant derivative is denoted $D_{\partial_\mu}\boldsymbol{\Psi} \equiv D_\mu \boldsymbol{\Psi}$.

We need now to calculate $\left.\dfrac{dg(t)}{dt}\right|_{t=0}$. In order to do that, recall that if $\dfrac{d}{dt}$ is a tangent to the curve σ in M, then $s_*\left(\dfrac{d}{dt}\right)$ is a tangent to $\hat{\sigma}$ the horizontal lift of σ, i.e., $s_*\left(\dfrac{d}{dt}\right) \in HP \subset TP$. As defined before $s = \Phi_i^{-1}(x, e)$ is the cross section associated to the trivialization Φ_i of P (see eq.(74). Then, as g is a mapping $U \to G$ we can write

$$\left[s_*(\frac{d}{dt})\right](g) = \frac{d}{dt}(g \circ \sigma). \tag{138}$$

To simplify the notation, introduce local coordinates $\langle x^\mu, g\rangle$ in $\pi^{-1}(U)$ and write $\sigma(t) = (x^\mu(t))$ and $\hat{\sigma}(t) = (x^\mu(t), g(t))$. Then,

$$s_*\left(\frac{d}{dt}\right) = \dot{x}^\mu(t)\frac{\partial}{\partial x^\mu} + \dot{g}(t)\frac{\partial}{\partial g}, \tag{139}$$

in the local coordinate basis of $T(\pi^{-1}(U))$. An expression like the second member of eq.(139) defines in general a vector tangent to P but, according to its definition, $s_*\left(\frac{d}{dt}\right)$ is in fact horizontal. We must then impose that

$$s_*\left(\frac{d}{dt}\right) = \dot{x}^\mu(t)\frac{\partial}{\partial x^\mu} + \dot{g}(t)\frac{\partial}{\partial g} = \alpha^\mu\left(\frac{\partial}{\partial x^\mu} + \bar{\omega}^a_\mu \mathcal{G}_a g\frac{\partial}{\partial g}\right), \tag{140}$$

for some α^μ.

We used the fact that $\frac{\partial}{\partial x^\mu} + \bar{\omega}^a_\mu \mathcal{G}_a g\frac{\partial}{\partial g}$ is a basis for HP, as can easily be verified from the condition that $\boldsymbol{\omega}(\mathbf{X}^h) = 0$, for all $\mathbf{X} \in HP$. We immediately get that

$$\alpha^\mu = \dot{x}^\mu(t), \tag{141}$$

and

$$\frac{dg(t)}{dt} = \dot{g}(t) = -\dot{x}^\mu(t)\bar{\omega}^a_\mu \mathcal{G}_a g, \tag{142}$$

$$\left.\frac{dg(t)}{dt}\right|_{t=0} = -\dot{x}^\mu(0)\bar{\omega}^a_\mu \mathcal{G}_a. \tag{143}$$

With this result we can rewrite eq.(137) as

$$(D^E_X \mathbf{\Psi})_{x_0} = \left.\frac{d\Psi_t}{dt}\right|_{t=0} - \rho'(e)\bar{\omega}(X)(\Psi_0), \qquad X = \left.\frac{d\sigma}{dt}\right|_{t=0}. \tag{144}$$

which generalizes trivially for the covariant derivative along a vector field $Y \in \sec TM$.

16. Suppose, e.g, that we take the tensor product $\mathcal{C}\ell(M) \otimes E$, where $\mathcal{C}\ell(M)$ is the Clifford bundle of differential forms[61] over M used in section 3 above, and E is an associated vector bundle to P, where the vector space of the (trivial) bundle is the linear space generated by $\{\mathcal{G}_a(x) = \mathcal{G}_a \in \mathfrak{G}\}$ and where $\rho(G) \equiv Ad(G)$. Consider the subbundle $\Lambda^2(M) \otimes E$ of $\mathcal{C}\ell(M) \otimes E$. It is obvious that we can identify $\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}$, the *local curvature* of the connection as defined in **13** above, with a section of $\Lambda^2(M) \otimes E$.

Written in local coordinates

$$\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}} = \left(\frac{1}{2}\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}{}^a_{\ \mu\nu}(x) dx^\mu \wedge dx^\nu\right) \otimes \mathcal{G}_a. \tag{145}$$

Now, we are working with a bundle that is a tensor product of two bundles. We restrict our attention in what follows to the case where it is possible (locally) to factorize the *functions* $\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}{}^{a}_{\mu\nu}(x)$ (supposed to be differentiable) as

$$\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}{}^{a}_{\mu\nu}(x) = f_{\mu\nu}(x)\eta^a(x), \tag{146}$$

where $f_{\mu\nu}(x)$ and $\eta^a(x)$ are also supposed to be differentiable functions.

If we denote by $\boldsymbol{\nabla}^{\Lambda^2(M)\otimes E}$ the covariant derivative acting on sections of $\Lambda^2(M)\otimes E$, then by definition[38],

$$\begin{aligned} \boldsymbol{\nabla}_X^{\Lambda^2(M)\otimes E}\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}} \quad = \quad & \boldsymbol{\nabla}_X\left(\tfrac{1}{2}f_{\mu\nu}(x(t))dx^\mu\wedge dx^\nu\right)\otimes(\eta^a(x(t))\mathcal{G}_a)+ \\ & +\left(\tfrac{1}{2}f_{\mu\nu}(x(t))dx^\mu\wedge dx^\nu\right)\otimes D_X^E(\eta^a(x(t))\mathcal{G}_a). \end{aligned} \tag{147}$$

where we take $\boldsymbol{\nabla}_X^{\Lambda^2(M)}\equiv\boldsymbol{\nabla}_X$ as the usual Levi-Civita connection acting on sections of $\mathcal{C}\ell(M)$, and D_X^E as the covariant derivative acting on sections of E. In eq.(137) we must calculate $\rho'(e)$ where ρ now refers to the adjoint representation of G. Now, as it is well known (see, e.g.,[39]) if $\mathfrak{f}\in\mathfrak{G}$, then since $Ad(g(t))\mathfrak{f} = g(t)\mathfrak{f}g(t)^{-1}$, we have

$$\frac{d}{dt}Ad(g(t))\mathfrak{f}\mid_{t=0} = \mathfrak{ad}(\mathfrak{g})(\mathfrak{f}) = [\mathfrak{g},\mathfrak{f}], \tag{148}$$

where

$$\mathfrak{g} = \left.\frac{dg(t)}{dt}\right|_{t=0} = -\dot{x}^\mu(0)\bar{\omega}^a_\mu\mathcal{G}_a. \tag{149}$$

and where the last equality in eq.(149) follows from eq.(143) modulo the isomorphism $T_eG\simeq\mathfrak{G}$.

Then, by using in eq.(137), $\Psi_0 = \eta^a(x(0))\mathcal{G}_a$, we have

$$\begin{aligned} \rho'(e)\left.\frac{dg(t)}{dt}\right|_{t=0} &= -\dot{x}^\mu(0)\eta^b(x(0))[\bar{\omega}^a_\mu\mathcal{G}_a,\mathcal{G}_b] \\ &= -\eta^a(x(0))[\bar{\boldsymbol{\omega}}(X),\mathcal{G}_a], \end{aligned} \tag{150}$$

and

$$D_X^E(\eta^a(x)\mathcal{G}_a) = \left.\frac{d\eta^a(x(t))}{dt}\right|_{t=0}\mathcal{G}_a + \eta^a(x(0))[\bar{\boldsymbol{\omega}}(X),\mathcal{G}_a]. \tag{151}$$

We can now trivially complete the calculation of $\boldsymbol{\nabla}_X^{\Lambda^2(M)\otimes E}\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}$ supposing that $\langle x^\mu\rangle$ are the usual Lorentz orthogonal coordinates of Minkowski spacetime. We have,

$$\boldsymbol{\nabla}_X^{\Lambda^2(M)\otimes E}\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}} = \frac{d}{dt}\left(\frac{1}{2}\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}{}^{a}_{\mu\nu}\right)dx^\mu\wedge dx\otimes\mathcal{G}_a + \left[\bar{\boldsymbol{\omega}}(X),\frac{1}{2}\bar{\mathbf{\Omega}}^{\boldsymbol{\omega}}{}^{a}_{\mu\nu}\right]dx^\mu\wedge dx\otimes\mathcal{G}_a. \tag{152}$$

This formula is trivially generalized for the covariant derivative in the direction of an arbitrary vector field $Y\in\sec TM$.

In particular when $Y = \dfrac{\partial}{\partial x^\rho} \equiv \partial_\rho$, it gives justification for the formula given by eq.(133) that we called the covariant derivative of the local curvature (or field strength). We have only to put

$$\boldsymbol{\nabla}_{\partial_\varrho}^{\Lambda^2(M)\otimes E}\bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}} \equiv D_\varrho \bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}} \stackrel{\circ}{=} D_\varrho \frac{1}{2}\bar{\boldsymbol{\Omega}}^{\boldsymbol{\omega}}{}_{\mu\nu} dx^\mu \wedge dx^\nu. \tag{153}$$

in order to complete the identification.

17. *Matter fields and the Higgs fields.*

(i) *Matter fields* are sections of $\Lambda^p(M) \otimes E$, where $E = P \times_\rho F$ are p-forms of the type (ρ, F).

(ii) *Spinor fields of spin 1/2* are sections of $S(M)$ and *generalized spinor fields of spin 1/2* and type (ρ, F) are sections of $S(M) \otimes E$, where $S(M)$ is a spinor bundle[39,60] of M.

(iii) *Higgs fields* are scalar matter fields of type (ρ, F), i.e., are sections of $\Lambda^0(M) \otimes E$.

observation 4. In $SU(2)$ gauge theory in order to formulate the Higgs mechanism, i.e., to give mass to some of the components of the field strength (i.e., the local curvature), ρ is taken as the vector representation of $SU(2)$ and F is taken as the linear space $\mathfrak{su}(2)$, the Lie algebra of $SU(2)$. This is exactly what is done in the formulation of the famous 't Hooft-Polyakov monopole theory, as described, e.g., in Ryder's book[44], a reference that *AIAS* authors used, but certainly did not undestand a single line.

6.2 Electromagnetism as a $U(1)$ gauge theory

We shall consider here a principal fiber bundle over the Minkowski spacetime M with structure group $U(1)$.

Recall that $U(1)$ is isomorphic to $SO(2)$, a fact that we denote as usual by $U(1) \simeq SO(2)$. This makes possible to parametrize $U(1)$ by elements of the unitary circle in a complex plane, i.e., we write

$$U(1) \simeq SO(2) = \{e^{-i\alpha}, \alpha \in R\}. \tag{154}$$

The Lie algebra $\mathfrak{u}(1)$ of $U(1)$ is then generated by the complex number $-i$. So a transition function $g_{jk} : U_j \cap U_k \to U(1)$ is given by $e^{i\psi(x)}$, where $\psi : U_i \cap U_j \to$ is a real function.

Now, given a local trivialization (gauge choice) $\Phi_V : \pi^{-1}(V) \to V \times U(1)$, $V \subset M$, we have a local section $\sigma_V : V \to P$. We associate to the connection $\boldsymbol{\omega}$ the gauge potential $\omega_V = \sigma_V^* \boldsymbol{\omega}$.

Since $\omega_V : V \to \mathfrak{u}(1) = \{-ia | a \in R\}$, we are able to write $\omega_V = -ieA_V$, where[44] $A_V \in \Lambda^1(U)$ is the *electromagnetic potential.*

Given another gauge choice Φ_W and his associated gauge potential ω_W, we have

$$\omega_W = g_{VW}\omega_V g_{VW}^{-1} + g_{VW}^{-1} dg_{VW},$$

[44] Here $e \in R - \{0\}$ is a constant which represents the electric charge.

where $g_{VW} : V \cap W \to U(1)$ is the corresponding transition function.

Since $U(1)$ is abelian it follows that

$$\omega_W = \omega_V + g_{VW}^{-1} dg_{VW}.$$

Therefore,

$$\omega_W = \omega_V + id\psi$$

and

$$A_W = A_V - \frac{1}{e} d\psi.$$

The fact that $U(1)$ is abelian also implies that

$$\boldsymbol{\Omega}^{\omega} = D^{\omega}\boldsymbol{\omega} = d\boldsymbol{\omega}.$$

Thus, the field strength of the electromagnetic field, in respect to the gauge potential ω_V, is given by

$$F_V = dA.$$

But it is easy to see that $d\omega_W = d(\omega_V + id\psi) = d\omega_V + idd\psi = d\omega_V$. Then $dA_V = dA_W$, leading to $F_V = F_W$ on $V \cap W$. This shows that F is globally defined[45] and we have

$$dF = 0$$

since $dF|_V = ddA_V = 0$ for all local trivialization Φ_V.

Of, course, without any additional hypothesis it is impossible to derive which is the value of δF. By means the definition of the current we are able to solve a variational problem on P which produces the desired equation[40]. By pulling back through a local section $\sigma_V : V \to P$ we obtain

$$\delta(F|_V) = J_e|_V ,$$

where $J_e|_V \in \Lambda^1(V)$ is the electric current pulled back to $V \subset M$. In the case when M is the Minkowski spacetime and for the vacuum $J_e = 0$ we have the pair of equations

$$\begin{aligned} dF &= 0, \\ \delta F &= 0. \end{aligned} \tag{155}$$

We discussed at length in **3** that this set of equations possess an infinite number of solutions which are *non transverse waves* in free space.

This shows that the statement that the existence of non transverse waves implies that electromagnetism cannot be described by a $U(1)$ theory is *false*.

[45] We put $F|_V = F_V$ to define it in all M.

6.3 $SU(2)$ gauge theory

In $SU(2)$ gauge theory, the connection 1-form $\boldsymbol{\omega} \in \Lambda^1(P, \mathfrak{su}(2))$ and the curvature 2-form $\boldsymbol{\Omega} \equiv \boldsymbol{\Omega}^{\omega} \in \Lambda^1(P, \mathfrak{su}(2))$, where $\mathfrak{su}(2)$ is the Lie algebra of $SU(2)$, are given by[46]

$$\boldsymbol{\Omega} = D\boldsymbol{\omega} = d\,\boldsymbol{\omega} + \frac{1}{2}[\boldsymbol{\omega}, \boldsymbol{\omega}], \tag{156}$$

$$D\boldsymbol{\Omega} = d\,\boldsymbol{\Omega} + \frac{1}{2}[\boldsymbol{\omega}, \boldsymbol{\Omega}]. \tag{157}$$

In a local trivialization Φ of the $SU(2)$ principal bundle, denoted $P_{SU(2)}$ and in local Lorentz orthogonal coordinates $\langle x^\mu \rangle$ of $U \subseteq M$, being s the cross section of $P_{SU(2)}$ associated to Φ the potential and field strength are given by

$$\boldsymbol{A} = s^*\boldsymbol{\omega} = A^a_\mu(x)dx^\mu \otimes \mathfrak{g}_{\mathfrak{a}} = A^a(x) \otimes \mathfrak{g}_{\mathfrak{a}} = \boldsymbol{A}_\mu(x)dx^\mu, \tag{158}$$

where $A^a(x) = A^a_\mu(x)dx^\mu$, $\boldsymbol{A}_\mu(x) = A^a_\mu(x)\mathfrak{g}_{\mathfrak{a}}$ and

$$\boldsymbol{F} = s\,^*\boldsymbol{\Omega} = \frac{1}{2}(F^a_{\mu\nu}(x)dx^\mu \wedge dx^\nu) \otimes \mathfrak{g}_{\mathfrak{a}}, \boldsymbol{F}_{\mu\nu}(x) = \frac{1}{2}F^a_{\mu\nu}(x)\mathfrak{g}_{\mathfrak{a}}, \tag{159}$$

where $F^a(x) = \frac{1}{2}F^a_{\mu\nu}(x)dx^\mu \wedge dx^\nu$, $\boldsymbol{F}_{\mu\nu}(x) = \frac{1}{2}F^a_{\mu\nu}(x)\mathfrak{g}_{\mathfrak{a}}$ and where

$$[\mathfrak{g}_a, \mathfrak{g}_b] = \epsilon^c_{ab}\mathfrak{g}_c, \quad a, b, c = 1, 2, 3. \tag{160}$$

express the structure constants given by the commutator relations of the Lie algebra $\mathfrak{su}(2)$. Keep in mind that A^a_μ, $F^a_{\mu\nu}(x) : M \to R$ (or $\mathcal{C}$) are scalar valued functions[47]. In the quantum version of the theory these objects are hermitian operators.

observation 5. in most physical textbooks the tensor product $\otimes$ is omitted. Here we keep it because it is our intention to show that section 3 of *AIAS***1** is full of mathematical nonsense[48]. Taking the pull back under s of eqs.(156), (157) we get

$$\boldsymbol{F}_{\mu\nu} = \partial_\mu\boldsymbol{A}_\nu - \partial_\nu\boldsymbol{A}_\mu + [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu] \tag{161}$$

$$= \partial_\mu\boldsymbol{A}_\nu - \partial_\nu\boldsymbol{A}_\mu + A^a_\mu(x)A^b_\nu(x)\epsilon^c_{ab}\mathfrak{g}_c, \tag{162}$$

$$F^c_{\mu\nu} = \partial_\mu A^c_\nu - \partial_\nu A^c_\mu + A^a_\mu(x)A^b_\nu(x)\epsilon^c_{ab}, \tag{163}$$

and

$$D_\rho\boldsymbol{F}_{\mu\nu} + D_\mu\boldsymbol{F}_{\nu\rho} + D_\nu\boldsymbol{F}_{\rho\mu} = 0, \tag{164}$$

$$D_\rho F^a_{\mu\nu} + D_\mu F^a_{\nu\rho} + D_\nu F^a_{\rho\mu} = 0, \tag{165}$$

[46] To simplify the notation we write in this section $D \equiv D^\omega$

[47] They can be real or complex functions depending, e.g., on the particular representation choose for the gauge group.

[48] Keep in mind that physicists in general put $\mathfrak{g}_a = -i\mathfrak{e}_a$ or use particular matrix representations for the $\mathfrak{g}_a$.

with

$$D_\rho \boldsymbol{F}_{\mu\nu} = \partial_\rho \boldsymbol{F}_{\mu\nu} + [\boldsymbol{A}_\rho, \boldsymbol{A}_{\mu\nu}], \tag{166}$$

$$D_\rho F^c_{\mu\nu} = \partial_\rho F^c_{\mu\nu} + A^a_\rho F^b_{\mu\nu}\epsilon^c_{ab}. \tag{167}$$

Eq.(164) (Bianchi identity) is the generalization of the homogeneous Maxwell equation $dF = 0$, which, as it is well known, reads $\partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0$, when written in components.

Now, which is the analogous of the inhomogeneous Maxwell equation $\delta F = -J_e$, which in local components reads $\partial^\mu F_{\mu\nu} = J_{e\,\nu}$?

As, in the case of Maxwell theory, the analogous equation for the $SU(2)$ gauge theory cannot be obtained without *extra* assumptions. For the vacuum case, i.e., when the gauge field is only interacting with itself, the analogous of $\delta A = 0$ is postulated[43] to be

$$D^\mu \boldsymbol{F}_{\mu\nu} = \partial^\mu \boldsymbol{F}_{\mu\nu} + [\boldsymbol{A}^\mu, \boldsymbol{F}_{\mu\nu}] = 0, \qquad \boldsymbol{A}^\mu = \eta^{\mu\nu}\boldsymbol{A}_\nu.$$

For the case where the gauge field is in interaction with some matter field which produces a conserved current $\boldsymbol{J}_\mu = J^a_\mu \mathfrak{g}_\mathfrak{a}$, and the theory is supposed to be derivable from an action principle, the analogous of the inhomogeneous Maxwell equations results

$$D^\mu \boldsymbol{F}_{\mu\nu} = \partial^\mu \boldsymbol{F}_{\mu\nu} + [\boldsymbol{A}^\mu, \boldsymbol{F}_{\mu\nu}] = \boldsymbol{J}_\mu. \tag{168}$$

7 Flaws in the "new electrodynamics"

In what follows we comments on some (unbelievable) mathematical flaws at the foundations of the "new electrodynamics" of the *AIAS* group and of Barrett.

To start, putting

$$[\boldsymbol{F}^{\mu\nu}] = \begin{bmatrix} 0 & -\boldsymbol{E}^1 & -\boldsymbol{E}^2 & -\boldsymbol{E}^3 \\ \boldsymbol{E}^1 & 0 & -\boldsymbol{B}^3 & \boldsymbol{B}^2 \\ \boldsymbol{E}^2 & \boldsymbol{B}^3 & 0 & -\boldsymbol{B}^1 \\ \boldsymbol{E}^3 & -\boldsymbol{B}^2 & \boldsymbol{B}^1 & 0 \end{bmatrix}, \tag{169}$$

we can write using the notations of section 3 and taking $A^a(x) = A^a_\mu(x)dx^\mu$ and $F^a(x) = \frac{1}{2}F^a_{\mu\nu}(x)dx^\mu \wedge dx^\nu$ as sections of the Clifford bundle $\mathcal{C}\ell(M)$,

$$\vec{\boldsymbol{E}} = \vec{\sigma}_i \otimes \boldsymbol{E}^i, \boldsymbol{E}^i = E^{ia}\mathfrak{g}_\mathfrak{a},$$

$$\vec{\boldsymbol{B}} = \vec{\sigma}_i \otimes \boldsymbol{B}^i, \boldsymbol{B}^i = B^{ia}\mathfrak{g}_\mathfrak{a},$$

$$(dx^\mu \otimes \boldsymbol{A}_\mu)\gamma_0 = \boldsymbol{A}_0 + \vec{\boldsymbol{A}}, \quad \boldsymbol{A}_0 = A^a_0\mathfrak{g}_\mathfrak{a}, \quad \vec{\boldsymbol{A}} = \vec{\sigma}_i \otimes \boldsymbol{A}^i, \quad \boldsymbol{A}^i = A^{ia}\mathfrak{g}_\mathfrak{a},$$

$$(dx^\mu \otimes \boldsymbol{J}_\mu)\gamma_0 = \boldsymbol{J}_0 + \vec{\boldsymbol{J}}, \quad \boldsymbol{J}_0 = J^a_0\mathfrak{g}_\mathfrak{a}, \quad \vec{\boldsymbol{J}} = \vec{\sigma}_i \otimes \boldsymbol{J}^i, \quad \boldsymbol{J}^i = J^{ia}\mathfrak{g}_\mathfrak{a}. \tag{170}$$

In eq.(170), the bold notation means a vector in isospace and the $\rightarrow$ notation, as in section 3 above, means an Euclidean vector.

By using the notations of eq.(170) we can write eqs.(164) and eq.(168) as a system of Maxwell-like equations in the vector calculus formalism. Choosing a matricial representation for the Lie algebra of $\mathfrak{su}(2)$ by putting $\mathfrak{g}_a = \hat{\boldsymbol{\sigma}}_a$, where $\hat{\boldsymbol{\sigma}}_a$ are the Pauli matrices we get, the following equations resembling the ones of classical electromagnetism[49]:

$$\nabla \cdot \vec{\boldsymbol{E}} = \boldsymbol{J}_0 - iq(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{E}} - \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}}), \tag{171}$$

$$\frac{\partial \vec{\boldsymbol{E}}}{\partial t} - \nabla \times \vec{\boldsymbol{B}} + i[\vec{A}_0, \vec{\boldsymbol{E}}] - iq(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} - \vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}) = -\vec{\boldsymbol{J}}, \tag{172}$$

$$\nabla \cdot \vec{\boldsymbol{B}} + i(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} - \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}) = \boldsymbol{0}, \tag{173}$$

$$\frac{\partial \vec{\boldsymbol{B}}}{\partial t} + \nabla \times \vec{\boldsymbol{E}} + i[\vec{A}_0, \vec{\boldsymbol{B}}] + i(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{E}} - \vec{\boldsymbol{E}} \times \vec{\boldsymbol{A}}) = 0 \tag{174}$$

At this point Barret presents what he called Harmuth's amended equations[6] (we write the equations with a correct notation),

$$\left\{ \begin{array}{ll} \nabla \cdot \vec{E} = \rho_e, & \nabla \times \vec{H} - \partial_t \vec{D} = \vec{J}_e, \\ \nabla \cdot \vec{B} = \rho_m, & \nabla \times \vec{E} + \partial_t \vec{B} = -\vec{J}_m. \end{array} \right. , \tag{175}$$

$$\vec{J}_e = \sigma \vec{E}, \qquad \vec{J}_m = s\vec{H} \tag{176}$$

Now, before proceeding it is very important to note that in[6−8] Barrett used the same symbols in both the non abelian Maxwell equations and the amended Harmuth's equations. He did not distinguish between the bold and arrow notations and indeed used no bold nor arrow notation at all. He then said[6]:

> "comparing the $SU(2)$ formulation of Maxwell equations and the Harmuth equations reveals the following identities"

and then presents the list. We write only one of these identities in what follows *using only here in the text* the same notation as the one used by Barrett in[6−8],

$\boldsymbol{U(1)}$ **symmetry**	$\boldsymbol{SU(2)}$ **symmetry**
$\rho_e = J_0$	$\rho_e = J_0 - iq(A \cdot E - E \cdot A)$

It is quite obvious that the equation in "$SU(2)$ symmetry" should be written as

$$\boldsymbol{\rho}_e = \boldsymbol{J}_0 - iq(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{E}} - \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{E}}). \tag{177}$$

[49] In the following equations we explicitly introduce the coupling constant q. Also, the dot product $\cdot$ and the vector product refers to these operations in the Euclidean part of the objects where the operations are applied.

Also, it is quite obvious that it is impossible to identify ρ_e with $\boldsymbol{\rho}_e$. The first is the zero component of a vector in Minkowski spacetime, being a real function, whereas the second is a real function (a zero-form) taking values in isotopic vector space.

It is moreover clear that trying to identify ρ_e with $\boldsymbol{\rho}_e$ amounts to identify also $\vec{\boldsymbol{E}}$ with $\vec{E}$, $\vec{\boldsymbol{B}}$ with $\vec{B}$, $\vec{\boldsymbol{A}}_0$ with A_0 and $\vec{\boldsymbol{A}}$ with $\vec{A}$, $\vec{\boldsymbol{J}}$ with $\vec{J}$, a sheer *nonsense* .

It is hard to believe that someone could do a confusion like the one above described. Unfortunately Barrett's notation seems to indicate that he did.

But, what was Barrett trying to do with the above identifications? Well, these "identifications" had among its objectives[50] to present a justification for Harmut's ansatz[51]. He wrote[6]:

> " Consequently, Harmutz's Ansatz can be interpreted as: (i) a mapping of Maxwell's ($U(1)$ symmetrical) equations into a higher order symmetric field (of $SU(2)$ symmetry) or covering space, where magnetic monopoles and charges exist; (ii) solving the equations for propagation velocities; and (iii) mapping the solved equations back into the $U(1)$ symmetrical field (thereby removing the magnetic monopole and charge)."

Now, the correct justification for Harmuth's ansatz is simply the very well known fact that Maxwell equations are invariant under duality rotations[16]and this has nothing to do with a $SU(2)$ symmetry of any kind.

Besides these misunderstandings by Barrett of Harmuth's papers, the fact is that there are other serious flaws in Barret's papers, and indeed in the section **7** we comment on a really *unacceptable* error for an author trying to correct Maxwell theory.

Now, we show that the *AIAS* group also did not understood the meaning of the "$SU(2)$ Maxwell's equations". The *proof* of this statement start when we give a look at page 313 of [0] in a note called "THE MEANING OF BARRETT'S NOTATION".

There the *AIAS* authors quoted that Rodrigues did not understand Barrett's notation[52], but now we prove that in fact are them who did not understand the meaning of the $SU(2)$ equations. Observe that eq.(1) at page 313 of [0] is wrongly

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[50]Other objectives were to "explain" electromagnetic phenomena that he claims (and also the *AIAS* group) that cannot be explained by $U(1)$ electrodynamics. In particular in[8] he arrived at the conclusion that a fidedigne explanation of the Sagnac effect requires that $U(1)$ electrodynamics be substituted by a covering theory, that he called the $SU(2)$ gauge electrodynamics theory. It is necessary to emphasize here that Sagnac effect is trivially explained by $U(1)$ electrodynamics and relativity theory. In particular, contrary to what is stated by Vigier (one of the *AIAS* authors) in[54−56] the Sagnac effect does not permit the identification of a preferred inertial frame. This will be discussed elsewhere. We call also the reader's attention on the Vigier statement in[54−56] that the phenomena of unipolar induction permits the identification of a preferred inertial frame is also completely misleading, as shown in[57].

[51]We show in the section 7 that Harmuth's "amended equations" constitute a legitimatized (and quite original) way to solve the original Maxwell equations for a particular physical problem.

[52]It is necessary to say here that violating what Evan's preach, the *AIAS* group quoted W.A. R., saying that he did not understood a capital point in this whole affair. *AIAS* group

printed, the right equation to start the discussion being eq.(171) above. This is a matrix equation and representing the Lie algebra of $\mathfrak{su}(2)$ in $\mathcal{C}(2)$) we have:

$$
\begin{aligned}
&\begin{bmatrix} \nabla\cdot\vec{\boldsymbol{E}}^{(3)} & \nabla\cdot\vec{\boldsymbol{E}}^{(1)} - i\nabla\cdot\vec{\boldsymbol{E}}^{(2)} \\ \nabla\cdot\vec{\boldsymbol{E}}^{(1)} + i\nabla\cdot\vec{\boldsymbol{E}}^{(2)} & -\nabla\cdot\vec{\boldsymbol{E}}^{(3)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_0^{(3)} & \boldsymbol{J}_0^{(1)} - i\boldsymbol{J}_0^{(2)} \\ \boldsymbol{J}_0^{(1)} + i\boldsymbol{J}_0^{(2)} & -\boldsymbol{J}_0^{(3)} \end{bmatrix} \\
&- iq \begin{bmatrix} \vec{\boldsymbol{A}}^{(3)} & \vec{\boldsymbol{A}}^{(1)} - i\vec{\boldsymbol{A}}^{(2)} \\ \vec{\boldsymbol{A}}^{(1)} + i\vec{\boldsymbol{A}}^{(2)} & -\vec{\boldsymbol{A}}^{(3)} \end{bmatrix} \cdot \begin{bmatrix} \vec{\boldsymbol{E}}^{(3)} & \vec{\boldsymbol{E}}^{(1)} - i\vec{\boldsymbol{E}}^{(2)} \\ \vec{\boldsymbol{E}}^{(1)} + i\vec{\boldsymbol{E}}^{(2)} & -\vec{\boldsymbol{E}}^{(3)} \end{bmatrix} \\
&+ iq \begin{bmatrix} \vec{\boldsymbol{E}}^{(3)} & \vec{\boldsymbol{E}}^{(1)} - i\vec{\boldsymbol{E}}^{(2)} \\ \vec{\boldsymbol{A}}^{(2)} + i\vec{\boldsymbol{E}}^{(2)} & -\vec{\boldsymbol{E}}^{(3)} \end{bmatrix} \cdot \begin{bmatrix} \vec{\boldsymbol{A}}^{(3)} & \vec{\boldsymbol{A}}^{(1)} - i\vec{\boldsymbol{A}}^{(2)} \\ \vec{\boldsymbol{A}}^{(1)} + i\vec{\boldsymbol{A}}^{(2)} & -\vec{\boldsymbol{A}}^{(3)} \end{bmatrix}
\end{aligned}
\tag{178}
$$

Now, let us write the equation corresponding to the 11 element of this matrix equation,

$$
\nabla\cdot\vec{\boldsymbol{E}}^{(3)} = \boldsymbol{J}_0^{(3)} + 2q(\vec{\boldsymbol{E}}^{(2)}\cdot\vec{\boldsymbol{A}}^{(1)} - \vec{\boldsymbol{E}}^{(1)}\cdot\vec{\boldsymbol{A}}^{(2)}) \tag{179}
$$

This equation in cartesian components read

$$
\begin{aligned}
\frac{\partial \boldsymbol{E}_x^{(3)}}{\partial x} + \frac{\partial \boldsymbol{E}_y^{(3)}}{\partial y} + \frac{\partial \boldsymbol{E}_z^{(3)}}{\partial z} = \boldsymbol{J}_0^{(3)} &+ 2q(\boldsymbol{E}_x^{(2)}\cdot\boldsymbol{A}_x^{(1)} + \boldsymbol{E}_y^{(2)}\cdot\boldsymbol{A}_y^{(1)} + \boldsymbol{E}_z^{(2)}\cdot\boldsymbol{A}_z^{(1)}) \\
&-2q(\boldsymbol{E}_x^{(1)}\cdot\boldsymbol{A}_x^{(2)} + \boldsymbol{E}_y^{(1)}\cdot\boldsymbol{A}_y^{(2)} + \boldsymbol{E}_z^{(1)}\cdot\boldsymbol{A}_z^{(2)})
\end{aligned}
\tag{180}
$$

This equation is to be compare with eq.(6) of page 313 of [0] derived by the *AIAS* group[53],

$$
\text{``}\frac{\partial E_z}{\partial z} = J_0 + 2q(E_y A_x - E_y A_x)\text{''} \tag{181}
$$

Comparison of equations (180) and (181) proves our claim that the *AIAS* group do not understand the equations they use!

8 Inconsistencies in section 3 of *AIAS*1

What has been said in the last sections proves that *AIAS* theory and (also Barrett's papers) are sheer nonsense. *AIAS* authors claims to have proven in section 3 of *AIAS***1** that their non-Abelian electrodynamics is equivalent to Barrett's non-Abelian electrodynamics. The fact is that section 3 of *AIAS***1** is simply wrong. It is a *pot-pourri* of inconsistent mathematics where the authors make confusions *worse* yet than the ones pointed above. We dennounce some of

did not acknowledged W.A.R. of that fact, and worse, did not inform their "unfortunate" readers from where they learned that W.A.R. did not understand Barrett's notation. Well, they learned that when reading the report that W.A.R. wrote for the *Found. Phys.* rejecting some papers that they submitted to that journal.

[53] This equation has been written between quotation marks in order to identify that it is a wrong equation. The same convention applies to all wrong equations quoted from other authors.

them in what follows. To show the "equivalence" between their approach and Barrett's, the *AIAS* authors introduce a theory where the $SU(2)$ gauge field interacts with a Higgs field. The interaction is given by the usual Lagrangian formalism, as given, e.g., in Ryder's book[43].

Recall that the Higgs field in this case (according to the general definition given in **17** of section 4.1 above) is a section of $\Lambda^0(M) \otimes E$ where the vector space of E is $F = \mathfrak{su}(2)$. Then, according to the notations introduced in the last section, $\boldsymbol{H}$ is an *isovector* and we can write

$$\boldsymbol{H} = H^a \mathfrak{e}_a \equiv (H^1, H^2, H^3), \tag{182}$$

where $H^a : M \to C$ are complex functions[54], and $\mathfrak{e}_a, a = 1, 2, 3$ are the generators of $\mathfrak{su}(2)$ (which is isomorphic to the Lie algebra of $SO(3)$) and satisfy

$$[\mathfrak{e}_a, \mathfrak{e}_b] = i\epsilon_{ab}^c \mathfrak{e}_c. \tag{183}$$

We now exhibit explicitly some of the mathematical nonsense of section 3 of *AIAS*$\mathbf{1}$.

(i) Eq.(43) of *AIAS*$\mathbf{1}$ is *wrong*. The right equation for F^a_{ij} is eq.(161) above. Note that, as emphasized in section 4 , F^a_{ij} and the A^b_j are scalar functions and so the commutator appearing in eq.(43) of *AIAS*$\mathbf{1}$ must be zero.

(ii) In eq.(67) *AIAS* authors changed their mind about the mathematical nature of the H^a, and in a completely *ad hoc* way, assumed that the H^a, $a = 1, 2$, are given by

$$\text{“}H^a = F^a_i \boldsymbol{e}^i\text{”}, \text{for } a = 1, 2. \tag{184}$$

The first remark here is that the *AIAS* authors do not explain in which space the $\boldsymbol{e}^i$ live. If we give a look at the equations following their eq.(56) it appears that the $\boldsymbol{e}^i$, $i = 1, 2, 3$, are basis vectors of a 3-dimensional vector space. This makes eq.(184) [eq.(43) in *AIAS*$\mathbf{1}$] sheer nonsense and invalidates all their calculations[55].

(iii) The inconsistency can be seen also with the definition of $\vec{B}^{(3)}$ given in eq.(66) of *AIAS*$\mathbf{1}$. Since until eq.(65) of *AIAS*$\mathbf{1}$ the H^a are functions (which is the case until eq.(67) in *AIAS*$\mathbf{1}$), the authors define

$$\text{“}\mathbf{A}^1 = \nabla H^1, \qquad \mathbf{A}^2 = \nabla H^2\text{”}. \tag{185}$$

Then, they write for "$\boldsymbol{B}^{(3)}$",

$$\begin{aligned} \text{“}\boldsymbol{B}^{(3)} &= -i\frac{e}{\hbar}\mathbf{A}^1 \times \mathbf{A}^2 \\ &= i\frac{e}{\hbar}\epsilon_{ijk}\partial_j H^1 \partial_k H^2 \text{ ”}. \end{aligned} \tag{186}$$

[54] Note that in eq.(41) of *AIAS*$\mathbf{1}$, the *AIAS* authors describe a situation where $H^1 = H^2 = 0$ and $H^3 = \sqrt{m}$, where m is the mass of the Higgs field. This shows clearly that they start their "theory" using a Higgs field which is a section of $\Lambda^0(M) \otimes E$, where the vector space of E is $F = \mathfrak{su}(2)$

[55] In [34], and also several times in[0] we are advised that the $\{\boldsymbol{e}^i\}$ are to be identified with the canonical basis of Euclidean (vector) space. Of course, this does not solve the inconsistences pointed above.

The equality in the second line of eq.(186) (which is eq.(66) of *AIAS***1**) is *obviously* wrong. Recall that the H^a are complex functions —which seems to be the case according to eq.(45) and eq.(52) and until eq.(67) of *AIAS***1.** After that equation authors change their mind as to the nature of the H_i, in order to try to give meaning to their eq.(66).

Then they write their eq.(68),

$$\text{“}\boldsymbol{B}^{(3)} = i\frac{e}{\hbar}\epsilon_{ijk}\partial_j F_m^1 \partial_k F_n^2 \boldsymbol{e}^m \boldsymbol{e}^n\text{”} \ . \tag{187}$$

Next a complete *ad hoc* rule is invoked—explicitly, *AIAS* authors wrote:

> " Since $\boldsymbol{e}^m$, $\boldsymbol{e}^n$ are orthogonal their product can only be cyclic, so if $\boldsymbol{e}^m \boldsymbol{e}^n = \epsilon^{mnr}\boldsymbol{e}_r$: ..."

After that the *AIAS* authors proceed by using some other illicit manipulations and very *odd* logic reasoning to arrive at their eq.(71),

$$\text{“}\boldsymbol{B}^{(3)} = i\frac{e}{\hbar}(\partial_j F_j^1 \partial_k F_k^2 - \partial_j F_k^1 \partial_k F_j^2)\text{”} \ . \tag{188}$$

Well, this equation is simply nonsense again, for the first member is a vector function and the second a scalar function.

9 A brief comment on Harmuth's papers

In the abstract of the first of Harmuth's papers[35] he said that there was never a satisfactory concept of propagation velocity of *signals* within the framework of Maxwell theory that is represented by Maxwell equations

$$\begin{cases} \nabla \cdot \vec{E} = \rho_e, & \nabla \times \vec{H} - \partial_t \vec{D} = \vec{J}_e, \\ \nabla \cdot \vec{B} = 0, & \nabla \times \vec{E} + \partial_t \vec{B} = 0. \end{cases} \tag{189}$$

$$\vec{D} = \varepsilon \vec{E}, \qquad \vec{B} = \mu \vec{H}. \tag{190}$$

He also said that the often mentioned group velocity fails on two accounts, one being that it is almost always larger than the velocity of light[56] in radio transmission through the atmosphere; the other being that its derivation implies a transmission rate of information equal to zero.

Harmuth recalls that he searched in vain in the literature for a solution of Maxwell equations for a wave with a beginning and an end (i.e., with compact support in the time domain), that could represent a signal[57], propagating in a

[56] There are now several experiments that show that superluminal group velocities have physical meaning as, e.g., [52,53]. A recent review of the status of what is superluminal wave motion can be found in [37].

[57] Sommerfeld and Brillouin[46], called *signals*: (i) electromagnetic waves such that, each one of its non null components is zero at $z = 0$, for $t < 0$ and equal to some function $f(t)$ for $t > 0$, or: waves with compact support in the time domain, i.e., at $z = 0$ the signal $f(t)$ is non zero only for $0 < t < T$.

lossy medium. He said also that "one might think that the reason is the practical difficulty of obtaining solutions, but this is only partly correct". He arrives at the conclusion that " the fault lies with Maxwell equations rather than their solutions". He said "in general, there can be no solutions for signals propagating in lossy media." and concludes "more scientifically, Maxwell equations fail for waves with nonnegligible relative frequency bandwidth propagating in a medium with non negligible losses". His suggestion to overcome the problem is to add a magnetic current density in Maxwell equations, thus getting[58]

$$\begin{cases} \nabla \cdot \vec{E} = \rho_e, & \nabla \times \vec{H} - \partial_t \vec{D} = \vec{J}_e, \\ \nabla \cdot \vec{B} = \rho_m, & \nabla \times \vec{E} + \partial_t \vec{B} = -\vec{J}_m. \end{cases} \tag{191}$$

He said that:

"but the remedy is even more surprising than the failure, since it is generally agreed that magnetic currents have not been observed and it is known from the study of monopoles that a magnetic current density can be eliminated or created by means of a so-called duality transformation. The explanation of both riddles is the singularities encountered in the course of calculation. If one chooses the current density zero before reaching the last singularity, one obtains no solution: if one does so after reaching the last singularity, one gets a solution."

Here we want to comment that with exception of the above inspired ansatz, the remaining mathematics of Harmuth's paper was already very well known at the time he published[35]. We repeat below some of his calculations to clearly separate the new and old knowledgment in his approach. This will be important in order to comment one more of Barrett's flaws. So, in what follows we show that using the duality invariance of *ME* and Harmuth's ansatz that in a lossy medium the dynamics of a wave with compact support in the spatial domain is such that its front propagates with the speed

$$c_1 = \frac{1}{\sqrt{\varepsilon\mu}}. \tag{192}$$

where ε, μ are the vacuum constants. Harmuth[35] studies the motion of a planar wave in a conducting medium. In eq.(7.3) he puts

$$\rho_e = \rho_m = 0. \tag{193}$$

and

$$\vec{J}_e = \sigma \vec{E}, \qquad \vec{J}_m = s\vec{H}. \tag{194}$$

He then considers a planar electromagnetic wave (*TEM*) propagating in the y direction. A *TEM* wave requires

$$E_y = H_y = 0. \tag{195}$$

[58]Contrary to what thinks Barrett, the formulation of a extended electrodynamics including phenomenological charges and phenomenological (i.e., non topological) monopoles do not lead to an $SU(2)$ gauge theory. Instead, we are naturally lead to a $U(1) \times U(1)$ gauge theory formulated in a spliced bundle[62].

With the above assumptions and putting moreover that

$$\mathcal{E} = E_x = E_z,$$
$$\mathcal{H} = H_x = H_z, \tag{196}$$

where $\mathcal{E}$ and $\mathcal{H}$ are functions only of t and y, the generalized Maxwell equations (191) reduce to the following system of partial differential equations

$$\frac{\partial \mathcal{E}}{\partial y} + \mu \frac{\partial \mathcal{H}}{\partial t} + s\mathcal{H} = 0,$$
$$\frac{\partial \mathcal{H}}{\partial y} + \varepsilon \frac{\partial \mathcal{E}}{\partial t} + \sigma\mathcal{E} = 0. \tag{197}$$

Harmuth proceeds solving the pair of equations first for $\mathcal{E}$. Eliminating $\mathcal{H}$ from the system we find the following second order equation for E,

$$\frac{\partial^2 \mathcal{E}}{\partial y^2} - \mu\varepsilon \frac{\partial^2 \mathcal{E}}{\partial t^2} - (\mu\sigma + \varepsilon s)\mathcal{E} - s\sigma\mathcal{E} = 0. \tag{198}$$

Of course, $\mathcal{H}$ also satisfies an equation identical to eq.(198). Anyway, after a solution of eq.(198) describing a signal is found we can trivially find the solution for $\mathcal{H}$. The rest of Harmuth's paper is dedicated to find such a solution and to show that even in the limit when $s = 0$, we still have a solution for the usual Maxwell system without the magnetic current.

The last statement can be proved as follows. Since in Harmuth's papers μ and ε are supposed constants, we can make a scale transformation in the generalized *ME* and write them in the Clifford bundle as

$$\partial \hat{F} = \hat{J}_e + \gamma^5 \hat{J}_m = \widehat{\mathcal{J}},$$
$$\hat{J}_e = \hat{\sigma}\hat{E}^i\gamma^i, \hat{J}_m = \hat{s}\hat{H}^i\gamma^i,$$
$$\hat{\sigma} = \frac{\sigma}{\varepsilon}, \qquad \hat{E}^i = \varepsilon E^i, \qquad \hat{s} = \frac{s}{\mu}, \qquad \hat{H}^i = \mu H^i \tag{199}$$

Now, *ME* (199) is invariant under duality transformations,

$$\hat{F} \mapsto e^{\gamma^5\beta}\hat{F}, \qquad \widehat{\mathcal{J}} \mapsto e^{-\gamma^5\beta}\widehat{\mathcal{J}} \tag{200}$$

It follows that starting with a solution $\hat{F}(t, x, s)$ of Maxwell equation with electric and magnetic currents describing a planar wave, have a solution with only the electric current if

$$\tan(\beta) = \frac{s}{\sigma}\vec{H}\vec{E}^{-1}, \qquad \vec{E} = E^i\vec{\sigma}_i, \qquad \vec{H} = H^i\vec{\sigma}_i \tag{201}$$

is a constant.

Then, in this case, if there exists the limit,

$$\lim_{s\to 0} \hat{F}(t, x, s) = \hat{F}_1(t, x), \tag{202}$$

it follows that, $\hat{F}_1(t,x)$ is a solution of ME only with the electric current term.

Now, we note by the remaining of the proof that the front of the signal travels with the velocity c_1 is known at least since 1876! Indeed, recall that the equations for a transmission line, where the variables are the potential $V(t,y)$ and the current $I(t,y)$, satisfy a system of partial differential equations that is identical to the system (198) since we have[47] for the equations describing the propagation of signals in the transmission line,

$$\begin{aligned}\frac{\partial V}{\partial y}+L\frac{\partial I}{\partial t}+RI&=0,\\ \frac{\partial I}{\partial y}+C\frac{\partial V}{\partial t}+GV&=0,\end{aligned}\tag{203}$$

which are known as the telegraphist equations and where, R,L,C,G, are respectively the resistance, the inductance, the capacitity and the lateral conductance, per unit length of the transmission line . Also, eliminating I in the system we get the following second order partial equation for V,

$$\frac{\partial^2 V}{\partial y^2}-LC\frac{\partial^2 V}{\partial t^2}-(GL+RC)V-RGV=0.\tag{204}$$

with an analogous equation for I.

A solution of system (203) with the following initial and boundary conditions

$$\begin{aligned}&V(t,y)=0 \text{ and } I(t,y)=0,\ \text{for } t=0 \text{ and } y\geq 0,\\ &V(t,0)=\begin{cases}0 & \text{for } t\leq 0\\ f(t) & \text{for } t>0.\end{cases}\end{aligned}\tag{205}$$

has been proposed and obtained by Heaviside in 1876[48], using his operator method. Heaviside operator method is not very rigorous. A rigorous proof of the fact that eq.(204) with conditions (205) possess solutions such that the front of the wave (the signal) propagates with a finite velocity, namely the velocity c_1, can be found in many textbooks. We like particularly the presentation of Oliveira Castro[47]. The identification of systems (197) and (203) is obvious under the following identifications

$$\begin{aligned}&\mathcal{E}\longleftrightarrow I, && \mathcal{H}\leftrightarrow V, && \varepsilon\longleftrightarrow L,\\ &\mu\longleftrightarrow C, && \sigma\longleftrightarrow\ , && s\longleftrightarrow G,\end{aligned}\tag{206}$$

We discuss Oliveira Castro's solution method for Harmuth's problem because it is very much pedagogical.

First recall that a solution of eq.(197) with initial and boundary conditions given by eq.(205) under the identifications (206) is a solution of eq.(198) with the same initial and boundary conditions, plus the additional conditions

$$\left.\frac{\partial\mathcal{E}}{\partial t}\right|_{t=0}=0,\quad \left.\frac{\partial\mathcal{H}}{\partial t}\right|_{t=0}=0\quad \text{for } y\geq 0.\tag{207}$$

Put

$$\kappa = \frac{1}{2}(\frac{\sigma}{\varepsilon} + \frac{s}{\mu}), \qquad \lambda = \frac{1}{2}(\frac{\sigma}{\varepsilon} - \frac{s}{\mu}), \tag{208}$$

and in system (197) make the substitutions

$$\mathcal{E}(t,y) = e^{-\kappa t}E(t,y), \qquad \mathcal{H}(t,y) = e^{-\kappa t}H(t,y). \tag{209}$$

Then, system (197) becomes

$$\begin{aligned} \frac{\partial E}{\partial y} + \mu\frac{\partial H}{\partial t} + sH = 0, \\ \frac{\partial H}{\partial y} + \varepsilon\frac{\partial E}{\partial t} + \sigma E = 0. \end{aligned} \tag{210}$$

and eq.(198) becomes,

$$\frac{\partial^2 E}{\partial y^2} - \frac{1}{c_1^2}\frac{\partial^2 E}{\partial t^2} + \frac{\lambda^2}{c_1^2}E = 0. \tag{211}$$

Eq.(211) is a *tachyonic* Klein-Gordon equation. It is a well known fact[49], that the *characteristics* of this equation are light cones (with light speed equal to c_1). It follows that, for Cauchy's problem, any initial field and normal field derivative configurations with compact support in the y-axis, will propagate along the characteristic. Heaviside problem, is different from Cauchy's problem, and the solution given in[47] obtained through Riemann's method, is:

$$E(t,x,s) = \begin{cases} e^{-\frac{\kappa t}{c_1}} f\left(t - \frac{y}{c_1}\right) + \frac{\lambda y}{c_1}\int_{\frac{y}{c_1}}^{t} du\, f(t-u)\frac{iJ_0'(i\sqrt{u^2 - y^2/c_1^2})}{\sqrt{u^2 - y^2/c_1^2}} & \text{if } t > y/c_1, \\ 0 & \text{if } t \leq y/c_1, \end{cases} \tag{212}$$

$$H(t,x,s) = \begin{cases} \sqrt{\frac{\mu}{\varepsilon}}e^{-\frac{\kappa t}{c_1}} f\left(t - \frac{y}{c_1}\right) - \lambda\int_{\frac{y}{c_1}}^{t} du f(t-u)\frac{i\mathrm{I}_0(\lambda\sqrt{u^2 - y^2/c_1^2})}{\sqrt{u^2 - y^2/c_1^2}} & \\ \qquad + \int_{\frac{y}{c_1}}^{t} du f(t-u)\frac{\lambda u\mathrm{I}_1(\sqrt{u^2 - y^2/c_1^2})}{\sqrt{u^2 - y^2/c_1^2}}, & \text{if } t > y/c_1, \\ 0 & \text{if } t \leq y/c_1. \end{cases} \tag{213}$$

where $\mathrm{I}_n(z) = i^{-n}J_n(iz)$.

Eqs. (212) and (213) have well defined limits when $s \to 0$, which are to be compared with Harmuth's solution.

Now, we can present another unpardonable Barrett's flaw in[8]. He explicitly wrote that it is possible to identify eq.(198), which he called "a two-dimensional *nonlinear* Klein-Gordon equation (without boundary conditions)" (sic) as a sine-Gordon equation, and gives for the equation the usual solitonic solution (hyperbolic tangent).

10 Conclusion

A. The *AIAS***1** paper should never be published by any serious journal because[59]:

(i) as proved above its section 2 is simply a (bad) review of Whittaker's paper theory and a trivial calculation of $\vec{B}^{(3)}$ in that formalism.

(ii) *AIAS* authors did not realize that Whittaker's formalism is a particular case of the more general Hertz potential method, which has been used in[19−22] to prove that Maxwell equations in vacuum possess *exact* arbitrary speeds ($0 \leq v < \infty$) *UPWs* solutions, and that in general the sub and superluminal solutions are not *transverse* waves. The existence of non transverse waves has also been proved by Kiehn[47]. Moreover, at least two of the authors of the present *AIAS* group knew these facts, namely Bearden and Evans and yet they quote no references[19−22].

(iii) It follows from (ii) that existence of non transverses waves in vacuum does not imply that electromagnetism is not a $U(1)$ gauge theory. Indeed, it is clear that *AIAS* authors simply do not know what a gauge theory is. This gave us the motivation for writing section 3 of this report. We hope it may be of some help for readers that want to know about the absurdities written by the *AIAS* group and also for those among that authors that want to know the truth.

(iv) Section 3 of *AISA***1** is a *pot-pourri* of inconsistent mathematics as we proved in section 6 above. Note that we did not comment on some odd and wrong statements like "The Higgs field is a mapping from $SU(2) \sim S(3) \rightarrow S(3)$", which show that indeed authors of *NS* did not understand what they read in p. 410 of Ryder's book[43][60].

B. The quotation that Barrett developed a consistent $SU(2)$ gauge theory of electromagnetism is non sequitur. Indeed, we proved that Barrett's papers[5−8] are as the *AIAS* papers, full of mathematical inconsistencies. Also, quotation of Harmuth's papers[35] by *AIAS authors* is out of context (and its use by Barrett is a complete nonsense).

We could continue pointing many other errors in the papers of the *AIAS* group published in the special issue of the *J. New Energy*[0] or in other publications, but after our analysis of *AIAS***1** it should be clear to our readers that such an enterprise should be given as exercises for the training of advanced mathematical and physical students in the identification of mathematical sophisms.

We think that our critical analysis of *AIAS***1** and of some other papers of the *AIAS* group and also of some papers by other authors quoted by them serves our proposal of clearly denouncing that very *bad* mathematics is being used in physics papers. Worse, these papers are being published in international journals and books. Someone must stop the proliferation of so much nonsense[61].

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[59] We must say that (unfortunately) this applies also to the other 59 papers of the *AISA* group published in the special issue: *J. New Energy* **4**(3), 1-335 (1999).

[60] Here we must comment also that (unfortunately) Ryder makes some confusion on this matter in his book.

[61] Believe you or not, the fact is that Evans "imagination" now is promoting his $\vec{B}^{(3)}$ theory

11 References

0. *AIAS* group[62], The New Maxwell Electrodynamics Equations, *J. New Energy* (special issue) **4**(3): 1-313 (1999).

1. E. T. Whittaker, *Math. Ann.* **57**, 333-355 (1903); reprinted in *J. New Energy* **4**, 18-39 (1999).

2. E. T. Whittaker, *Proc. London Math. Soc.* **1**, 367-372 (1904); reprinted in *J. New Energy* **4**, 40-44 (1999).

3. M. W. Evans, *Physica B* **182**, 227-236; ibidem 237-243-211 (1992).

4. M. W. Evans and L.B. Crowell, *Classical and Quantum Electrodynamics and the* $\boldsymbol{B}^{\mathbf{(3)}}$ *Field*, World Sci. Publ. Co., Singapore, 2000.

5. T. W. Barrett, *Ann. Fond. L. de Broglie* **14**, 37-76 (1989).

6. T. W. Barrett, *Ann. Fond. L. de Broglie* **15**,143-161: ibidem **15**, 253-285 (1990).

7. T. W. Barrett, in A. Lakhatia (ed.), *Essays on the Formal Aspects of Electromagnetic Theory*, 6-86, World Sci. Publ. Co., Singapore, 1993.

8. T. W. Barrett, in T. W. Barrett and D. M. Grimes (eds.), *Advanced Electromagnetism*, 278-313, World Sci. Publ. Co., Singapore, 1995.

9. J. P. Vigier, *Phys. Lett. A* **234**, 75-85 (1997).

10. R. Huth, *Optik* **78**, 12 (1987): ibidem **99**, 113 (1995).

11. B. Lehnert, *Spec. Sci. and Tech.* **17**, 259-264 (1994).

12. B. Lehnert, *Phys. Scripta* **53**, 204-211 (1996).

13. P. K. Anastasovski, T. E. Bearden, C. Ciubotariu, et al. (*AIAS* group), *Found. Phys. Lett.* **12**, 187-192 (1999).

14. P. K. Anastasovski, T. E. Bearden, C. Ciubotariu, et al. (*AIAS* group), *Found. Phys. Lett.* **12**, 251-265 (1999).

15. P. K. Anastasovski, T. E. Bearden, C. Ciubotariu, et al. (*AIAS* group), *Found. Phys. Lett.* **12**, 579-584 (1999)

16. P. K. Anastasovski, T. E. Bearden, C. Ciubotariu, et al. (*AIAS* group), *Optik* **111**, 53-56 (2000).

17. P. K. Anastasovski, T. E. Bearden, C. Ciubotariu, et al. (*AIAS* group), *Phys. Scripta* **61**, 79-82 (2000).

18. J. D. Jackson, *Classical Electrodynamics (*second edition), J. Willey & Sons, Inc., N. York, 1975.

19. W. A. Rodrigues, Jr. and J. Vaz, Jr., Proc. of the Int. Conf: on the Theory of the Electron, Mexico City, Sept. 1995, *Adv. Applied Clifford Algebras* **7**, 453-462 (1997).

20. W. A. Rodrigues, Jr. and J. E. Maiorino, *Random Oper. Stochastic Equ.* **4**, 355-400 (1996).

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as a $SU(2) \times SU(2)$ gauge theory. This new theory is described in[63], and this fact constitutes proof that the quality of many articles published in standard orthodox journals is very *bad* indeed.

[62] *AIAS* members that signed the papers of[0] are: P. K. Anastasoviski, T. E. Bearden, C. Ciubotariu, W. T. Coffey, L. B. Crowell, G. J. Evans, M. E. Evans, R. Flower, S. Jeffers, A. Labounsky, B. Lehnert, M. Mészáros, P. R. Molnár, J. P. Vigier and S. Roy. *AIAS* means *Alpha Foundation's Institute for Advanced Study*

21. W. A. Rodrigues, Jr and J. Y. Lu, *Found. Phys.* **27**, 435-508 (1997).
22. E. Capelas de Oliveira and W. A. Rodrigues, Jr., *Ann. der Phys.* **7**, 654-659 (1998).
23. R. Donnelly and R. Ziolkowski, *Proc. R. Soc. London A*, **437**, 673-692 (1992)
24. R. Donnelly and R. Ziolkowski, *Proc. R. Soc. London A* **440**, 541-565 (1993)
25. M. W. Evans, *Found. Phys. Lett.* **9**, 191-204 (1996).
26. E. Recami, *Physica* **252**, 586-610 (1998).
27. H. Hertz, *Ann. der Phys.* **36**, 1-10 (1888).
28. M. E. Evans, *Found. Phys. Lett.* **9**, 175-181 (1996)
29. M. E. Evans, *Found. Phys. Lett.* **9**, 587-593 (1996)
30. J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941.
31. K. K. Lee, *Phys. Rev. Lett.* **50**, 138-138 (1982).
32. C. Chu and T. Ohkwava, *Phys. Rev. Lett.* **48**, 837-838 (1982).
33. K. Shimoda et al, *Am. J. Phys. 58*, 194-396 (1990).
34. M. W. Evans, *Found. Phys. Lett.* **8**, 253-259 (1995).
35. H. F. Harmuth, *IEEE Trans. Electromag. Compat.* **28**, 250-258: ibidem **28**, 259-266: ibidem **28**, 267-272 (1986).
36. P. Saari and K. Reivelt, *Phys. Rev. Lett.* **21**, 4135-4138 (1997).
37. J. E. Maiorino and W. A. Rodrigues, Jr., *What is Superluminal Wave Motion* (177 pages), *Sci. and Tech. News* **4**, 1999 (electronic journal of CPTEC-UNISAL, SP, Brazil, http://www.cptec.br/stm/pdf/wavem.pdf).
38. C. Nash and S. Sen, *Topology and Geometry for Physicists,* Academic Press, London, 1983.
39. Y. Choquet-Bruhat, C. DeWitt-Morette and M. Dillard-Bleick, *Analysis, Manifolds and Physics* (revised version), North Holland Publ. Co., Amsterdam, 1982.
40. D. Bleecker, *Gauge Theory and Variational Principles*, Addison-Wesley Publ. Co., Inc., Reading, MA, 1981.
41. M. Nakahara, *Geometry, Topology and Physics*, Inst. Phys. Publ., Bristol and Philadelphia, 1990.
42. E. Prugovečki, *Quantum Geometry, A Framework for Quantum General Relativity*, Kluwer Acad. Publ., Dordrecht, 1992.
43. A. L. Trovon de Carvalho, *Mathematical Foundations of Gauge Theories*, M. Sc. thesis, IMECC UNICAMP, 1992.
44. L. H. Ryder, *Quantum Field Theory* (second edition), Cambridge Univ. Press, Cambridge, 1996.
45. M. A. F. Rosa and W. A. Rodrigues, Jr., *Mod. Phys. Lett. A* **4**, 175-184 (1989).
46. L. Brillouin, *Wave Propagation and Group Velocity*, Academic Press, New York, 1960.
47. F. M. de Oliveira Castro, *Waves in Transmition Line. Fundamental Problem*, National School of Eng., Univ. of Brazil, 1949.
48. O. Heaviside, *Phil. Mag.* **2**, 5-28 (1876).

49. R. Courant and D. Hilbert, *Methods of Mathematical Physics*, vol.2, J. Wiley & Sons, New York, 1966.

50. R. M. Kiehn, *Topology and Topological Evolution of Electromagnetic Fields and Currents*, preprint Univ. Houston, 1997.

51. V. Barashenkov and W. A. Rodrigues, Jr., *N. Cimento B* **113**, 329-338 (1998).

52. R. Y. Chiao, A. E. Kozhekin and G. Kuruzi, *Phys. Rev. Lett.* **77**, 1254-1257 (1996).

53. G. Nimtz, *Europhysics J. B* **7**, 523-525 (1999).

54. J. P. Vigier, in G. Hunter, S. Jeffers and J. P. Vigier (eds.), *Causality and Locality in Modern Physics*, Fund. Theories of Phys. **97**, 1-22, Kluwer Acad. Publ, Dordrecht, 1998.

55. J P. Vigier, *Phys. Lett. A* **235**, 419-431 (1997).

56. J. P. Vigier, *Phys. Lett. A* **234**, 75-85 (1997).

57. D. S. Thober, *Math. Reviews* **62**:78003 (2000).

58. G. Hunter, *Chem. Phys.* **242**, 331-339 (1999).

59. G. Hunter, *Apeiron* **7**, 17-28 (2000).

60. L. Landau and E. M. Lifschitz, *Classical Theory of Fields* (fourth revised englsh edition), Pergamon Press, Oxford, 1975.

61. W. A. Rodrigues, Jr, .Q. A. G. de Souza, J. Vaz, Jr., and P. Lounesto, *Int. J. Teor. Phys.* **35**, 1849-1900 (1996).

62. M. A. F. Rosa and W. A. Rodrigues, Jr., *Mod. Phys. Lett. A* **4**, 175-184 (1989).

63. L. B. Crowell and M.W. Evans, *Found. Phys. Lett.* **12**, 373-382 (1999).

64. M. P. Silverman, *Waves and Grains: Refelections on Light and Learning*, Princeton Univ. Press, Princeton, NJ, 1998.

65. E. T. Whittaker, *The Theories of Aether and Electricity*, Humannities Press, 1974.

66. M. W. Evans, J. P. Vigier, *The Enigmatic Photon,* vol. **1**: *The Field* $\mathbf{B}^{(3)}$, Kluwer Acad. Publ., Dordrecht, 1994.

67. M. W. Evans, J. P. Vigier, *The Enigmatic Photon,* vol. **2**: *Non Abelian Electrodynamics*, Kluwer Acad. Publ., Dordrecht, 1995.

68. M. W. Evans, J. P. Vigier, S. Roy and S. Jeffers,*The Enigmatic Photon,* vol. **3**: *Theory and Practice of the* $\mathbf{B}^{(3)}$ *Field*, Kluwer Acad. Publ., Dordrecht, 1996.

69. M. W. Evans, J. P. Vigier, S. Roy and G. Hunter, *The Enigmatic Photon,* vol. **4**: *New Directions*, Kluwer Acad. Publ., Dordrecht, 1998.

70. M. W. Evans, J. P. Vigier, *The Enigmatic Photon,* vol. **5**: $O(3)$ *Electrodynamics*, Kluwer Acad. Publ., Dordrecht, 1999.

71. V. M. Bradis, V. L. Minkowskii and A. K. Kharcheva, *Lapses in Mathematical Reasoning*, Dover Publ., Inc, Mineola, New York, 1999.

72. W. A. Rodrigues, Jr., *The Many Faces of Maxwell and Dirac Equations and Their Interpretation*, in preparation (2000).

73. J. M. Jauch and F. Horlich, *The Theory of Photons and Electrons*, Springer-Verlag, Berlin, 1976.

74. L. de Broglie, *Ondes Electromagnétiques & Photons*, Gauthier Villars, Paris, 1968.

75. P. S. Pershan, J. P. van der Ziel and L. D. Malmostrom, *Phys. Rev.* **143**, 574-583 (1966).

76. G. L. J. A. Rikken, *Opt. Lett.* **20**, 846-847 (1995).

77. M. Y. Akhatar Raja, W. N. Sisk, M. Yousaf, D. Allen, *Appl. Phys. B* **64**, 79-84 (1997).

78. M. W. Evans (guest editor) et al, *Apeiron* **4** , 35-79 (1997).

79. W. A. Rodrigues, Jr. and M. A. F. Rosa, *Found. Phys.* **19,** 705-724 (1989).

80. A. K. Dewdney, *Yes, We Have No Neutrons*, J. Wiley &Sons Inc., New York, 1997.

81. L. D. Baron, *Physica B* **190**, 307-309 (1993).

82. A. Lakhtakia, *Physica B* **191**, 362-366 (1993).

83. D. M. Grimes, *Physica B* **191**, 367-371 (1993).