Thoughtful comments on 'Bessel beams and signal propagation'*

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Abstract

In this paper we present thoughtful comments on the paper 'Bessel beams and signal propagation' showing that the main claims of that paper are wrong. Moreover, we take the opportunity to show the non trivial and indeed surprising result that a scalar pulse (i.e., a wave train of compact support in the time domain) that is solution of the homogeneous wave equation (vector $(\vec{E}, \vec{B})$ pulse that is solution of Maxwell equations) is such that its wave front in some cases does travel with speed greater than c, the speed of light . In order for a pulse to posses a front that travels with speed c, an additional condition must be satisfied, namely the pulse must have finite energy. When this condition is fulfilled the pulse still can show peaks propagating with superluminal (or subluminal) velocities, but now its wave front travels at speed c. These results are important because they explain several experimental results obtained in recent experiments, where superluminal velocities have been observed, without implying in any breakdown of the Principle of Relativity.

In this paper we present some thoughtful comments ($\mathbf{C}_1 - \mathbf{C}_4$) concerning statements presented in the paper 'Bessel beams and signal propagation' [1] and also some non trivial results concerning superluminal propagation of peaks in particular electromagnetic pulses in nondispersive media.

In [1] the author recalls that the experimental results presented in [2] showed that Bessel beams generated at microwave frequencies have a *group velocity* greater than the velocity

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of light c (in what follows we use units such that $c = 1$)[1]. His intention was then to show that the signal velocity, defined according to Brillouin and Sommerfeld (*B&S*) was also superluminal. We explicitly shows that the particular example used by the author of [1], given by the Bessel beam of his eq.(3) does not endorse his claim. Contrary to the author's conclusion this beam has no fronts in both space and time domains, hence cannot satisfy *B&S* defintion of a signal. Moreover, the beam given by eq.(3) of [1] travels rigidly with a superluminal speed. We prove then that there are two classes of general Bessel pulses satisfying *B&S* definition of signal. A solution of the *HWE* corresponding to class I is such that the group speed is always less than c whereas its front moves with speed c.[2] A solution of the *HWE* of the class II travels rigidly at superluminal speed if care is not taken of the energy content of the pulse. We present also some necessary comments concerning solutions of Maxwell equations associated with Bessel beams of classes I and II.

We start by recalling the general solution of the *HWE* $\Box\Phi = 0$ in Minkowski spacetime (M, η, D) [10-12]. In a given Lorentz reference frame [10-12] $I = \partial/\partial t \in \sec TM$, we choose cylindrical coordinates (ρ, φ, z) naturally adapted to the I reference frame, where $\rho = (x^2 + y^2)^{\frac{1}{2}}$ and $x = \rho\cos\varphi$ and $y = \rho\sin\varphi$, with (x, y, z) being the usual cartesian coordinates naturally adapted to I. Writting

$$\Phi(t, \rho, \varphi, z) = f_1(\rho) f_2(\varphi) f_3(t, z), \tag{1}$$

and substituting eq.(1) in the *HWE* we get the following equations (where ν and Ω are *separation parameters*),

$$\begin{cases} \left[\rho^2 \frac{d^2}{d\rho^2} + \rho\frac{d}{d\rho} + (\rho^2\Omega^2 - \nu^2)\right] f_1 = 0, \\ \left(\frac{d^2}{d\varphi^2} + \nu^2\right) f_2 = 0, \\ \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} + \Omega^2\right) f_3 = 0. \end{cases} . \tag{2}$$

The first of eqs.(2) is Bessel's equation, the second one implies that ν must be an integer and the third is a Klein-Gordon equation in two dimensional Minkowski spacetime.[3] In what follows (without loss of generality for the objectives of the present paper) we choose $\nu = 0$

[1] In [3] we scrutinized the experimental results of [2]. We presented there a simple model showing that all particulars of the data (including the slowing of the superluminal velocity of the peak along the propagation direction) can be qualitatively and quantitatively understood as a scissor's like effect. Moreover in [3] we called the readers attention that in [4] peaks of *finite aperture approximations (FAA)* to particular acoustical Bessel pulses called X-waves (first discoverd by Lu and Greenleaf ([5,6]) have been see to travel at supersonic speed i.e., with velocity greater than c_s , the sound speed parameter appearing on the homogenous wave equation (*HWE*). In [4] and [7] it is also predicted the possibilty of launching *FAA* to superluminal electromagnetic X-waves, a fact that has been confirmed experimentally in the microwave region in [2] and in the optical region in [8]. A review concerning the different facets of 'superluminal' wave motion under different physical conditions can be found in [9].

[2] Of course, this is a kind of generalized reshaping phenomena which cannot endures for ever. It lasts until the peak of the wave catches the front.

[3] In 4-dimensional spacetime the Klein-Gordon equation possess families of luminal and superluminal solutions, besides subluminal solutions. See [4] and references therein.

(and also $\Omega > 0$). Then, we obtain as a solution of eqs.(2) a wave propagating in the z-direction, i.e.,

$$\Phi_{J_0}(t,\rho,z) = J_0(\rho\Omega)\exp[-i(\omega t - \bar{k}z)], \tag{3}$$

where the following dispersion relation must necessarily be satisfied,

$$\omega^2 - \bar{k}^2 = \Omega^2. \tag{4}$$

The dispersion relation given by eq.(4) may look strange at first sight, but there are evidences that it can be realized in nature (see below) in some special circunstances.

C$_1$. It is quite clear that the wave described by eq.(3), called in [1] a Bessel beam[4], has phase velocity $v_{ph} = \omega/\bar{k} > 1$. However, we point out that the statement done in [1] is *false*, namely: 'As known, in the absence of dispersion the group velocity v_{gr} of a Bessel pulse is equal to the phase one [4,5][5] since all the components at different frequencies propagate with the same velocity'. To prove its falsity recall that there exists a Lorentz reference frame [10-12]

$$I' = (1 - v_{gr}^2)^{\frac{1}{2}}\,(\partial/\partial t + v_{gr}\partial/\partial z) \in \sec TM, \tag{5}$$

which is moving with velocity $v_{gr} = d\omega/d\bar{k} < 1$ in relation to the frame I in the z-direction. In the coordinates naturally adapted to the frame I' the frequency of the wave is $\omega' = \Omega$, which means that in the frame I' the Bessel beam is stationary. This proves our statement that for Bessel beam the group velocity is always less than the velocity of light c.

C$_2$. Now, we show how to build two different classes (I and II) of solutions of the *HWE* by appropriate linear superpositions of waves of the form given by our eq.(3).

Class I. Suppose, following *B&S* [13,14] that a signal is defined as a pulse with a *finite* time duration at the origin $z = 0$ where a physical device generated it. We model our problem as a *Sommerfeld problem* [15] for the *HWE* (with cylindrical symmetry), i.e., we want to find the solution of the *HWE* with the following conditions (called in what follows Sommerfeld conditions),

$$\begin{aligned}
\Phi(t,\rho,0) &= AJ_0(\rho\Omega)[\Theta(t) - \Theta(t-T)]\sin\omega_0 t \\
&= AJ_0(\rho\Omega)\frac{1}{2\pi}\,\mathrm{Re}\int_\Gamma d\omega \mathrm{e}^{-i\omega t}\frac{\left\{\mathrm{e}^{i\omega T} - 1\right\}}{\omega - \omega_0}, \\
\left.\frac{\partial\Phi(t,\rho,z)}{\partial z}\right|_{z=0} &= AJ_0(\rho\Omega)\frac{1}{2\pi}\,\mathrm{Re}\int_\Gamma d\omega\ \bar{k}(\omega)\ \mathrm{e}^{-i\omega t}\frac{\left\{\mathrm{e}^{i\omega T} - 1\right\}}{\omega - \omega_0}.
\end{aligned} \tag{6}$$

In eq.(6) $\Theta(t)$ is the Heaviside function, A and $\omega_0 = \Omega$ are constants, Re means real part and $\bar{k}(\omega)$ is given below and for simplicity we take $T = N\tau_0 = 2\pi N/\omega_0$, with N an integer. Now, to solve our problem it is enough to get a solution of the third of eqs.(2). We have,

$$f_3(t,z) = \frac{1}{2\pi}\,\mathrm{Re}\int_\Gamma \frac{d\omega}{\omega - \omega_0}\left\{\mathrm{e}^{-i\omega(t-T-v_{gr}z)} - \mathrm{e}^{-i\omega(t-v_{gr}z)}\right\} \tag{7}$$

[4]Note that in [1] the author writes $\Omega = \omega\sin\theta$ and $\bar{k} = \omega\cos\theta$.

[5]The references [4,5] in [1] are the references [8,13] in the present paper.

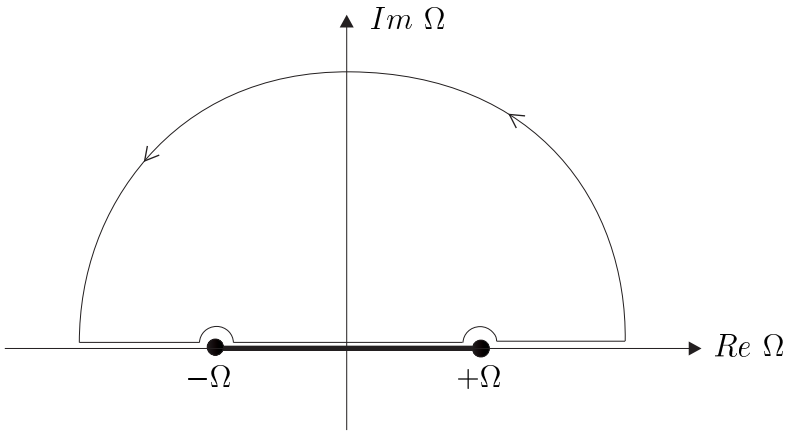

Figure 1: Contour for integration of Eq.(7) for $t - z < 0$.

where $v_{gr} = \bar{k}(\omega)/\omega$ and Γ is an appropriate path in the complex ω-plane. We note $\lim_{\omega\to\infty} v_{gr} = 1$. Putting eq.(7) into the third of eqs.(2) we see that the dispersion relation given by eq.(4) must be satisfied. To continue we write,

$$\bar{k}(\omega) = \sqrt{(\omega + \Omega)(\omega - \Omega)}. \tag{8}$$

There are two branch points at $\omega = \pm\Omega$. The corresponding branch cuts can be taken as the segment $(-\Omega, \Omega)$ in the real ω-axis. Following Γ from positive values of $\Re\omega$ above and close to the real axis, the root in eq.(8) acquires a phase factor $e^{i\pi} = -1$ when passing from $\mathrm{Re}\,\omega > \Omega$ to $\mathrm{Re}\,\omega < -\Omega$. Then, on the real ω-axis we have,

$$\bar{k}(\omega) = \begin{cases} |\sqrt{\omega^2 - \Omega^2}|, & \omega > \Omega \\ -|\sqrt{\omega^2 - \Omega^2}|, & \omega < -\Omega \end{cases} \tag{9}$$

a result that is necessary in order to calculate the value of f_3 for $(t - v_{gr}z) > 0$. We are not going to investigate this case here, since we are interested in the behavior of f_3 for the case where $(t - z < 0)$. In this case, we must close the contour Γ in the upper half plane. Since there are no poles inside the contour we get that

$$f_3(t, z) = 0 \qquad \text{for} \qquad t - z < 0. \tag{10}$$

Now, it is easy to verify the intensity of the wave which is solution of the *HWE* and satisfies the Sommerfeld conditions given by eq.(6) has a maximum for $\omega = \omega_0$, i.e., the waves with frequency near ω_0 have always a much greater amplitude than all others. Under these conditions let us write,

$$\omega t - \bar{k}z = (\omega_0 t - \bar{k}_0 z) + (t - \frac{z}{v_{gr_0}})(\omega - \omega_0), \tag{11}$$

where $v_{gr_0} = (d\omega/d\bar{k})|_{\omega=\omega_0} < 1$ and $v_{ph_0} = \omega_0/\bar{k}_0 > 1$. We can write an approximation for the function $f_3(t,z)$ denoted by $\tilde{f}_3(t,z)$ as,

$$\tilde{f}_3(t,z) = \frac{1}{2\pi}\,\mathrm{Re}\left\{ \mathrm{e}^{-i\omega_0(t-z/v_{ph\,0})} \int\limits_{\omega_0-\triangle\omega}^{\omega_0+\triangle\omega} \frac{d\omega}{\omega-\omega_0} \left\{ \mathrm{e}^{-i\omega(t-T-z/v_{gr0})} - \mathrm{e}^{-i\omega(t-z/v_{gr0})} \right\} \right\}. \qquad (12)$$

We see that $\tilde{f}_3(t,0)$ is equal to $f_3(t,0)$ if we suppress in the expression for this function the frequencies very different from ω_0. Now, $\tilde{f}_3(t,0)$ has support on the whole temporal axis, i.e., in the interval $-\infty < t < \infty$, but it is taken by some authors (like, e.g., [16]) as representing a wave that begin gradually at $t = 0$ and ends gradually at $t = T$. Of course, no wave of the kind of $\tilde{f}_3$ can be build by any physical device. The importance of the function $\tilde{f}_3(t,z)$ is that, as emphasized by *B&S* [13,14] it shows that we can associate a *group velocity* to pulse peaks in general (and of Bessel beams in particular) satisfying the Sommerfeld conditons (eq.(6)) and that the group velocity in this case is *less* than the velocity of light. This means that after a while the *back* end of the wave that is travelling at speed $c(=1)$ will catch the peak. The wave reshapes even when propagating in vacuum.

A general subluminal J_0-Bessel beam can be written as,

$$\Phi_B(t,\rho,z) = J_0(\rho\omega)\mathcal{F}^{-1}[T(\omega)]\mathrm{e}^{i\bar{k}z} \qquad (13)$$

where $T(\omega)$ is an appropriate transfer function and $\mathcal{F}^{-1}$ is the inverse Fourier transform. Now, the *peaks* of *FAA* to acoustical pulses of the form given by eq.(13) (i.e., the waves at $z = 0$ are not zero only in the time interval $0 < t < T$) have been seen travelling at *subluminal* speed[6] in an experiment described in [4], thus endorsing the above analysis.

Class II. We now return to the dispersion relation given by eq.(4) and write,

$$\bar{k} = k\cos\theta, \quad \Omega = k\sin\theta, \qquad (14)$$

where θ is a constant called *axicon* angle [5,6,17]. It results that

$$\omega = \pm k. \qquad (15)$$

We immediately verify that

$$J_0(\omega\rho\sin\theta)\mathrm{e}^{-i(\omega t - kz\cos\theta)}, \qquad (16)$$

is a solution of the *HWE* whose beam width is proportional to $1/\omega\sin\theta$, thus being frequency dependent. The dependency of the beam width on frequency will cause the beam to have a pulse response that is independent of position. Indeed, suppose that the source is driven by a frequency distribution $B(\omega)$, i.e., we have a pulse

$$\Phi_X(t,\rho,z) = \int\limits_{-\infty}^{\infty} d\omega B(\omega) J_0(\omega\rho\sin\theta)\mathrm{e}^{-i(\omega t - kz\cos\theta)},\ \omega = k. \qquad (17)$$

[6]Of course, in this case the speed paramenter appearing in the *HWE* must be c_s, the sound speed in the medium, and the word subluminal speed used must be understood as a speed less than c_s.

If J_0 were not dependent on frequency the integral in eq.(17) would be simply the inverse Fourier transform of the source spectrum and we return to class I solutions. However, here J_0 is dependent on frequency and also on position and consequently modifies the pulse spectrum in such a way to make the time response of the pulse dependent on radial position. We put an index X in the wave given by eq.(17) because pulses of this kind have been named X-waves by Lu and Greenleaf since 1992 [5,6]. Even more, taking $B(\omega) = A\mathrm{e}^{-a_0|\omega|}$ (A and $a_0 > 0$ being constants), we can easily verify (c.r., pages 707 and 763 of [18]) that we can write for $\sin\theta > 0$,

$$\Phi_X(t,\rho,z) = A\int_{-\infty}^{\infty} d\omega \mathrm{e}^{-a_0|\omega|} J_0(\omega\rho\sin\theta)\mathrm{e}^{-i\omega(t-z\cos\theta)} \tag{18a}$$

$$= A\int_0^{\infty} d\omega \mathrm{e}^{-a_0\omega} J_0(\omega\rho\sin\theta)\cos(\omega\mu)$$

$$= \frac{A}{\left[\rho^2\sin^2\theta + [a_0+i\mu]^2\right]^{\frac{1}{2}}} + \frac{A}{\left[\rho^2\sin^2\theta + [a_0-i\mu]^2\right]^{\frac{1}{2}}} \tag{18b}$$

$$= \frac{A\sqrt{2}\left\{\left[\left[\rho^2\sin^2\theta + a_0^2 - \mu^2\right]^2 + 4a_0^2\mu^2\right]^{\frac{1}{2}} + \rho^2\sin^2\theta + a_0^2 - \mu^2\right\}^{\frac{1}{2}}}{\left\{\left[\rho^2\sin^2\theta + a_0^2 - \mu^2\right]^2 + 4a_0^2\mu^2\right\}^{\frac{1}{2}}}, \tag{18c}$$

where $\mu = (\ t - z\cos\theta)$.

Eq.(18c) shows that this wave is a real solution of the *HWE*. We recall that if in eq.(18a) we use as integration interval $0 < \omega < \infty$, we get only the first term in eq.(18b). In this case we have a complex wave that has been called the *broad band* X-wave in [4-6]. These waves and the more general ones given by eq.(18b) propagate without distortion with superluminal velocity given by $1/\cos\theta$, but of course they cannot be produced in the physical world because (like the plane wave solutions of the *HWE*) they have *infinity energy*, as it is easy to verify. Waves that are solutions of the linear relativistic wave equations and that propagate in a distortion free mode, have been called *UPWs* (undistorted progressive waves) in [4].

Now, we show that a X-pulse even if it has compact support in the time domain (thus being of the form of a *B&S* signal) is such that its front propagates with superluminal speed. To prove our statement we look for a solution of the *HWE* satisfying the following Sommerfeld conditions[7],

$$\begin{aligned} \Phi_X(t,\rho,0) &= [\Theta(t+T) - \Theta(t-T)]\int_{-\infty}^{\infty} d\omega B(\omega) J_0(\omega\rho\sin\theta)\mathrm{e}^{-i\omega t}, \\ \left.\frac{\partial\Phi_X(t,\rho,z)}{\partial z}\right|_{z=0} &= i\,[\Theta(t+T) - \Theta(t-T)]\cos\theta\int_{-\infty}^{\infty} d\omega B(\omega) J_0(\omega\rho\sin\theta)k(\omega)\mathrm{e}^{-i\omega t}, \end{aligned} \tag{19}$$

[7] $B(\omega)$ is taken in this example as a function such that $\int_{-\infty}^{\infty} d\omega B(\omega)J_0(\omega\rho\sin\theta)\mathrm{e}^{-i\omega t}$ has support in the interval $-\infty < t < \infty$.

and $k(\omega) = \omega$. Proceeding in the same way as in the Sommerfeld problem of class I solution presented above we obtain as a solution of the *HWE* (for $z > 0$),

$$\begin{aligned}\Phi_X(t,\rho,z) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} d\bar{\omega}B(\bar{\omega})J_0(\bar{\omega}\rho\sin\theta) \\ &\quad \times \int_{-\infty}^{\infty} d\omega e^{-i\omega(t-z\cos\theta)}\left[\frac{\mathrm{e}^{i(\omega-\bar{\omega})T}-\mathrm{e}^{-i(\omega-\bar{\omega})T}}{i(\omega-\bar{\omega})}\right] \qquad (20) \\ &= \begin{cases} \int_{-\infty}^{\infty} d\omega B(\omega)J_0(\omega\rho\sin\theta)\mathrm{e}^{-i\omega(t-z\cos\theta)} & \text{for} \quad |t-z\cos\theta| < T \\ 0 & \text{for} \quad |t-z\cos\theta| > T \end{cases}. \qquad (21)\end{aligned}$$

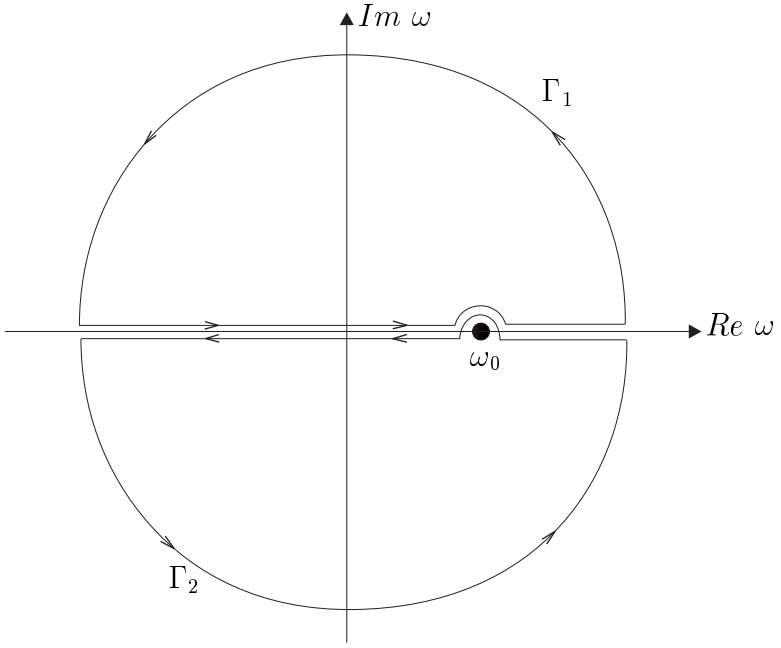

Figure 2: Contour for integration of Eq.(20). Γ_1 for $|t - z\cos\theta| > T$ and Γ_2 for $|t - z\cos\theta| < T$.

We see that for $|t - z\cos\theta| < T$ the integral in eq.(20) is *not* zero. Since the axicon angle $\theta > 0$, then $1 > \cos\theta < 0$ and it follows that the pulse is not zero for $z > t$ and $t > T$, what means that the wave front of our pulse propagates with *superluminal* speed! Of course, the pulse is zero for $z < (t-T)/\cos\theta$ or $z > (t+T)/\cos\theta$. We observe that the above result is true even a single Bessel pulse, i.e., when $B(\omega) = \delta(\omega - \omega_0)$, a result that we mentioned in [3].

How to compare this finding with the famous *B&S* result [13,14] stating that a wave pulse which propagates in a dispersive medium with loss has a front propagating at maximum speed c? Some things are to be recalled in order to get a meaningful answer. The first is that *B&S* example refers to a propagation of a 'plane' wave truncated in time (which, of course, has *infinite* energy) satisfying the Sommerfeld conditions (analogous to eq.(6)) and propagating in a dispersive medium with loss. A careful analysis [19] shows that the same problem in a dispersive medium with gain reveals that in this case we can find two kinds of solutions (both of of infinite energy). In one of these kinds, by appropriately choosing the integration path in the complex ω-plane we obtain as result that the front of the wave may travel with superluminal speed. This situation is somewhat analogous to what happen with some possible mathematical solutions of the tachyonic Klein-Gordon equation in two dimensional Minkowski spacetime [20,21]. This equation is important because it can be associated with the so called telegraphist equation.

The reason for our finding that the X-pulse propagating in a nondispersive medium, although of compact support in the time domain, is such that its front travel at superluminal speed is the following; the solution given by eq.(20) is not of *compact support* in the space domain and as such has infinite energy as can be easily verified. Only for a pulse of finite energy we can warranty that its front always travel with a speed that cannot be greater than maximum speed. Indeed, suppose we produce on the plane $z = 0$ a pulse like the one given by eq.(20), except that it has a finite lateral circular width of radius a, i.e.,it is taken as zero for $\rho > a$. Such a pulse is called a *FAA* to the pulse given by eq.(20) and as can be easily verified has *finite energy*. If such a pulse does not spread with infinite velocity during its build up, then after it is ready, i.e., at $t = T$ it occupies a region of compact support in space given by $|\vec{x}| < R$, where R is the maximum linear dimension involved. Such a field configuration can then be taken as part of the initial conditions for a *strictly* hyperbolic Cauchy problem at $t = T$. For such a problem it is well known the mathematical theorem that stablishes that [22,23] the time evolution of the pulse must be such that it is *null* for $|\vec{x}| > R + c(t - T)$. In conclusion, it is not sufficient for a wave to be of compact support in the time domain (i.e., to be a pulse) to assure that the wave front of the pulse moves in a nondispersive medium at maximum speed c. In order for the wave front to move with velocity c it is necessary that the pulse possess *finite energy*, and in order for this condition to be satisfied the pulse must have compact support in the space domain after its build up. We recall here that in [4] the peaks of *FAA* to acoustical pulses given by eq.(18) (with appropriated $B(\omega)$) have been seen traveling with velocities $c_s/\cos\theta$, thus confirming the theory developed above.

$\mathbf{C}_3$. We now examine the claim of [1] that a wave given by our eq.(17), with $B(\omega) = 1$, i.e.,

$$U(t, \rho, z) = \int_{-\infty}^{\infty} d\omega J_0(\omega\rho \sin\theta) \mathrm{e}^{-i(\omega t - kz\cos\theta)}, \ \omega = k. \tag{22}$$

is a pulse with support only in the z-axis at points $z = t/\cos\theta$ and with value at that points $\delta(0)$. The calculations presented in [1] are wrong. Before we prove our statement let us recall

that [1] quotes Brillouin: 'a signal can be defined as a pulse of finite temporal extension, that is, of infinite extension in the frequecy domain'.[8] The wave given by eq.(21) has an infinite extension in the frequency domain but it is not a pulse of finite time domain (for a fixed z). Indeed, as theorem 11 on page 22 in Sneddon's book [24] stablishes: a function which is bounded in the time domain has an infinite extension in the frequency domain, but it is not true that a function with an infinite frequency spectrum is necessarily bounded in the time domain. A trivial example of the last statement is the case of a Gaussian pulse, whose Fourier transform is itself a Gaussian. In the particular case of the wave given by eq.(21) it is immediate to realize that the integral is nothing more than the *Fourier transform* of a J_0 function, and the value of the integral is given in many books, in particular on page 523 of Sneddon's book [24]. We have,

$$\int_{-\infty}^{\infty} d\omega J_0(\omega\rho\sin\theta)\mathrm{e}^{-i(\omega t - kz\cos\theta)} \tag{23a}$$

$$= \begin{cases} \frac{2}{\sqrt{\rho^2\sin^2\theta - (t - z\cos\theta)^2}} & \text{for } |t - z\cos\theta| < \rho\sin\theta \\ 0 & \text{for } |t - z\cos\theta| > \rho\sin\theta \end{cases} \tag{23b}$$

Eq.(22b) shows that $U(t,\rho,z)$ has support in the entire time axis provided that $|t - z\cos\theta| < \rho\sin\theta$. When $\rho = 0$, since U is real (as can be seen directly from eq.(22a) we must have that $|t - z\cos\theta| = 0$ and the function U is singular. We see that the result expressed by eq.(22b) is compatible with the one given by eq.(18b) if we take the limit for $a_0 \to 0$.

$\mathbf{C}_4$. Finally, we investigate the claim (done in [1] and attributed to [8]) that the wave function given by eq.(3) represents an electric field. This claim is a nonsequitur. Indeed, the scalar solutions of the *HWE* can be used to generated solutions of the Maxwell system using the *Hertz* potential method (see, e.g.[25,26]). In particular, superluminal solutions of the *HWE* can be used to produce superluminal solutions of Maxwell equations [4,7,9]. If we choose a *magnetic* Hertz potential $\vec{\Pi}_m = \Phi_{J_0}\hat{z}$ it is a simple exercise to show that the transverse electric and magnetic fields do not show any dependence on J_0. Only the B_z component of the electromagnetic field configuration has a J_0 dependence. Explicitly we have from the well known formulas $\vec{E} = -\partial/\partial t(\nabla\times\vec{\Pi}_m)$ and $\vec{B} = \nabla\times\nabla\times\vec{\Pi}_m$ that,

$$\begin{aligned} E_\rho &= 0,\ E_\varphi = i\omega\Omega J_1(\Omega\rho)\mathrm{e}^{-i(\omega t - \bar{k}z)},\ E_z = 0, \qquad (24)\\ B_\rho &= -i\bar{k}\Omega J_1(\Omega\rho)\mathrm{e}^{-i(\omega t - \bar{k}z)},\\ B_z &= \Omega^2\left[\frac{J_1(\Omega\rho)}{\Omega\rho} + \frac{J_0(\Omega\rho)}{2} - \frac{J_2(\Omega\rho)}{2}\right]\mathrm{e}^{-i(\omega t - \bar{k}z)},\\ \omega^2 - \bar{k}^2 &= \Omega^2. \qquad (25) \end{aligned}$$

With an electric Hertz potential we obtain a solution where only the E_z component has a J_0 dependence. As such, we conclude that the electromagnetic beams observed in [2] and

[8] This definition is due to Sommerfeld. See [13,14].

also in [8,17] are not J_0 beams. A careful analysis of the solutions of Maxwell equations in cylindrical symmetry shows that there are not J_0 solutions representing transverse electric fields. The existence of only one peak observerd in the experiments done in [2] is due to a J_1/ρ like term in the diffracted E_φ term of the real experiment. A more detailed analysis will be reported elsewhere.

Our conclusions are as follows: (i) our results show that the main claims of [1] are wrong and/or misleading and leads to equivocated conclusions concerning recent experimental results showing superluminal motion of peaks of particular electromagnetic field configurations in *nondispersive* media; (ii) we also prove a non *trivial* result, namely that the condition that a wave is of *finite* time duration is not a *sufficient* condition for its front to propagate with the speed c. It is necessary in order for the front to travel with speed c that the pulse possess *finite* energy, and thus as explained above it must (after being prepared by the launching device) have support only in a compact space region when ready;[9] (iii) only *FAA* to superluminal solutions of the *HWE* (acoustical case) and to superluminal solutions of Maxwell equations can be produced in nature, because only waves of this kind have finite energy. These *FAA* exhibit peaks propagating with superluminal speeds even in the vacuum, but since their fronts propagate with speed c this kind of phenomenom does not implies in any danger for the Theory of Relativity.

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