Large time behavior of vortex dynamics

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Abstract. In this paper we prove two results regarding the large-time behavior of vortex dynamics in the full plane. In the first result we show that the total integral of vorticity is confined in a region of diameter growing at most like the square-root of time. In the second result we show that if a dynamic rescaling of the absolute value of vorticity with spatial scale growing linearly with time converges weakly, then it must converge to a discrete sum of Dirac masses. This last result extends in scope a previous result by the authors, valid for nonnegative initial vorticity on a half-plane.

Key words: Vorticity, confinement, incompressible flow, ideal flow.

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1. Introduction

Let ω_0 be a compactly supported function in $L^p(\mathbb{R}^2)$, with $p > 2$, and let $\omega = \omega(x,t)$ be the vorticity associated to a weak solution of the incompressible two-dimensional Euler equations in the full plane, with initial vorticity ω_0. In vorticity form, the Euler equations may be written as an active scalar transport equation:

$$\begin{cases} \omega_t + (K * \omega) \cdot \nabla \omega = 0, \\ \omega(x,0) = \omega_0, \end{cases} \tag{1.1}$$

with K the Biot-Savart vector kernel for the full plane, given by

$$K(x) = K(x_1, x_2) = \frac{1}{2\pi|x|^2}(-x_2, x_1) = \frac{x^\perp}{2\pi|x|^2} \tag{1.2}$$

We are interested in obtaining information on the behavior of the solution $\omega(\cdot,t)$ as $t \to \infty$, particularly with regards to the spatial distribution of the vorticity. We consider a self-similar rescaling of vorticity of the form:

$$\widetilde{\omega}(x,t) \equiv t^{2\alpha}\omega(t^\alpha x, t),$$

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with $\alpha \in (0,1]$. This scaling preserves the integral of vorticity and its L^1 norm. The large time behavior of $\widetilde{\omega}$ carries information on the distribution of vorticity, focusing on a certain asymptotic scale determined by the parameter α. The purpose of this note is to prove two results. The first result is that, for any initial data ω_0, we have $\widetilde{\omega} \rightharpoonup m\delta_0$, where $m = \int \omega_0$ and δ_0 is the dirac measure at the origin. The second result is a generalization of a previous result by the authors. The main result in [4] can be reformulated as stating that, if: (i) the initial vorticity ω_0 is odd with respect to the horizontal axis, (ii) its restriction to the upper half-plane has a distinguished sign and (iii) $\alpha = 1$, then the hypothesis that $|\widetilde{\omega}(x,t)| \rightharpoonup \mu$, where μ is a measure (which must be supported in $\{|x_1| \leq M\} \times \{x_2 = 0\}$ for confinement reasons), implies that μ must consist of an at most countable sum of Dirac masses whose supports may only accumulate at the origin. Our second result in this article is to remove conditions (i) and (ii) on ω_0, keeping the same conclusion.

In 1994, C. Marchioro proved that the solution $\omega = \omega(x,t)$ of equation (1.1) with an initial vorticity bounded and nonnegative, with support contained in a disk of radius $R_0 > 0$ centered at the origin, has support contained in a disk of radius $(R_0^a + bt)^{1/a}$, with $a = 3$, for some constant $b > 0$, see [7]. This first result in confinement of vorticity has been improved and extended in several ways. The exponent $1/a$ has been improved to $1/4+$ by P. Serfati, see [12] and independently by Iftimie, Sideris and Gamblin, see [5]. Other extensions and improvements include unbounded initial vorticity [6, 2], flows in exterior domains [8], slightly viscous flows [9] and axisymmetric flows, [10, 11]. Confinement results basically control the rate at which vorticity is spreading. The present work is an attempt to go beyond controlling this rate, actually describing the way in which vorticity is spreading.

If the initial vorticity does not have a distinguished sign, the best confinement one may expect in general is at the rate $a = 1$, see [5]. In fact, [5] contains the construction of an example of smooth. compactly supported vorticity for which the support grows precisely in a linear fashion. This means that the self-similar scale of interest is $\alpha = 1$, and the time asymptotic behavior of $|\widetilde{\omega}|$ is what would give a reasonably complete description of the vorticity scattering in this case.

The remainder of this article is divided into three sections. In the first section we discuss the result on the asymptotic behavior of $\widetilde{\omega}$. The second section contains the result on $|\widetilde{\omega}|$ and the third section contains comments and conclusions.

2. Confinement of the net vorticity

Let $\omega_0 \in L^p_c(\mathbb{R}^2)$, for some $p > 2$ and consider $\omega = \omega(x,t)$ a solution of (1.1) with initial data ω_0. Our basic problem is to describe the spatial distribution of the vorticity $\omega(\cdot,t)$ for large t. For $\alpha \in (0,1]$ we introduce the rescaled vorticity:

$$W_\alpha = W_\alpha(x,t) \equiv t^{2\alpha}\omega(t^\alpha x, t).$$

Clearly, if ω_0 is single-signed, the known results on confinement tell us that, for any $\alpha > 1/4$, the support of W_α is contained in a disk centered at the origin whose radius vanishes as $t \to \infty$. What happens when the vorticity is allowed to change sign?

Let $U_\alpha \equiv K * W_\alpha$, with K given by (1.2). It is a straightforward calculation to verify that W_α and U_α satisfy the equation

$$\frac{\partial W_\alpha}{\partial t} - \frac{\alpha}{t} \operatorname{div} (x W_\alpha) + \frac{1}{t^{2\alpha}} \operatorname{div} (U_\alpha W_\alpha) = 0. \tag{2.1}$$

We are now ready to state and prove our first result.

Theorem 2.1. *Let $\alpha > 1/2$ and set $m = \int \omega_0(x)\, dx$. Then $W_\alpha(\cdot, t) \rightharpoonup m\delta_0$ weak-$*$ in $\mathcal{BM}(\mathbb{R}^2)$ as $t \to \infty$.*

Proof. We will begin by considering the linear part of the evolution equation (2.1) with initial condition at $t = 1$:

$$\begin{cases} \dfrac{\partial f}{\partial t} - \dfrac{\alpha}{t} \operatorname{div} (x f) = 0 \\ \\ f(x, 1) = g(x). \end{cases}$$

The solution f is given by the (multiplicative) semigroup $f(x,t) = S_t[g](x) \equiv t^{2\alpha} g(t^\alpha x)$, interpreted in the sense of distributions. We then write (2.1) as an inhomogeneous version of this linear equation, with source term given by

$$h(x,t) \equiv -\frac{1}{t^{2\alpha}} \operatorname{div} (U_\alpha W_\alpha).$$

With this we can write the solution W_α of (2.1), with initial data $W_\alpha(x, 1) = \omega(x, 1) \equiv g(x)$, using Duhamel's formula:

$$W_\alpha(x,t) = S_t[g](x) + \int_1^t S_{t/s}[h](x,s)\, \mathrm{d}s. \tag{2.2}$$

(In the integral above the semigroup is acting in the spatial variable only.) Of course (2.2) must be interpreted in the sense of distributions. We now turn to the analysis of each term in (2.2). Let $\varphi \in C_c^\infty(\mathbb{R}^2)$. We then have:

$$\int_{\mathbb{R}^2} \varphi(x) W_\alpha(x,t)\, \mathrm{d}x = \int_{\mathbb{R}^2} \varphi\left(\frac{y}{t^\alpha}\right) g(y)\, \mathrm{d}y + \int_1^t \int_{\mathbb{R}^2} \varphi\left(\frac{s^\alpha y}{t^\alpha}\right) h(y,s)\, \mathrm{d}y\, \mathrm{d}s \equiv I_1 + I_2.$$

First note that, as $t \to \infty$,

$$I_1 \to \left(\int_{\mathbb{R}^2} g(y)\, \mathrm{d}y \right) \varphi(0),$$

by the Lebesgue Dominated Convergence Theorem. Next, recall that the total integral of vorticity is conserved and hence the proof will be concluded once we establish that $I_2 \to 0$.

We compute directly, integrating by parts and using the relation between U_α and W_α:

$$\begin{aligned}
I_2 &= -\int_1^t \int_{\mathbb{R}^2} \varphi\left(\frac{s^\alpha y}{t^\alpha}\right) \frac{1}{s^{2\alpha}} \operatorname{div}\,(U_\alpha W_\alpha)(y,s)\,\mathrm{d}y\,\mathrm{d}s \\
&= \int_1^t \int_{\mathbb{R}^2} \frac{1}{s^\alpha t^\alpha} \nabla\varphi\left(\frac{s^\alpha y}{t^\alpha}\right) \cdot (U_\alpha W_\alpha)(y,s)\,\mathrm{d}y\,\mathrm{d}s \\
&= \int_1^t \frac{1}{s^\alpha t^\alpha} \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \nabla\varphi\left(\frac{s^\alpha y}{t^\alpha}\right) \cdot K(y-z) W_\alpha(z,s) W_\alpha(y,s)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}s.
\end{aligned}$$

We now use the antisymmetry of the Biot-Savart kernel K to obtain:

$$I_2 = \frac{1}{2}\int_1^t \frac{1}{s^\alpha t^\alpha} \int_{\mathbb{R}^2}\int_{\mathbb{R}^2} H_\varphi(s,t,z,y) W_\alpha(z,s) W_\alpha(y,s)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}s,$$

where

$$H_\varphi(s,t,z,y) \equiv \left(\nabla\varphi\left(\frac{s^\alpha y}{t^\alpha}\right) - \nabla\varphi\left(\frac{s^\alpha z}{t^\alpha}\right)\right) \cdot K(y-z).$$

Let us observe that

$$|H_\varphi| \leq \frac{s^\alpha}{t^\alpha}\|D^2\varphi\|_{L^\infty}|y-z||K(y-z)| \leq C(\varphi)\frac{s^\alpha}{t^\alpha}.$$

Hence we arrive finally at

$$|I_2| \leq C(\varphi)\left(\int_{\mathbb{R}^2}|\omega_0|\right)^2 \frac{t-1}{t^{2\alpha}},$$

which clearly converges to 0 as $t \to \infty$ as long as $2\alpha > 1$. This concludes the proof. □

Remark 2.1. This result does not say anything new if the initial vorticity has a distinguished sign. As we mentioned in the introduction, if the vorticity has a distinguished sign, the support of vorticity is contained in a ball whose radius grows like $\mathcal{O}(t^\alpha)$, with $1/4 < \alpha$. From that, Theorem 2.1 follows immediately.

Remark 2.2. What new information is contained in the conclusion of Theorem 2.1? Imagine that we are given initial vorticity $\omega_0 = \omega_0^+ - \omega_0^-$, which are the positive and negative parts of the initial vorticity. Let $\omega = \omega^+ - \omega^-$ be the solution of 2D Euler with initial vorticity ω_0. Due to the nature of vortex dynamics, both ω^+ and ω^- are time-dependent rearrangements of ω_0^+ and ω_0^- respectively, and hence their integrals, which we may call m^+ and m^-, are constant in time. One consequence of Theorem 2.1 is that the integral of vorticity in a ball of radius t^α converges to $m^+ - m^-$, for any $\alpha > 1/2$. This is weak confinement of the *imbalance* between the positive and negative parts of vorticity in a ball of sublinear radius. This is consistent with the conjectural picture that the only way for the support of vorticity to grow fast is through the shedding of vortex pairs.

3. Vortex scattering

The aim of this section is to prove the following theorem.

Theorem 3.1. *Suppose that the initial vorticity* $\omega_0 \in L^p_c(\mathbb{R}^2)$*,* $p > 2$ *is such that the absolute value of the rescaled vorticity* $|\widetilde{\omega}(y,t)| = t^2|\omega(ty,t)|$ *converges weakly to some measure* μ *as* $t \to \infty$*. Then* μ *must be of the form:*

$$\mu = \sum_{i=1}^{\infty} m_i \; \delta_{\alpha_i}$$

where:

(a) $\alpha_i \neq \alpha_j$ *if* $i \neq j$ *and* $\alpha_i \to 0$ *as* $i \to \infty$*;*
(b) *the masses* m_i *are nonnegative and verify* $\sum_{i=1}^{\infty} m_i = \|\omega_0\|_{L^1}$*;*
(c) *for all* i*,* $|\alpha_i| \in [0, M]$*, where* $M = \|u\|_{L^\infty([0,\infty)\times\mathbb{R}^2)}$*;*
(d) *there exists a constant* $D > 0$*, depending solely on* p*, such that, for all* i *with* $m_i \neq 0$ *we have*

$$|\alpha_i| \leq D\|\omega_0\|_{L^p}^{\frac{p'}{2}} \; m_i^{1-\frac{p'}{2}}.$$

Remark 3.1. In the statement above, the masses m_i are allowed to vanish only to include the case when the limit measure contains a finite number of Diracs. For notational convenience, in the case when there are only a finite number of Dirac masses, we artificially added a countable number of Dirac masses with zero masses and positions converging to 0.

To prove Theorem 3.1 we first note that since ω is transported by the velocity u, so is $|\omega|$:

$$\partial_t|\omega| + \operatorname{div}(u|\omega|) = 0$$

so that the equation for the absolute value of the rescaled vorticity is

$$\partial_t|\widetilde{\omega}(y,t)| - \frac{1}{t}\operatorname{div}\big[y|\widetilde{\omega}(y,t)|\big] + \frac{1}{t^2}\operatorname{div}\big[\widetilde{u}(y,t)|\widetilde{\omega}(y,t)|\big] = 0,$$

where $\widetilde{u}(y,t)$ denotes the rescaled velocity $\widetilde{u}(y,t) = tu(ty,t)$.

Let us take the product with a test function $\varphi \in C^1(\mathbb{R}^2)$ and integrate in space:

(3.1)
$$\partial_t \int |\widetilde{\omega}(y,t)|\,\varphi(y)\,\mathrm{d}y = -\frac{1}{t}\int |\widetilde{\omega}(y,t)|\,y\cdot\nabla\varphi(y)\,\mathrm{d}y + \frac{1}{t^2}\int |\widetilde{\omega}(y,t)|\,\widetilde{u}(y,t)\cdot\nabla\varphi(y)\,\mathrm{d}y.$$

We now recall the following argument that was used in [4]. The left-hand side of (3.1), when integrated from 0 to t, is uniformly bounded in t. By the hypothesis we know that

$$\lim_{t\to\infty}\int |\widetilde{\omega}(y,t)|\,y\cdot\nabla\varphi(y)\,\mathrm{d}y = \big\langle y\mu, \nabla\varphi\big\rangle$$

so that the integral from 0 to t of the first term on the right-hand side of (3.1) will behave in general like $\big\langle y\mu,\nabla\varphi\big\rangle \log t$. As for the third term, it is not difficult to see that it is

$\mathcal{O}(1/t)$. The dominant part of the third term must balance the logarithmic blow-up in time of the second term. This argument implies as in [4, Lemma 3.3] that the following inequality must hold:

$$\limsup_{t\to\infty}\Big(\frac{1}{t}\int |\,\widetilde{\omega}(y,t)|\,\widetilde{u}(y,t)\cdot\nabla\varphi(y)\,\mathrm{d}y\Big) \geq \big\langle y\mu, \nabla\varphi\big\rangle. \tag{3.2}$$

On the other hand, it was also proved in [4, Proposition 3.1] a key estimate that in our case reads

$$\limsup_{t\to\infty}\Big|\frac{1}{t}\int |\,\widetilde{\omega}(y,t)|\,\widetilde{u}(y,t)\cdot\nabla\varphi(y)\,\mathrm{d}y\Big| \leq D\|\omega_0\|_{L^p}^{\frac{p'}{2}}\sum_{i=1}^{\infty} m_i^{2-\frac{p'}{2}}|\nabla\varphi(\alpha_i)| \tag{3.3}$$

where $\sum_{i=1}^{\infty} m_i\delta_{\alpha_i}$ is the discrete part of the measure μ. The proof of [4, Proposition 3.1] valid in the case of the half-plane carries over to the case of the full plane with straightforward modifications. Indeed, that proof only uses the following estimate relating the rescaled velocity to the rescaled vorticity

$$|\widetilde{u}(x,t)| \leq \int \frac{C}{|x-y|}\,|\,\widetilde{\omega}(y,t)|\,\mathrm{d}y.$$

This estimate trivially holds in the case of the full space, too.

We now deduce from (3.2) and (3.3) that

$$\big\langle y\mu, \nabla\varphi\big\rangle \leq D\|\omega_0\|_{L^p}^{\frac{p'}{2}}\sum_{i=1}^{\infty} m_i^{2-\frac{p'}{2}}|\nabla\varphi(\alpha_i)|.$$

Writing the same relation with φ replaced by $-\varphi$ we finally get

$$|\big\langle y\mu, \nabla\varphi\big\rangle| \leq D\|\omega_0\|_{L^p}^{\frac{p'}{2}}\sum_{i=1}^{\infty} m_i^{2-\frac{p'}{2}}|\nabla\varphi(\alpha_i)|. \tag{3.4}$$

It is a simple matter to deduce from (3.4) that

$$|\alpha_i| \leq D\|\omega_0\|_{L^p}^{\frac{p'}{2}} m_i^{1-\frac{p'}{2}}. \tag{3.5}$$

Indeed, let us fix $i_0\in\mathbb{N}$ and choose $\varphi\in C_0^\infty(\mathbb{R}^2)$ such that $\nabla\varphi(0)=\alpha_{i_0}$. Define next $\varphi_\varepsilon(x)=\varepsilon\varphi\big(\frac{x-\alpha_{i_0}}{\varepsilon}\big)$ and use it as test function in (3.4) to obtain

$$\big|\big\langle y\mu, \nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\big\rangle\big| \leq D\|\omega_0\|_{L^p}^{\frac{p'}{2}}\sum_{i=1}^{\infty} m_i^{2-\frac{p'}{2}}\big|\nabla\varphi\big(\frac{\alpha_i-\alpha_{i_0}}{\varepsilon}\big)\big|. \tag{3.6}$$

By the dominated convergence theorem the right-hand side converges to $D\|\omega_0\|_{L^p}^{\frac{p'}{2}} m_{i_0}^{2-\frac{p'}{2}}|\alpha_{i_0}|$ as $\varepsilon\to 0$. As for the left-hand side, we write

$$\big\langle y\mu, \nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\big\rangle = \alpha_{i_0}\big\langle \mu, \nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\big\rangle + \big\langle \mu, (y-\alpha_{i_0})\nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\big\rangle$$

and we notice that

$$(y-\alpha_{i_0})\nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\to 0 \quad \text{in } L^\infty \text{ strongly}$$

and

$$\nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\rightharpoonup \alpha_{i_0}\chi_{\{\alpha_{i_0}\}} \quad \text{in } L^\infty \text{ weak} *$$

as $\varepsilon\to 0$ so that

$$\big\langle y\mu,\nabla\varphi\big(\frac{y-\alpha_{i_0}}{\varepsilon}\big)\big\rangle\to |\alpha_{i_0}|^2 m_{i_0} \quad \text{as } \varepsilon\to 0.$$

Taking the limit $\varepsilon\to 0$ in (3.6) now yields (3.5).

We just proved part (d) of Theorem 3.1. Part (a) also follows at once by remarking that we have $m_i\to 0$ so, by (3.5), $\alpha_i\to 0$ as $i\to\infty$ too. Part (c) is a trivial consequence of the fact that the support of the vorticity is transported by the flow of u. Finally, part (b) is a direct consequence of the nonnegativity of the measure μ and also from the conservation of the L^1 norm of $|\widetilde{\omega}|$.

We now go to the last part of the argument, i.e. the proof that the continuous part of the measure μ vanishes.

Let D be a strip of the form $D=\{c\le ay_1+ay_2\le d\}$ disjoint of the set $A\equiv\{0\}\bigcup_{i\ge 1}\{\alpha_i\}$. We prove that the measure μ must necessarily vanish in the interior of such a strip. First, since $0\ne D$ we have that $cd>0$. We assume without loss of generality that $c,d>0$. Let $[c',d']$ a subinterval of (c,d) and choose a smooth function $h\in C^\infty(\mathbb{R})$ such that $h'\in C_0^\infty(c,d)$, $h'\ge 0$ and $h'(s)=1/s$ for all $s\in[c',d']$. Choose now $\varphi(y_1,y_2)=h(ay_1+by_2)$ as test function in (3.4). Since $\operatorname{supp}\varphi\subset D$ we have that $\operatorname{supp}\varphi\cap A=$ which implies in turn that the right-hand side of (3.4) vanishes for this choice of test function. Therefore the left-hand side must vanish too:

$$0=\big\langle y\mu,\nabla\big(h(ay_1+by_2)\big)\big\rangle=\big\langle\mu,(ay_1+by_2)h'(ay_1+by_2)\big\rangle. \tag{3.7}$$

The function $y\mapsto(ay_1+by_2)h'(ay_1+by_2)$ is nonnegative and equal to 1 on the strip $\{c'\le ay_1+by_2\le d'\}$. Since the measure μ is nonnegative too, we deduce from (3.7) that μ vanishes on the strip $\{c'\le ay_1+by_2\le d'\}$. And since $[c',d']$ was an arbitrary subinterval of (c,d) we finally deduce that μ vanishes in the interior of the strip D.

In order to conclude the proof of Theorem 3.1, we only need to show that the measure μ vanishes in the neighborhood of each point of A^c. Let $y_0\in A^c$. Since the only possible accumulation point of the set A is 0, there exists a line $\{ay_1+by_2=c\}$ passing through y_0 and which does not cross A. A continuity argument using again that the points α_i can only accumulate at $\{0\}$ shows that there exists a strip $\{c-\varepsilon\le ay_1+by_2\le c+\varepsilon\}$ disjoint of A. But we proved that the measure μ must vanish on such a strip. This implies that μ vanishes in the neighborhood of y_0 and this completes the proof of Theorem 3.1.

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