INFLUENCE DIAGNOSTICS FOR STRUCTURAL ERRORS-IN-VARIABLES MODEL UNDER THE STUDENT-t DISTRIBUTION

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Abstract

The influence of observations on the parameter estimates for the simple structural errors-in-variables model with no equation error, under the Student-t distribution, is investigated using the local influence approach. The likelihood displacement approach is useful for outlier detection especially when a masking phenomenon is present. The diagnostics are illustrated with two examples.

Key Words: Diagnostics; influential observations; local influence; t-distribution.

1 Introduction

The main object of this paper is the study of local influence and diagnostic in the structural errors-in-variables models. It is assumed that the observed variables follow a bivariate Student-t distribution. The detection of outliers and influential observations in the error-in-variables model (EVM), under the normality assumption, has been considered by some authors. For example, Kelly (1984) derived the influence functions of the model parameters. Wellman and Gunst (1991) showed the need for influence diagnostics in such models using the influence function. Abdullah (1995) applied some diagnostic methods in regression analysis to the functional model. Lee and Zhao (1996) employed the local influence approach to some linear and nonlinear measurement error models. Recently, Kim (2000) applied the local influence method in the structural EVM. However, no applications of local influence has been considered for structural EVM under Student t-distributions. Thus, the main object of the this paper is to apply the approach of local influence to structural EVM under Student-t distributions. The perturbation schemes considered here are schemes in which the scale matrix is modified to allow convenient perturbations of the model.

In the section 2, along with the notation, the structural EVM, under the Student-t distribution, is defined. The local influence method is reviewed in section 3. Section 4 deals with the derivation of the diagnostics procedures for the structural errors-in-variables Student-t model. Two illustrative examples are given in the last section.

2 The structural Student-t errors-in-variables model

In this paper, we consider the simple structural EVM with no equation error given by (Fuller; 1987)

$$\begin{aligned} Y_i &= y_i + e_i, \\ X_i &= x_i + u_i, \\ y_i &= \alpha + \beta x_i, \quad i = 1, ..., n, \end{aligned} \tag{2.1}$$

where Y_i and X_i are the ith observations whose true values are y_i and x_i, respectively, and e_i and u_i are measurement errors. As in Bolfarine and Arellano-Valle (1994), it is assumed that the unobservable vectors $(x_i, e_i, u_i)^\top$ are independently distributed as a trivariate Student t-distribution with location vector $(\mu_x, 0, 0)^\top$ and diagonal scale matrix $\mathrm{diag}(\phi_x, \phi_e, \phi_u)$. Further assume that $\lambda = \phi_e/\phi_u$ is known. In this case, we may consider that $\phi_e = \phi_u = \phi$, $(\lambda = 1)$, without loss of generality. Then the joint distribution of $Z_i = (Y_i, X_i)^\top$ becomes a bivariate Student-t distribution with location vector $\boldsymbol{\mu} = (\alpha + \beta\mu_x, \mu_x)^\top$ and scale matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \beta^2\phi_x + \phi & \beta\phi_x \\ \beta\phi_x & \phi_x + \phi \end{pmatrix},$$

that is, $\boldsymbol{Z}_i \overset{\text{iid}}{\sim} t_2(\boldsymbol{\mu}, \boldsymbol{\Sigma}; \nu)$, $\nu > 0$, $i = 1, ..., n$. The density function of $\boldsymbol{Z}_i$ is given by:

$$f(\boldsymbol{z}_i; \boldsymbol{\theta}) = \frac{1}{2\pi}|\boldsymbol{\Sigma}|^{-1/2}(1 + \frac{1}{\nu}d_i(\boldsymbol{\theta}))^{-\frac{1}{2}(\nu+2)}, \tag{2.2}$$

where $d_i(\boldsymbol{\theta}) = (\boldsymbol{z}_i - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{z}_i - \boldsymbol{\mu})$ and $\boldsymbol{\theta} = (\alpha, \beta, \phi, \mu_x, \phi_x)^\top$. To obtain the maximum likelihood estimator of the vector $\boldsymbol{\theta}$, we use the EM-algorithm as considered in Bolfarine and Arellano-Valle (1994).

Several authors have considered the Student-t distribution as an alternative to the normal distribution because it can naturally acomodate outliers present in the data. For example, Lange et. al. (1989) and Galea et al. (1997) discuss the use of the Student-t distribution in regression models and in problems related to multivariate analysis; Bolfarine and Arellano-Valle (1994) introduce Student-t functional and structural measurement error models and Bolfarine and Galea (1996) use the Student-t distribution in comparative calibration models.

The Student-t distribution incorporates an additional parameter, ν, namely the degrees of freedom, which allows adjusting for the kurthosis of the distribution. This parameter can be fixed previously and Lange et al. (1989) and Berkane et al. (1994) recommend taking $\nu = 4$ or, otherwise, get information for it from the data set. For some difficulty in the estimation of ν, see Fernández and Steel (1999).

3 Influence diagnostics for parameter estimates

Detecting outliers and influential observations is an important step in the analysis of data sets. Several approaches exist to asses the influence of data and model perturbations on the parameter estimates. Overviews can be find in the books by Cook and Weisberg (1982) and Chatterjee and Hadi (1988) and the paper by Cook (1986).

Case deletion is a popular way to asses the individual impact of cases on the estimation process. This approach is global influence analysis, namely the effect of an observation is assessed by completely removing it. An alternative approach, local influence, is based on differential geometry instead of complete deletion. It employs a differential comparison of parameter estimates before and after perturbation to data values or model assumptions. We apply local influence methods to the EVM Student-t model. As in Cook (1986), the displacement in log-likelihood function was taken as the metric to evaluate local influence.

The log-likelihood function of model (2.1) is given by

$$L(\boldsymbol{\theta}) = \sum_{i=1}^{n} l_i(\boldsymbol{\theta}), \tag{3.1}$$

where $l_i(\boldsymbol{\theta}) = -\log(2\pi) - \frac{1}{2}\log|\boldsymbol{\Sigma}| - \frac{1}{2}(\nu+2)\log(1+d_i(\boldsymbol{\theta})/\nu)$, $i = 1, ..., n$ and $\boldsymbol{\theta} = (\alpha, \beta, \phi, \mu_x, \phi_x)^\top$

Small perturbations are introduced into the Student-t EVM through a vector $\boldsymbol{\omega}$. We write $L(\boldsymbol{\theta}|\boldsymbol{\omega})$ for the log-likelihood (3.1) corresponding to the perturbed data or model and let $\widehat{\boldsymbol{\theta}}_\omega$ be the maximum likelihood estimates from the perturbed model. Specific perturbation schemes are described below. For each scheme there is a point $\boldsymbol{\omega}_0$ representing no perturbation. The influence of $\boldsymbol{\omega}$ can be assessed by the log-likelihood displacement

$$LD(\boldsymbol{\omega}) = 2[L(\widehat{\boldsymbol{\theta}}) - L(\widehat{\boldsymbol{\theta}}_\omega)], \tag{3.2}$$

where $\widehat{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}}_{\omega_0}$. Because evaluation of $LD(\boldsymbol{\omega})$ for all $\boldsymbol{\omega}$ is practically unfeasible, Cook (1986) proposes studying the local behaviour of $LD(\boldsymbol{\omega})$ around $\boldsymbol{\omega}_0$. This was done using the normal curvature C_l of $LD(\boldsymbol{\omega})$ at $\boldsymbol{\omega}_0$ in the direction of some unit vector $\boldsymbol{l}$.

Cook (1986) showed that the normal curvature in the direction $\boldsymbol{l}$ takes the form

$$C_l = 2|\boldsymbol{l}^\top \boldsymbol{\Delta}^\top \ \boldsymbol{I}^{-1} \boldsymbol{\Delta} \boldsymbol{l}|, \tag{3.3}$$

where $\|\boldsymbol{l}\| = 1$, $\boldsymbol{\Delta} = \dfrac{\partial^2 L(\boldsymbol{\theta}/\boldsymbol{\omega})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\omega}^\top}$ and the 5×5 observed information matrix $\boldsymbol{I} = -\dfrac{\partial^2 L(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^\top}$ are both evaluated at $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$ and $\boldsymbol{\omega} = \boldsymbol{\omega}_0$.

Let $\boldsymbol{l}_{\max}$ be the direction of maximum normal curvature, which is the perturbation that produces the greatest local change in $\widehat{\boldsymbol{\theta}}$. The most influential elements of the data may be identified by their large component of the vector $\boldsymbol{l}_{\max}$. Furthermore, $\boldsymbol{l}_{\max}$, is just the eigenvector corresponding to the largest eigenvalue of $\boldsymbol{\Delta}^\top \ \boldsymbol{I}^{-1}\boldsymbol{\Delta}$. Other important direction is $\boldsymbol{l} = \boldsymbol{e}_{in}$, which corresponds to the ith position, where there is a one. In that case, the normal curvature, called the total local influence of individual i, is given by $C_i = 2\boldsymbol{\Delta}_i^\top \boldsymbol{I}^{-1} \boldsymbol{\Delta}_i$, where $\boldsymbol{\Delta}_i^\top$ is the ith column of $\boldsymbol{\Delta}$, $i = 1, ..., n$. We use $\boldsymbol{l}_{\max}$ and C_i as diagnostics for local influence. From (3.1), it follows that $\boldsymbol{I}$ takes the form

$$\boldsymbol{I} = -\left[\left(\frac{\partial^2 L(\boldsymbol{\theta})}{\partial \gamma \partial \tau} \right) \right], \tag{3.4}$$

where, $\gamma, \tau = \alpha, \beta, \phi, \mu_x, \phi_x$. The elements of the matrix $\boldsymbol{I}$ are presented in the appendix.

When a subset $\boldsymbol{\theta}_1$ from the partition $\boldsymbol{\theta} = (\boldsymbol{\theta}_1^\top, \boldsymbol{\theta}_2^\top)^\top$ is of interest, diagnostics for influence can be based on (Cook, 1986)

$$\boldsymbol{\Delta}^\top (\boldsymbol{I}^{-1} - \boldsymbol{B}_{22}) \boldsymbol{\Delta},$$

where

$$\boldsymbol{B}_{22} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{22}^{-1} \end{pmatrix},$$

and $\boldsymbol{I}_{22}$ is determined from partitioning $\boldsymbol{I}$ conformably with the partition of $\boldsymbol{\theta}$.

As in Kim (2000) we considered a perturbed model in which the vectors $\boldsymbol{Z}_i = (Y_i, X_i)^\top$ are independently distributed as the Student t-distribution; $t_2(\boldsymbol{\mu}, \boldsymbol{\Sigma}/\omega_i; \nu)$, $i = 1, ..., n$. Here $\boldsymbol{\omega} = (\omega_1, ..., \omega_n)^\top$ and $\boldsymbol{\omega}_0 = \mathbf{1}_n = (1, ..., 1)^\top$. This perturbation scheme puts a weight on the scale matrix for each observation and provides similar result as the case-weights perturbation.

In this case the $\boldsymbol{\Delta}$ matrix in given by,

$$\boldsymbol{\Delta} = D_{\boldsymbol{\theta}} D(\boldsymbol{a}), \tag{3.5}$$

where $D_{\boldsymbol{\theta}} = [\boldsymbol{d}_{1\theta} \dots \boldsymbol{d}_{n\theta}]$ and $D(\boldsymbol{a}) = \mathrm{diag}(a_1, ..., a_n)$, with $a_i = -\frac{1}{2}\frac{\nu(\nu+2)}{(\nu+d_i(\theta))^2}$, $\boldsymbol{d}_{i\theta} = (d_{i\alpha}, d_{i\beta}, d_{i\phi}, d_{i\mu_x}, d_{i\phi_x})^\top$, $d_{i\gamma} = \dfrac{\partial d_i(\boldsymbol{\theta})}{\partial \gamma}$ as in the Appendix $i = 1, ..., n$, evaluated at $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}$.

Note that for the normal model ($\nu \longrightarrow \infty$), $\boldsymbol{\Delta} = (-1/2)D_{\boldsymbol{\theta}}$.

4 Applications

4.1 Serum kanamycin data

To illustrate the methodology described in this paper we consider first the data set reported by Kelly (1984). The data set consists of paired measurements of serum kanamycin levels in blood samples drawn from 20 babies. The measurements were obtained by two distinct methods. Diagnostics based on the influence fuction (Kelly, 1984) detected babies 2 and 16 as influential in the estimation of $(\alpha, \beta)^\top$.

According to estudies reported in Bolfarine and Arellano-Valle (1994), a Student-t model with ν=10 degress freedom seems to provide the best fit. Further, $\nu = 100$ seems to provide a good approximation for the normal model. Figure 1 considers local influence of the observations for $\nu = 1$, $\nu = 10$ and $\nu = 100$ degrees of freedom. Observations 2 and 17 seems to moderatily influence the estimation of $\boldsymbol{\theta}$ in the normal model. In the Cauchy case ($\nu = 1$) and in the Student-t model with low degrees of freedom, this influence seems to be substantially reduced. Local influence in the normal model is investigated in Kim (2000) with similar results as the ones reported above. Further, in the normal model, homocedasticity seems to be plausible since variance pertubation yields $|\boldsymbol{l}_{\max}|$=$(0.70681, 0.70740)^\top$. This seems also to be case in the Student-t model with low degrees of freedom, as also considered by Kelly (1984), Kim (2000) and Bolfarine and Arellano-Valle (1994).

4.2 Concrete data

The concrete data was studied in Wellman and Gunst (1991). The data set contains comprehensive strengh measurements of 41 samples of concrete. It was desired to use a linear regression model to predict the strength of concrete 28 days after pourning from the strength measurements taken two days after pouring. Wellman and Gunst (1991) consider a normal linear measurement error model. In this application we consider a Student-t model with varying degrees of freedom parameter ν.

The local influence of observations is investigated in Figure 2. As Figure 2 indicate, no influencial observations arise with low degrees of freedom that is, the Cauchy and Student-t model with low degrees of freedom are quite able to incorporate well the possible outlying observations in the data. However, according to Figure 2 as we move in the direction of the normal model, when $\nu = 100$ or higher, the model is quite influenced by observation 21, which, is this case, certainly is the most influential. The local influence resulting from other perturbation shemes were also investigated and since they give similar results to Figure 2 they are not presented. Similar results were also obtained for individual influence using the quantity $C_i, \quad i = 1, \ldots, n$.

Figure 1: Index plots, for the kanamycin levels with, $\nu = 1, 10, 100$ degrees of freedom

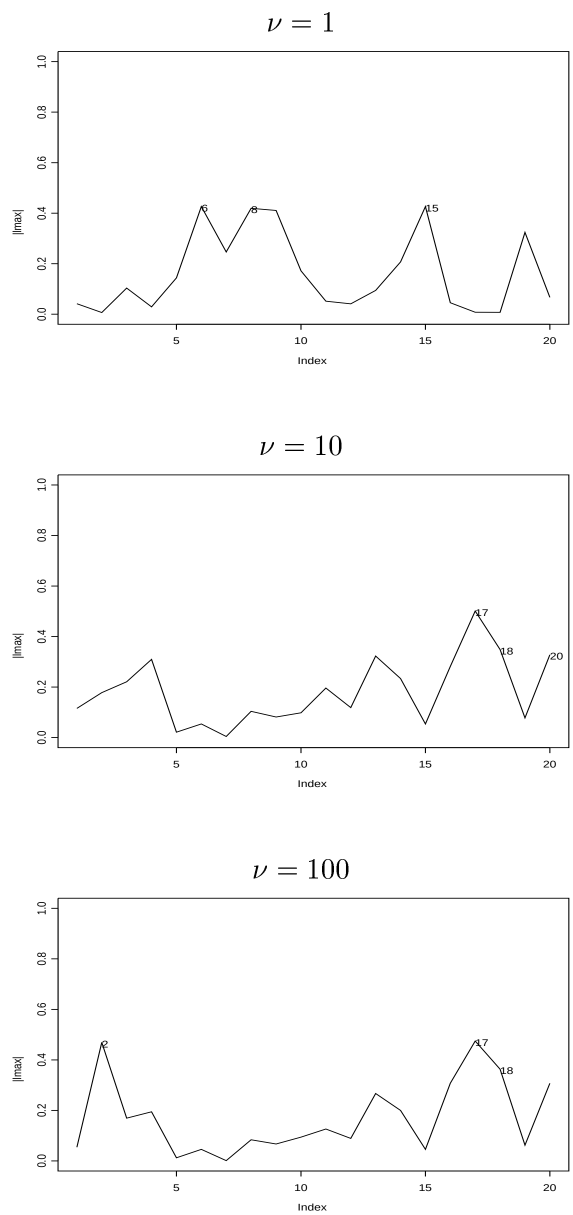
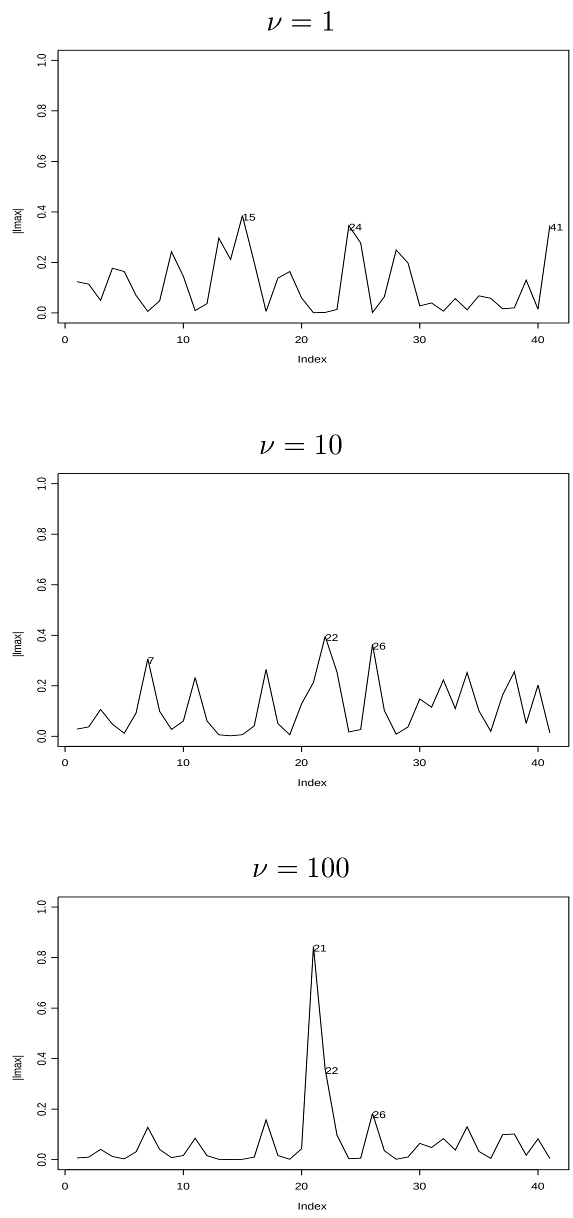


Figure 2: Index plots, for the Concrete data with, $\nu = 1, 10, 100$ degrees of freedom



Appendix: Computing the observed information matrix in the Student-t structural model

In this appendix we present the elements of the observed information matrix. From (3.1), it follows that

$$\frac{\partial l_i(\boldsymbol{\theta})}{\partial \gamma} = -\frac{1}{2}\frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \gamma} - \frac{1}{2}\frac{\nu+2}{(\nu+d_i(\boldsymbol{\theta}))} d_{i\gamma}, \tag{A.1}$$

with $d_{i\gamma} = \dfrac{\partial d_i(\boldsymbol{\theta})}{\partial \gamma}$, $\gamma = \alpha, \beta, \phi, \mu_x$ and ϕ_x, $d_i(\boldsymbol{\theta})$ as in (2.2), $i = 1, ..., n$. We have that

$$\begin{aligned} \frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \beta} &= 2\beta a, \quad \frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \phi} = \frac{1}{\phi} + \frac{a}{\phi_x}, \\ \frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \phi_x} &= \frac{ac}{\phi_x}, \quad \frac{\partial \log|\boldsymbol{\Sigma}|}{\partial \gamma} = 0, \quad \gamma = \alpha, \mu_x, \end{aligned} \tag{A.2}$$

$$d_{i\alpha} = \frac{2}{\phi}\{a\beta q_{2i} - (Y_i - \alpha - \beta\mu_x)\} \tag{A.3}$$

$$d_{i\beta} = \frac{2}{\phi}\{a^2\beta q_{2i}^2 - a q_{2i}(Y_i - \alpha - 2\beta\mu_x) - \mu_x(Y_i - \alpha - \beta\mu_x)\} \tag{A.4}$$

$$d_{i\phi} = \frac{a}{\phi} q_{2i}^2 \left(\frac{1}{\phi} + \frac{a}{\phi_x}\right) - \frac{q_{1i}}{\phi^2} \tag{A.5}$$

$$d_{i\mu_x} = \frac{2}{\phi} q_{2i}(ac - 1) \tag{A.6}$$

$$d_{i\phi_x} = \frac{a q_{2i}^2}{\phi\phi_x}(ac - 1), \tag{A.7}$$

where $c = 1 + \beta^2$, $a = \phi_x/(\phi + c\phi_x)$, $q_{1i} = (Y_i - \alpha - \beta\mu_x)^2 + (X_i - \mu_x)^2$ and $q_{2i} = \beta(Y_i - \alpha - \beta\mu_x) + (X_i - \mu_x)$, $i = 1, ..., n$.
¿From (A.1) it follows that the per element observed information matrix is given by

$$I_i = -\left[\left(\frac{\partial^2 l_i}{\partial \gamma \partial \tau}\right)\right], \tag{A.8}$$

where

$$\frac{\partial^2 l_i}{\partial\gamma\partial\tau} \quad = \quad -\frac{1}{2}\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\gamma\partial\tau} - \frac{1}{2}\frac{\nu+2}{\nu+d_i(\boldsymbol{\theta})}\left\{d_{i\gamma\tau} - \frac{d_{i\gamma}d_{i\tau}}{\nu+d_i(\boldsymbol{\theta})}\right\}$$

with $d_{i\gamma}$, $\gamma = \alpha, \beta, \phi, \mu_x, \phi_x$ as in (A.3)-(A.7) and $d_{i\gamma\tau} = \dfrac{\partial^2(d_i)}{\partial\gamma\partial\tau}$ $\gamma, \tau = \alpha, \beta, \phi, \mu_x, \phi_x$, where

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\alpha\partial\gamma} = \frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\mu_x\partial\gamma} = 0, \quad \gamma = \alpha, \beta, \phi, \mu_x, \phi_x, \tag{A.9}$$

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\beta\partial\beta} = 2a(1 - 2a\beta^2), \tag{A.10}$$

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\beta\partial\phi} = -2\beta a^2/\phi_x, \tag{A.11}$$

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\beta\partial\phi_x} = 2\frac{a\beta}{\phi_x}(1 - ac), \tag{A.12}$$

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\phi\partial\phi} = -(\frac{1}{\phi^2} + \frac{a^2}{\phi_x^2}), \tag{A.13}$$

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\phi\partial\phi_x} = -\frac{a^2 c}{\phi_x^2}, \tag{A.14}$$

$$\frac{\partial^2 \log|\mathbf{\Sigma}|}{\partial\phi_x\partial\phi_x} = -(\frac{ac}{\phi_x})^2, \tag{A.15}$$

$$d_{i\alpha\alpha} \quad = \quad \frac{2}{\phi}(1 - a\beta^2), \tag{A.16}$$

$$d_{i\alpha\beta} \quad = \quad \frac{2}{\phi}\{a\beta(Y_i - \alpha - 2\beta\mu_x) + aq_{2i}(1 - 2a\beta^2) + \mu_x\}, \tag{A.17}$$

$$d_{i\alpha\phi} \quad = \quad \frac{2}{\phi}\{\frac{1}{\phi}(Y_i - \alpha - \beta\mu_x) - a\beta q_{2i}(\frac{1}{\phi} + \frac{a}{\phi_x})\}, \tag{A.18}$$

$$d_{i\alpha\mu_x} \quad = \quad \frac{2\beta}{\phi}(1 - ac), \tag{A.19}$$

$$
\begin{aligned}
d_{i\alpha\phi_x} &= \frac{2a\beta}{\phi\phi_x} q_{2i}(1-ac), && \text{(A.20)}\\
d_{i\beta\beta} &= \frac{2}{\phi}\{\mu_x^2 + 2a\mu_x q_{2i} - a(Y_i - \alpha - 2\beta\mu_x)^2 + 4a^2\beta q_{2i} && \text{(A.21)}\\
&\quad (Y_i - \alpha - 2\beta\mu_x) + a^2(1-4a\beta^2)q_{2i}^2\},\\
d_{i\beta\phi} &= \frac{2}{\phi}\{aq_{2i}(Y_i - \alpha - 2\beta\mu_x)(\frac{1}{\phi} + \frac{a}{\phi_x}) - a^2\beta q_{2i}^2(\frac{1}{\phi} + \frac{2a}{\phi_x}) && \text{(A.22)}\\
&\quad + \frac{\mu_x}{\phi}(Y_i - \alpha - \beta\mu_x)\},\\
d_{i\beta\mu_x} &= \frac{2(1-ac)}{\phi}\{2a\beta q_{2i} - (Y_i - \alpha - 2\beta\mu_x)\}, && \text{(A.23)}\\
d_{i\beta\phi_x} &= \frac{2a(1-ac)}{\phi\phi_x}\{2a\beta q_{2i}^2 - q_{2i}(Y_i - \alpha - 2\beta\mu_x)\}, && \text{(A.24)}\\
d_{i\phi\phi} &= \frac{2}{\phi^2} d_i(\boldsymbol{\theta}) - \frac{2a^2}{\phi\phi_x} q_{2i}^2(\frac{1}{\phi} + \frac{a}{\phi_x}), && \text{(A.25)}\\
d_{i\phi\mu_x} &= \frac{2}{\phi^2}\{q_{2i} - ac(1 + \frac{a\phi}{\phi_x})q_{2i}\}, && \text{(A.26)}\\
d_{i\phi\phi_x} &= \frac{a}{\phi\phi_x}(\frac{1-ac}{\phi} + \frac{a - 2a^2c)}{\phi_x})q_{2i}^2, && \text{(A.27)}\\
d_{i\mu_x\mu_x} &= 2c(1-ac)/\phi, && \text{(A.28)}\\
d_{i\mu_x\phi_x} &= \frac{2ac(1-ac)}{\phi\phi_x} q_{2i}, && \text{(A.29)}\\
d_{i\phi_x\phi_x} &= \frac{2a^2c(1-ac)}{\phi\phi_x^2} q_{2i}^2, && \text{(A.30)}
\end{aligned}
$$

$i = 1, ..., n$. Thus, the complete observed information matrix is $I_{ob}(\boldsymbol{\theta}/\boldsymbol{Y}) = \sum_{i=1}^{n} I_i(\boldsymbol{\theta}/\boldsymbol{Y}_i)$. Evaluating the observed information matrix at $\widehat{\boldsymbol{\theta}}$ it follows that $I_{ob}(\widehat{\boldsymbol{\theta}}/\boldsymbol{Y}) = -\boldsymbol{I}$ given in (3.4).

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