NEW COINCIDENCE RESULTS FOR ABSOLUTELY SUMMING MULTILINEAR MAPPINGS

DANIEL M. PELLEGRINO

Abstract. In this paper we explore the latest advances in the theory of absolutely summing multilinear mappings in order to prove new coindence theorems. We also show that our results of coincidence can not be improved in some natural ways.

1. Introduction

It is well known that every continuous scalar valued linear operator between Banach spaces is absolutely p-summing. It is also easy to check that this result, in general, is no longer valid for absolutely $(p;p,...,p)$-summing multilinear mappings. However, an unpublished result, due to A. Defant and J. Voigt, sates that every scalar valued multilinear mapping is absolutely $(1;1,...,1)$-summing (see [1],[5]). Several other coincidence theorems for multilinear mappings can be found in [2],[3],[5],[7],[9],[11]. In Section 3, using the concept of cotype and the Rademacher functions, we obtain two new results of coincidence. In Section 4, we prove some multilinear results, sketched in [7],[8], in order to show that our Coincidence Theorems can not be improved in many natural ways.

2. Preliminaries

Throughout this paper p is a real number not smaller than 1 and $E, E_1, ..., E_n, F$ are Banach spaces. The scalar field $\mathbb{K}$ can be either $\mathbb{R}$ or $\mathbb{C}$. The linear space of all sequences $(x_j)_{j=1}^{\infty}$ in E such that

$$\|(x_j)_{j=1}^{\infty}\|_p = (\sum_{j=1}^{\infty} \|x_j\|^p)^{\frac{1}{p}} < \infty$$

will be denoted by $l_p(E)$. We will also denote by $l_p^w(E)$ the linear subspace of $l_p(E)$ composed by the sequences $(x_j)_{j=1}^{\infty}$ in E such that $(<\varphi, x_j>)_{j=1}^{\infty} \in l_p(\mathbb{K})$ for every bounded linear functional $\varphi : E \to \mathbb{K}$. We define $\|.\|_{w,p}$ in $l_p^w(E)$ by

$$\|(x_j)_{j=1}^{\infty}\|_{w,p} := \sup_{\varphi \in B_{E'}} (\sum_{j=1}^{\infty} |<\varphi, x_j>|^p)^{\frac{1}{p}}.$$

One can see that $\|.\|_p$ ($\|.\|_{w,p}$) is a norm in $l_p(E)$ ($l_p^w(E)$).

Recall that if $2 \leq q \leq \infty$ and $(r_j)_{j=1}^{\infty}$ are the Rademacher functions, E has cotype q if there exists $C \geq 0$ such that, no matter how we choose $k \in \mathbb{N}$ and

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$x_1, ..., x_k \in E$,

$$(\sum_{j=1}^{k} \|x_j\|^q)^{\frac{1}{q}} \leq C(\int_0^1 \| \sum_{j=1}^{k} r_j(t) x_j \|^2 dt)^{\frac{1}{2}}.$$

To cover the case $q = \infty$ we replace $(\sum_{j=1}^{k} \|x_j\|^q)^{\frac{1}{q}}$ by $max_{j \leq n} \|x_j\|$. We will define the cotype of E by

$$\cot E = \inf\{2 \leq q \leq \infty; E \text{ has cotype } q\}.$$

The infimum of the constants C is denoted by $C_q(E)$.

Another important concept concerning the local study of Banach spaces, and broadly used in the study of absolutely summing operators, is the definition of $\mathcal{L}_{p,\lambda}$-spaces, due to Lindenstrauss and Pełczyński [4]. A Banach space E is said to be an $\mathcal{L}_{p,\lambda}$-space if every finite dimensional subspace E_1 of E is contained in a finite dimensional subspace F of E for which there exists an isomorphism $v_{E_1} : F \to l_p^{\dim F}$ with $\|v_{E_1}\| \|v_{E_1}^{-1}\| < \lambda$. We say that E is an $\mathcal{L}_p$-space if it is an $\mathcal{L}_{p,\lambda}$-space for some $\lambda > 1$.

The definition of absolutely summing polynomials and multilinear mappings we will work with is a natural generalization of the linear case, due to Alencar and Matos.

Definition 1. *(Alencar-Matos [1]) A continuous multilinear mapping*

$$T : E_1 \times ... \times E_n \to F$$

is absolutely $(p; q_1, ..., q_n)$-summing if

$$(T(x_j^{(1)}, ..., x_j^{(n)}))_{j=1}^{\infty} \in l_p(F)$$

for all $(x_j^{(s)})_{j=1}^{\infty} \in l_{q_s}^w(E)$, $s = 1, ..., n$. We will write $\mathcal{L}_{as(p;q_1,...,q_n)}(E_1, ..., E_n; F)$ to denote the space of all absolutely $(p; q_1, ..., q_n)$-summing multilinear mappings from $E_1 \times ... \times E_n$ into F.

As in the linear case, we also have a Characterization Theorem:

Theorem 1. *(Matos [5]) Let T be an n-linear mapping from $E_1 \times ... \times E_n$ into F. The following statements are equivalent:*

(1) T is absolutely $(p; q_1, ..., q_n)$-summing.

(2)There exists $C > 0$ such that

$$(\sum_{j=1}^{k} \|T(x_j^{(1)}, ..., x_j^{(n)})\|^p)^{\frac{1}{p}} \leq C \|(x_j^{(1)})_{j=1}^{k}\|_{w,q_1} ... \|(x_j^{(1)})_{j=1}^{k}\|_{w,q_n} \ \forall k \in \mathbb{N}$$

(3)There exists $C > 0$ such that

$$(\sum_{j=1}^{\infty} \|T(x_j^{(1)}, ..., x_j^{(n)})\|^p)^{\frac{1}{p}} \leq C \|(x_j^{(1)})_{j=1}^{\infty}\|_{w,q_1} ... \|(x_j^{(1)})_{j=1}^{\infty}\|_{w,q_n} \forall (x_j^{(l)})_{j=1}^{\infty} \in l_{q_l}^w(E).$$

The infimum of the $C > 0$ for which the last inequality holds defines a norm for the space of absolutely $(p; q_1, ..., q_n)$-summing multilinear mappings. This norm will be denoted by $\|.\|_{as(p;q_1,...,q_n)}$ and $\mathcal{L}_{as(p;q_1,...,q_n)}(E_1, ..., E_n; F)$ endowed with this norm is a Banach space.

3. Coincidence Theorems

The following result generalizes a theorem of C.A. Soares [10] and will play a fundamental role to achieve our new results of coincidence.

Theorem 2. *Let $A \in \mathcal{L}(E_1, ..., E_n; F)$. Suppose that there exists $K > 0$ so that for any $x_1 \in E_1, ..., x_r \in E_r$, the s-linear ($s = n - r$) mapping*

$$A_{x_1 ... x_r}(x_{r+1}, ..., x_n) = A(x_1, ..., x_n)$$

is absolutely $(1; q_1, ..., q_s)$-summing and besides

$$\|A_{x_1 ... x_r}\|_{as(1;q_1,...,q_s)} \leq K \|A\| \|x_1\| ... \|x_r\| .$$

Then A is absolutely $(1; 1, ..., 1, q_1, ..., q_s)$-summing.

Proof: For $x_1^{(1)}, ..., x_1^{(m)} \in E_1, ..., x_n^{(1)}, ..., x_n^{(m)} \in E_n$, let us consider $\varphi_j \in B_{F'}$ such that

$$\left\| A(x_1^{(j)}, ..., x_n^{(j)}) \right\| = \varphi_j(A(x_1^{(j)}, ..., x_n^{(j)})) \text{ for every } j = 1, ..., m.$$

Thus, defining by $r_j(t)$ the Rademacher functions on $[0, 1]$ and denoting by λ the Lebesgue measure in $I = [0, 1]^r$,we have

$$\begin{aligned}
&\int_I \sum_{j=1}^m \left(\prod_{l=1}^r r_j(t_l) \right) \varphi_j A\Big(\sum_{j_1=1}^m r_{j_1}(t_1) x_1^{(j_1)}, ..., \sum_{j_r=1}^m r_{j_r}(t_r) x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)}\Big) d\lambda \\
&= \sum_{j,j_1,...j_r=1}^m \varphi_j A(x_1^{(j_1)}, ..., x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)}) \int_I r_j(t_1)...r_j(t_r) r_{j_1}(t_1)...r_{j_r}(t_r) d\lambda \\
&= \sum_{j,j_1,...j_r=1}^m \varphi_j A(x_1^{(j_1)}, ..., x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)}) \int_0^1 r_j(t_1) r_{j_1}(t_1) dt_1 ... \int_0^1 r_j(t_r) r_{j_r}(t_r) dt_r \\
&= \sum_{j=1}^m \sum_{j_1=1}^m ... \sum_{j_r=1}^m \varphi_j A(x_1^{(j_1)}, ..., x_r^{(j_r)}, x_{r+1}^{(j)}, ..., x_n^{(j)}) \delta_{jj_1} ... \delta_{jj_r} \\
&= \sum_{j=1}^m \varphi_j A(x_1^{(j)}, ..., x_n^{(j)}) = \sum_{j=1}^m \left\| A(x_1^{(j)}, ..., x_n^{(j)}) \right\| = (*).
\end{aligned}$$

So, for each $l=1,...,r$, assuming $z_l=\sum\limits_{j=1}^{m} r_j(t_l)x_l^{(j)}$ we obtain

$$\begin{aligned}
(*) &= \int_I \sum_{j=1}^{m}\left(\prod_{l=1}^{r} r_j(t_l)\right)\varphi_j A(\sum_{j_1=1}^{m} r_{j_1}(t_1)x_1^{(j_1)},...,\sum_{j_r=1}^{m} r_{j_r}(t_r)x_r^{(j_r)},x_{r+1}^{(j)},...,x_n^{(j)})d\lambda \\
&\leq \int_I \left|\sum_{j=1}^{m}\left(\prod_{l=1}^{r} r_j(t_l)\right)\varphi_j A(\sum_{j_1=1}^{m} r_{j_1}(t_1)x_1^{(j_1)},...,\sum_{j_r=1}^{m} r_{j_r}(t_r)x_r^{(j_r)},x_{r+1}^{(j)},...,x_n^{(j)})\right| d\lambda \\
&\leq \int_I \sum_{j=1}^{m}\left\|A(\sum_{j_1=1}^{m} r_{j_1}(t_1)x_1^{(j_1)},...,\sum_{j_r=1}^{m} r_{j_r}(t_r)x_r^{(j_r)},x_{r+1}^{(j)},...,x_n^{(j)})\right\| d\lambda \\
&\leq \sup_{t_l\in[0,1],l=1,...,r}\sum_{j=1}^{m}\left\|A(\sum_{j_1=1}^{m} r_{j_1}(t_1)x_1^{(j_1)},...,\sum_{j_r=1}^{m} r_{j_r}(t_r)x_r^{(j_r)},x_{r+1}^{(j)},...,x_n^{(j)})\right\| \\
&\leq \sup_{t_l\in[0,1],l=1,...,r}\left\|A_{z_1...z_r}\right\|_{as(1;q_1,...,q_s)}\left\|(x_{r+1}^{(j)})_{j=1}^m\right\|_{w,q_1}...\left\|(x_n^{(j)})_{j=1}^m\right\|_{w,q_s} \\
&\leq \sup_{t_l\in[0,1],l=1,...,r} K\left\|A\right\|\left\|z_1\right\|...\left\|z_r\right\|\left\|(x_{r+1}^{(j)})_{j=1}^m\right\|_{w,q_1}...\left\|(x_n^{(j)})_{j=1}^m\right\|_{w,q_s} \\
&= K\left\|A\right\|\sup_{t\in[0,1]}\left(\prod_{l=1}^{r}\left\|\sum_{j=1}^{m} r_j(t)x_l^{(j)}\right\|\right)\left(\prod_{l=1}^{s}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,q_l}\right) \\
&\leq K\left\|A\right\|\left(\prod_{l=1}^{r}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,1}\right)\left(\prod_{l=1}^{s}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,q_l}\right).\ \text{Q.E.D.}
\end{aligned}$$

We have the following straightforward consequence:

Corollary 1. *If*

$$\mathcal{L}(E_1,...,E_m;F)=\mathcal{L}_{as(1;q_1,...,q_m)}(E_1,...,E_m;F)$$

then, for any Banach spaces $E_{m+1},...,E_n$, we have

$$\mathcal{L}(E_1,...,E_n;F)=\mathcal{L}_{as(1;q_1,...,q_m,1,...,1)}(E_1,...,E_n;F).$$

Note that an particular case of this result is the aforementioned coincidence result of Defant and Voigt. Another outcome of Theorem 2 is the following:

Corollary 2 (Coincidence Theorem I)**.** *If $E_1,...\ E_k$ are $\mathcal{L}_\infty$-spaces then, for any choice of Banach spaces $E_{k+1},...,E_n$, we have*

$$\mathcal{L}(E_1,...,E_k,...,E_n;\mathbb{K})=\mathcal{L}_{as(1;q_1,....,q_n)}(E_1,...,E_k,...,E_n;\mathbb{K}),$$

where $q_1=...=q_k=2$ e $q_{k+1}=....=q_n=1$.

Proof. Immediate consequence of the last Corollary and of a result of Perez [9] which states that every scalar valued n-linear mapping defined on $\mathcal{L}_\infty$-spaces is absolutely $(1;2,...,2)$-summing.

Corollary 3. *If* $\cot F=q<\infty$ *and*

$$\mathcal{L}(E_1,...,E_s;\mathbb{K})=\mathcal{L}_{as(1;q_1,....,q_s)}(E_1,...,E_s;\mathbb{K}),$$

then, for any choice of Banach spaces $E_{s+1},...,E_n$, we have

$$\mathcal{L}(E_1,...,E_n;F)=\mathcal{L}_{as(q;q_1,....,q_s,1,....,1)}(E_1,...,E_n;F),$$

Proof. Since F has finite cotype q, using the estimates of Theorem 2, we obtain

$$\left(\sum_{j=1}^{m}\left\|A(x_1^{(j)},...,x_n^{(j)})\right\|^q\right)^{\frac{1}{q}} \leq C_q(F)\sup_{\varphi\in B_{F'}}\left(\sum_{j=1}^{m}\left|<\varphi,A(x_1^{(j)},...,x_n^{(j)})>\right|\right)$$

$$\leq C_q(F)\sup_{\varphi\in B_{F'}}K\left\|\varphi A\right\|\left(\prod_{l=1}^{s}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,q_l}\right)\left(\prod_{l=s+1}^{n}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,1}\right)$$

$$\leq KC_q(F)\left\|A\right\|\left(\prod_{l=1}^{s}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,q_l}\right)\left(\prod_{l=s+1}^{n}\left\|(x_l^{(j)})_{j=1}^m\right\|_{w,1}\right).$$

Corollary 4 (Coincidence Theorem II)**.** *If* $\cot F = q < \infty$ *and* $E_1,...,E_k$ *are* $\mathcal{L}_\infty$*-spaces, then, regardless of the Banach spaces* $E_{k+1},...,E_n$*, we have*

$$\mathcal{L}(E_1,...,E_k,...,E_n;F) = \mathcal{L}_{as(q;q_1,....,q_n)}(E_1,...,E_k,...,E_n;F),$$

where $q_1 = ... = q_k = 2$ *and* $q_{k+1} = ... = q_n = 1$.

4. Related questions

It is obvious that Corollary 2 is still true if we replace $\mathbb{K}$ by any finite dimensional Banach space. A natural questoin is whether Corollary 2 can be improved for infinite dimesional $\mathcal{L}_\infty$-spaces, $E_1,...,E_k$ and some infinite dimensional Banach space in the place of $\mathbb{K}$. Precisely, the question is:

- If $E_1,...,E_k$ are infinite dimensional $\mathcal{L}_\infty$-spaces, is there some infinite dimensional Banach space F such that

  $$\mathcal{L}(E_1,...,E_k,...,E_n;F) = \mathcal{L}_{as(1;q_1,....,q_n)}(E_1,...,E_k,...,E_n;F),$$

  where $q_1 = ... = q_k = 2$ and $q_{k+1} = ... = q_n = 1$, regardless of the Banach spaces $E_{k+1},...,E_n$?

The answer to this question, surprisingly, is no. The proof will be a consequence of the next result which proof (sketched in [7], [8]), we will give in details, below:

Theorem 3. *Let* F *be an infinite dimensional Banach space and* $E_1,...,E_m$ *denote infinite dimensional Banach spaces with unconditional Schauder basis. If* q *is so that* $\frac{1}{m} \leq q < 2$ *and* $\mathcal{L}_{as(q;1,...,1)}(E_1,...,E_m;F) = \mathcal{L}(E_1,...,E_m;F)$ *we conclude that for any normalized unconditional Schauder basis* $\{x_j^{(1)}\}_{j=1}^\infty,...,\{x_j^{(m)}\}_{j=1}^\infty$ *for* $E_1,...,E_m$*, respectively, the natural mapping*

$$\psi : E_1\times...\times E_m \to l_\infty : (\sum_{i=1}^{\infty}a_i^{(1)}x_i^{(1)},...,\sum_{i=1}^{\infty}a_i^{(m)}x_i^{(m)}) \to (a_i^{(1)}...a_i^{(m)})_{i=1}^\infty$$

is such that $\psi(E_1\times...\times E_m)\subset l_{\frac{2q}{2-q}}$.

Proof. The Open Mapping Theorem yields the existence of $K > 0$ so that $\|T\|_{as(q;1,...,1)} \leq K\|T\|$ for all continuous m-linear mappings $T : E_1\times...\times E_m \to F$.

By the main Lemma of the well known Dvoretzky-Rogers Theorem (see [4]), for every n, there exist normalized $y_1,...,y_n$ in F such that

$$\|\sum_{j=1}^{n}\lambda_j y_j\| \leq 2(\sum_{j=1}^{n}\mid\lambda_j\mid^2)^{1/2}. \tag{4.1}$$

regardless of the scalars $\lambda_1, ..., \lambda_n$.

For each $k = 1, ..., m$, consider $z_k = \sum_{i=1}^{\infty} a_i^{(k)} x_i^{(k)}$ and for each natural n, let $\{\mu_i\}_{i=1}^{n}$be such that $\sum\limits_{j=1}^{n} \mid \mu_j \mid^s = 1$ with $s = \frac{2}{q}$. Define $T : E_1 \times ... \times E_m \to F$ by

$$T(z_1, ..., z_m) = \sum_{j=1}^{n} \left|\mu_j\right|^{\frac{1}{q}} a_j^{(1)} ... a_j^{(m)} y_j,$$

where we chose y_j satisfying (4.1).

Since each $\{x_j^{(k)}\}_{j=1}^{\infty}$ is an unconditional basis, there exists $\rho_k > 0$ such that, for all $z_k = \sum\limits_{j=1}^{\infty} a_j^{(k)} x_j^{(k)} \in E_k$,

$$\| \sum_{j=1}^{\infty} \varepsilon_j a_j^{(k)} x_j^k \| \leq \rho_k \| \sum_{j=1}^{\infty} a_j^{(k)} x_j^k \| = \rho_k \|z_k\| \text{ for all } \varepsilon_j \in \{1, -1\}.$$

Hence $\| \sum\limits_{j=1}^{r} \varepsilon_j a_j^{(k)} x_j^{(k)} \| \leq \rho_k \|z_k\|$ for all natural r and any $\varepsilon_j = 1$ or -1. We thus have

$$\begin{aligned}
\|T(z_1, ..., z_k)\| &= \| \sum_{j=1}^{n} \left|\mu_j\right|^{\frac{1}{q}} a_j^{(1)} ... a_j^{(m)} y_j \| \leq 2 (\sum_{j=1}^{n} \left|\mu_j\right|^{\frac{2}{q}} \mid a_j^{(1)} ... a_j^{(m)} \mid^2)^{1/2} \\
&\leq 2 (\sum_{j=1}^{n} \mid \mu_j \mid^{2/q} \rho_1^2 ... \rho_m^2)^{1/2} \|z_1\| ... \|z_m\| \\
&\leq 2 \rho_1 ... \rho_m \|z_1\| ... \|z_m\| (\sum_{j=1}^{n} \mid \mu_j \mid^{2/q})^{1/2} \\
&= 2 \rho_1 ... \rho_m \|z_1\| ... \|z_m\| (\sum_{j=1}^{n} \mid \mu_j \mid^{s})^{1/2} \\
&\leq 2 \rho_1 ... \rho_m \|z_1\| ... \|z_m\|.
\end{aligned}$$

Then $\|T\| \leq 2\rho_1 ... \rho_m$ and $\|T\|_{as(q;1,...,1)} \leq 2K\rho_1 ... \rho_m$. Therefore

$$\begin{aligned}
\left[\sum_{j=1}^{n} \left(\left|\mu_j\right|^{\frac{1}{q}} \left|a_j^{(1)} ... a_j^{(m)}\right| \right)^q \right]^{1/q} &= \left(\sum_{j=1}^{n} \left\| T(a_j^{(1)} x_j^{(1)}, ..., a_j^{(m)} x_j^{(m)}) \right\|^q \right)^{1/q} \\
&\leq \|T\|_{as(q;1,...,1)} \prod_{k=1}^{m} \|(a_j^{(k)} x_j^{(k)})_{j=1}^{n}\|_{w,1} \\
&= \|T\|_{as(q;1,...,1)} \prod_{k=1}^{m} \max_{\varepsilon_j \in \{1,-1\}} \{ \| \sum_{j=1}^{n} \varepsilon_j a_j^{(k)} x_j^{(k)} \| \} \\
&\leq \|T\|_{as(q;1,...,1)} \prod_{k=1}^{m} (\rho_k \|z_k\|) \\
&\leq 2K \rho_1^2 ... \rho_m^2 \|z_1\| ... \|z_m\|.
\end{aligned}$$

Recall that the last inequality holds whenever $\sum\limits_{j=1}^{n} \mid \mu_j \mid^s = 1$. Hence

$$\begin{aligned}\left[\sum_{j=1}^{n}(\mid a_j^{(1)}...a_j^{(m)} \mid^{\frac{s}{s-1}q})\right]^{1/(\frac{s}{s-1})} &= \left\|\left(\left|a_j^{(1)}...a_j^{(m)}\right|^q\right)_{j=1}^n\right\|_{\frac{s}{s-1}} \\ &= \sup\left\{\left|\sum_{j=1}^{n}\mu_j\left|a_j^{(1)}...a_j^{(m)}\right|^q\right|; \sum_{j=1}^{n}\mid \mu_j \mid^s = 1\right\} \\ &\leq \sup\left\{\sum_{j=1}^{n}(\mid \mu_j \mid \left|a_j^{(1)}...a_j^{(m)}\right|^q ; \sum_{j=1}^{n}\mid \mu_j \mid^s = 1\right\}\end{aligned}$$

and thus

$$[\sum_{j=1}^{n}(\mid a_j^{(1)}...a_j^{(m)} \mid^{\frac{s}{s-1}q})]^{1/(\frac{s}{s-1})} \leq (2K\rho_1^2...\rho_m^2\|z_1\|...\|z_m\|)^q.$$

Finally,

$$[\sum_{j=1}^{n}(\mid a_j^{(1)}...a_j^{(m)} \mid^{\frac{s}{s-1}q})]^{1/(\frac{s}{s-1})q} \leq 2K\rho_1^2...\rho_m^2\|z_1\|...\|z_m\|.$$

Since $\frac{s}{s-1}q = \frac{2q}{2-q}$, and n is arbitrary, the proof is done. Q.E.D.

Corollary 5. *Suppose that $E_1, ..., E_k$ are infinite dimensional $\mathcal{L}_\infty$-spaces. If $q_1 = ... = q_k = 2$, $q_{k+1} = ... = q_n = 1$ and*

$$\mathcal{L}(E_1, ..., E_k, ..., E_n; F) = \mathcal{L}_{as(1;q_1,....,q_n)}(E_1, ..., E_k, ..., E_n; F),$$

regardless of the Banach spaces $E_{k+1}, ..., E_n$, then $\dim F < \infty$.

Proof. By a standard localization argument, it suffices to prove that

$$\mathcal{L}(^n c_0; F) \neq \mathcal{L}_{as(1;q_1,....,q_n)}(^n c_0; F),$$

where $q_1 = ... = q_k = 2$ and $q_{k+1} = ... = q_n = 1$. But Theorem 3 yields that

$$\mathcal{L}(^n c_0; F) \neq \mathcal{L}_{as(q;q_1,....,q_n)}(^n c_0; F),$$

regardless of the $q < 2$ and $q_1 = ... = q_n \geq 1$. Q.E.D.

Another natutal question is whether our Coincidence Theorem II can be improved to $p < q$, ie,

- If $\cot F = q < \infty$ and E_1,..., E_k are infinite dimensional $\mathcal{L}_\infty$-spaces, is there some $p < q$ for which, regardless of the Banach spaces $E_{k+1}, ..., E_n$,

  $$\mathcal{L}(E_1, ..., E_k, ..., E_n; F) = \mathcal{L}_{as(p;q_1,....,q_n)}(E_1, ..., E_k, ..., E_n; F),$$

  where $q_1 = ... = q_k = 2$ and $q_{k+1} = ... = q_n = 1$?

Again, a result sketched in [7] and [8], similar to Theorem 3, yields, using the same reasoning, a negative answer to this question.

Theorem 4. *If $\cot F = q < \infty$, $\dim F = \dim E_1 = ... = \dim E_m = \infty$ and each E_j has unconditional Schauder basis, then whenever p is such that $\frac{1}{m} \leq p < q$ and $\mathcal{L}_{as(p;1,...,1)}(E_1, ..., E_m; F) = \mathcal{L}(E_1, ..., E_m; F)$ we conclude that for any normalized* *unconditional Schauder basis* $\{x_j^{(1)}\}_{j=1}^{\infty}, ..., \{x_j^{(m)}\}_{j=1}^{\infty}$ *for* $E_1, ..., E_m$*, respectively, the natural mapping*

$$\psi : E_1 \times ... \times E_m \to l_\infty : (\sum_{i=1}^{\infty} a_i^{(1)} x_i^{(1)}, ..., \sum_{i=1}^{\infty} a_i^{(m)} x_i^{(m)}) \to (a_i^{(1)} ... a_i^{(m)})_{i=1}^{\infty}$$

is such that $\psi(E_1 \times ... \times E_m) \subset l_{\frac{pq}{q-p}}$.

Proof. Similar to the proof of Theorem 3. The only difference is that, exploring cotype we have a finner estimate, due to Maurey and Pisier [6], replacincing the Dvoretzky Rogers Lemma. We can find $y_1, ..., y_n$ in F such that $\|y_j\| \leq 1$ and

$$\| \sum_{j=1}^{n} \lambda_j y_j \| \leq (\sum_{j=1}^{n} | \lambda_j |^q)^{1/q}.$$

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(Daniel M. Pellegrino) DME- UFCG- Caixa Postal 10044 Campina Grande-PB- Brasil
Current address: IMECC- UNICAMP Caixa Postal 6065 Campinas-SP- Brasil
E-mail address: `dmp@dme.ufpb.br`