

Calculating Capillary Pressure Curve from Single-Speed Centrifuge Experiments

Cristina Cunha
Antonio Carlos Moretti
Instituto de Matemática, Estatística e Ciência da Computação
Universidade Estadual de Campinas
Caixa Postal 6065
13081-970 - Campinas - SP - Brazil

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Abstract

We present a method for the calculation of capillary pressure curve in porous media using centrifuge single-speed experimental data. The mathematical model is based on the hydrostatic equilibrium equation for capillary pressure and the inverse problem for finding the saturation distribution along the sample from the displaced fluid volume when all the flow ceased.

The capillary effects are interpreted as constraint forces which maintain the oil trapped into the pores of the sample. We use the Lagrangian formulation to find the equilibrium configuration of the oil in the sample. The great advantage of this formulation is that the explicit inclusion of forces is not necessary. Mathematical properties of capillary pressure curve and centrifuge data are used as constraints of the kinetic energy minimization procedure. The discretized problem is solved using a nonlinear programming procedure using only the single-speed experimental data. Numerical results are confronted with calculations in the frequently used multi-speed centrifuge tests.

1 Introduction

A knowledge of how the capillary pressure is related to saturation in a porous medium, the capillary pressure curve, is useful to characterize pore structure of the medium and to predict of how one phase displaces the other. The centrifuge method for measuring capillary pressure in porous medium was proposed by Hassler and Brunner [3] in 1945. In the centrifuge method a cylindrical sample initially filled with a liquid spins about an axis of rotation. The sample is mounted so that the axis of the cylinder is orthogonal to the axis of rotation. This method entails increasing the centrifuge speed in steps and measuring, in each step, the amount of displaced phase produced from the rock when all flow ceased. The requirement that all flow cease is crucial. Much literature is available on techniques for deriving capillary pressure from centrifuge data (see for instance, Ayappa et al [1]).

In this work we will assume that the gas phase displaces the oil phase in the experiments, the water is an immobile phase and the fluids are incompressible. Determination of the displaced fluid saturation, the relative volume of this phase, plays the central role in these methods.

The basic equations in the calculations use the stabilized fluid produced and the hydrostatic equilibrium capillary pressure distribution. For each centrifuge speed, the fluid produced when the flow ceased is related to the oil saturation, $s(x)$, in a point x distant from the core inflow end, by the integral equation:

$$\text{prod}(w) = \phi A \int_0^L (1 - s_{w_i} - s(x)) dx \quad (1)$$

where A is the sample cross section area, L is the core length, ϕ is the medium porosity and s_{w_i} is the residual water saturation. All these values are assumed to be known.

The hydrostatic equilibrium equation, see Lake [5] for example, is obtained by combining the Darcy's laws (gas and oil phases) in equilibrium (in which gas and oil velocities vanish), and using capillary pressure definition (the difference between gas and oil pressures). In this way, the differential equation for the capillary pressure is,

$$\frac{dp_c}{dx} + \Delta\rho\omega^2 R\alpha(x) = 0 \quad (2)$$

where R is the distance from axis of rotation to middle of the L long core sample, $\Delta\rho$ is the oil and gas densities difference and the $\alpha(x)$ function accounts for the variation of the centrifugal force along the core:

$$\alpha(x) = 1 - \frac{L}{2R} + \frac{x}{R}. \quad (3)$$

Drainage mode capillary pressure exhibits an entry pressure, p_e , a pressure which must be exceeded before any gas phase can enter the core. In this case capillary pressure is not a unique function of saturation insofar as it can have values between zero and the entry pressure when the rock is fully saturated with the oil phase. The implication of this for a centrifuge experiment is that the invading phase (gas) never penetrates into the rock any further than the point at which the capillary pressure equals the entry pressure. From this point, which we denote as he , to the outflow end the oil phase saturation is $1 - s_{w_i}$. By integrating the hydrostatic equilibrium equation, from an arbitrary point x to he , and using the boundary condition $p_c(he) = p_e$, we obtain the expression for the capillary pressure distribution along the core:

$$p_c(x) = p_e + \Delta\rho\omega^2 Rz(x) \quad (4)$$

where

$$z(x) = (he - x)\left(1 - \frac{L}{2R} + \frac{he + x}{2R}\right) \quad (5)$$

Equations (1) and (4) will be used to find the capillary pressure curve, $p_c(s)$. In fact, given the stabilized fluid produced in a single centrifuge speed we have an integral equation of

the first kind, which must be inverted to find $s(x)$. On the other hand, the capillary pressure distribution along the sample is given in Equation (4). Both $s(x)$ and $p_c(x)$ are one-to-one functions, therefore, they can be combined to find the capillary pressure curve $p_c(s)$. The usual method used to invert the integral equation (1) requires a discretization procedure in which multi-speed centrifuge production data are used (see for instance, [1] for details). A discretization procedure is needed because we do not know how to solve the inverse problem described by Equation (1), i.e., how to find the saturation distribution from the stabilized production. If it were possible to find this saturation distribution, these values of $s(x)$ could be coupled with the values of $p_c(x)$ calculated by Equation (4), to define the capillary pressure curve.

We propose here a variational formulation for this problem. We show that this formulation yields a new procedure to solve the inverse problem defined by Equation (1). Using this formulation and a discrete approximation scheme we can calculate the saturation profile along the sample. We compute the capillary pressure curve with this saturation. Our computational results matched with multi-speed centrifuge tests.

2 Variational Formulation

2.1 The Lagrangian Functional

The asymptotic production in centrifuge tests is determined only by the capillary forces. Therefore, it is natural to interpret the capillary forces as constraint forces which maintain the oil trapped in the core sample pores. This interpretation is crucial to our arguments about the advantages of the variational formulation of the problem.

In order to circumvent the impossibility of obtaining explicit expressions for the pores level of constraint forces we take minimal principles in our approach. The search of minimal principles has a long history and is based on the notion that nature acts in such a way that important quantities are minimized when a physical process takes place. The mathematical foundation of these principles was given by Lagrange in 1760. Since then the variational methods and extremum principles have been extensively used to solve problems in physics and mathematics, see Segel [6], for example.

Most applications of the variational statement of the Hamilton's Principle in dynamics require that the Lagrangian functional be minimum at the stable configuration. The scalar Lagrangian functional is the difference between the kinetic and potential energies of the system, expressed in convenient generalized coordinates.

In centrifuge experiments, the kinetic energy predominates, in thousands order of magnitude, over the potential energy. In this case the Lagrangian may be assumed equal to the total kinetic energy of the oil left on the core.

We will choose the saturation distribution into the sample to calculate the kinetic energy of the oil trapped into the pore of the core sample. The saturation at an inner core point, $s(x)$, measures the relative oil volume in the strip volume $A\phi\Delta x$, where A is the sample cross section area, ϕ is the porosity of the medium and Δx is the height of this elemental volume.

To calculate the kinetic energy we observe that the mass into the elemental volume is given by

$$\Delta m(x) = \rho A \phi s(x) \Delta x \quad (6)$$

where ρ is the oil density. Using the angular speed, ω , we calculate the velocity of a generic point x by

$$v(x) = \omega R \alpha(x).$$

Therefore, the kinetic energy of the oil into this elemental volume is given by

$$\Delta \tilde{T}(x) = \frac{1}{2} \rho A \phi \omega^2 R^2 s(x) \alpha^2(x) \Delta x. \quad (7)$$

As we said before, from $x = he$ to the outflow end, the core is saturated with oil and the residual water. Since the value of he is uniquely determined by the centrifuge speed, as shown by O'Meara and Crump [4], the oil saturation varies from $x = 0$ to $x = he$. Therefore, the total kinetic energy of the oil trapped into this region of the core sample is the integral of Equation (7):

$$\tilde{T}(s) = \frac{1}{2} \rho A \phi \omega^2 R^2 \int_0^{he} \alpha^2(x) s(x) dx. \quad (8)$$

To simplify we use the dimensionless variable $x' = \frac{x}{L}$, denoted by x from now on. As shown by O'Meara and Crump [4],

$$he = \frac{1}{2} - \frac{R}{L} + \sqrt{\left(\frac{R}{L} - \frac{1}{2}\right)^2 + \frac{2R}{L}(1-N)} \quad (9)$$

where $N = \frac{p_e}{\omega^2 R \Delta \rho}$.

Since the constant values that multiply the integral (8) do not modify the minimum, the functional to be minimized is

$$T(s) = \int_0^{he} \alpha^2(x) s(x) dx. \quad (10)$$

2.2 The constraints

Since the saturated region begins at $x = he$ the boundary condition for the saturation distribution is :

$$s(he) = 1 - s_{w_i}. \quad (11)$$

We now describe two constraints arising from mathematical properties of $p_c(s)$. Analytical expressions for drainage capillary pressure curves are based on the assumption that the capillary pressure at any cross section within the porous medium is uniquely determined by the saturation at that cross section, and that the porous medium is homogeneous and isotropic. Moreover, these expressions are formulated to meet mathematical restrictions

imposed by the physics of the fluid displacement in a porous medium, Bentsen and Anli [2]. From the mathematical point of view the assumptions are that the capillary pressure curve is a monotonic decreasing convex function of the saturation. Therefore, the first derivative of this function is negative and the second is positive, i.e.,

$$p'_c(s) < 0 \quad \text{and} \quad p''_c(s) > 0. \quad (12)$$

Since saturation itself is a function of the position, $s(x)$, we use the chain rule to compute the first and second derivatives of capillary pressure, as function of the position, which are, respectively:

$$\frac{dp_c(s(x))}{dx} = p'_c(s)s'(x) \quad (13)$$

$$\frac{d^2p_c(s(x))}{dx^2} = p''_c(s)s'(x)^2 + p'_c(s)s''(x). \quad (14)$$

Using Equation (4) we have, in dimensionless x :

$$\frac{dp_c}{dx} = -\Delta\rho\omega^2 RL\alpha(x) < 0. \quad (15)$$

By the derivation of Equation (4), we also have

$$\frac{d^2p_c}{dx^2} = -\Delta\rho\omega^2 L^2 < 0. \quad (16)$$

Since $p'_c(s) < 0$, combining Equations (13) and (15) we have

$$s'(x) > 0. \quad (17)$$

On the other hand, using Equations (14) and (16) we have

$$-\Delta\rho\omega^2 L^2 = p''_c(s)(s'(x))^2 + p'_c(s)s''(x) < 0. \quad (18)$$

Now, we can use Equation (12) to obtain

$$s''(x) > 0. \quad (19)$$

Finally, we observe that the stabilized production, described by Equation (1), is also a constraint since it means that the area under all feasible curves $s(x)$ must be the same, a known value calculated from stabilized production, i.e.,

$$\int_0^{he} s(x)dx = C, \quad (20)$$

for a known C .

We may summarize our approach in the following way. To overcome the difficulty of solving the integral (1) we present a new approach for the capillary curve calculation. The idea is based on Hamilton's Principle which uses the kinetic energy of the remaining oil in a single

speed centrifuge test. We also describe the constraints used in the minimization procedure. The porous and fluids interface interaction is strongly used in the constraints (12), (17) and (19). The stabilized production is used to obtain the constraint (20). Therefore, we want to determine a function $s(x), 0 < x < he$, that minimizes

$$T(s) = \int_0^{he} \alpha^2(x)s(x)dx \quad (21)$$

among all sufficiently smooth functions that satisfy the boundary condition,

$$s(he) = 1 - s_{w_i} \quad (22)$$

and the constraints

$$s'(x) > 0 \quad (23)$$

$$s''(x) > 0 \quad (24)$$

$$\int_0^{he} s(x)dx = C \quad (25)$$

where C is a known constant and he is given by (8).

As we emphasized in the beginning of this work, the saturation values $s(x)$ are coupled with the values of $p_c(x)$, which are computed using Equation (4), to calculate $p_c(s)$, the capillary pressure curve.

3 Discrete Model and the Computational Scheme

The discrete approximation scheme for solving the problem replaces the derivatives and integrals, in Equations (21) through Equation (25), by appropriate difference quotient approximations and numerical quadratures.

Before we explain how the approximation scheme was done, we want to say that the problem described by the objective function (21) and the constraints (22) through (25) represents the situation we want to model, but, it does not answer all the questions since we observed that there are an infinitely many curves that satisfy the constraints but the final fitting of the capillary pressure obtained by our model with the capillary pressure obtained by laboratory experiments were not good enough. Therefore, there is a missing information in the process and to overcome this situation we forced the capillary pressure to have a specific shape. To do that, we regularize the objective function to minimize the kinetic energy and a penalization of the quadratic error of the obtained capillary pressure and a model function we believe the capillary pressure must be (we based this information from the experimental data).

We want to emphasize here that we do not use the experimental data obtained from the multi-speed experiments, but, we kept our previous approach as a single-experiment and at each iteration of our algorithm once we have a value for $s(x)$, we compute $p_c(s(x))$ by the formula (4) and an auxiliary capillary pressure, $\bar{p}_c(s(x))$, given by, for instance,

$\bar{p}_c(s(x)) = \frac{1}{as(x)+b}$. Then, the quadratic error is given by $\sum_{i=1}^n (p_c(s(x)) - \bar{p}_c(s(x)))^2$. Using this approach we obtain as final result a curve which is convex, monotone decreasing and fits well the experimental data.

The approximation scheme is defined as follows. First we select an integer $N > 0$ and divide the interval $[0, he]$ into $2N$ equal subintervals, whose end points are $x_i = ih$, for $i = 0, 1, \dots, 2N$, with $h = \frac{1}{2} \frac{he}{N}$. Choosing constant values for h make the calculations easier, but we could also use different values h_i .

In our calculations we used the composite Simpsons rule to compute the integrals, and finite difference quotients to approximate the derivatives.

Denoting by s_i the approximation of $s(x_i)$ and by α_i the approximation of $\alpha(x_i)$ where $\alpha(x)$ is given by Equation (3) we can write the discrete formulation of the problem.

$$\text{Minimize } J(s) = \frac{h}{3} \left\{ \alpha_0^2 s_0 + 2 \sum_{i=1}^{N-1} \alpha_{2i}^2 s_{2i} + 4 \sum_{i=1}^N \alpha_{2i-1}^2 s_{2i-1} + \alpha_{2N}^2 s_{2N} \right\} + \sum_{i=1}^{2N} (p_c(s_i) - \bar{p}_c(s_i))^2 \quad (26)$$

with the constraints

$$s_{2N} = 1 - s_{w_i} \quad (27)$$

$$s_{i+1} - s_i \geq 0 \quad (28)$$

$$s_{i+1} - 2s_i + s_{i-1} \geq 0 \quad (29)$$

$$\frac{h}{3} \left\{ s_0 + 2 \sum_{i=1}^{N-1} s_{2i} + 4 \sum_{i=1}^N s_{2i-1} \right\} = C - \frac{h}{3} (1 - s_{w_i}) \quad (30)$$

We solve this nonlinear programming problem using the solver “fmincon” from the Optimization Toolbox of Matlab version 5.3.1 in a SUN Ultra 1 with 256 MegaBytes of RAM and 200 Mhz of clock. We results obtained were pretty good and they are presented in next section.

4 Computational Results

Numerical experiments were performed in two samples. The multi-speed centrifuge production data for each speed were known, therefore, we could test the procedure using two different speed production in both samples. Although, the rock used is the same for all samples, the fluids are different.

It was observed that in a low speed testing, the first value of the saturation profile, s_0 , may be high. As a consequence the capillary curve may begin in high values of saturation. We present here the results of two experiments: Samples 1 and 2 with $\omega = 1000\text{rpm}$ and $\omega = 1250\text{rpm}$. Fluid and rock properties are presented in Table (1).

Numerical results from Sample 1 calculations are in Figures (1) and (2). The final saturation distribution using both speeds obtained by Equations (27) through (30) are plotted

in Figures (1b) and (2b). The curve presents a sharp derivative near the saturated region, where the saturation is $s(h_e) = 1 - s_{w_i}$.

Capillary pressure values are calculated with these values of $s(x)$ and (4). The results are presented as a solid line in Figures (1a) and (2a). In the figures we also present the multi-speed test result. In the multi-speed test the calculation follows Skuse [7]. The Figures (1a) (2a) show that our calculations are in accordance with the available experimental results.

The same convention is used to present the Sample 2 results. We guess that the difference between experimental results and our calculation is due to the fact that few speeds were used in this experiment. As observed before, the multi-speed calculations are based on the discretization of an integral equation and therefore, few speeds in the tests may cause poor final results.

5 Conclusions

We presented a variational formulation and a numerical procedure to calculate capillary curves in porous medium from a single-speed centrifuge experiments.

The integral to be minimized in this formulation is the kinetic energy of the oil left in the sample after all flow ceased in an ω -speed test.

The capillary effects enter as restrictions to be satisfied by the feasible functions. The stabilized production of the displaced fluid is also used as a restriction in the formulation. As far as we know, the interpretation of capillary effects as constraints forces is new. Although the idea is simple, it allows a formulation in which the great advantage is that the explicit inclusion of forces of constraint is not necessary.

Linear programming techniques were used in the effective calculation of the final distribution of the displaced phase saturation.

Our calculations are in accordance with the results obtained in the frequently used multi-speed centrifuge tests.

We believe that this new insight of the problem may be useful in new methods for calculating the capillary pressure curve from single speed centrifuge experiments.

References

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Captions for Figures 1 and 2

Figure 1 : Sample 1 results: (a) Capillary Pressure curve obtained (full curve) and from a multi-speed centrifuge experiment ('*'). (b) The saturation distribution, $s(x)$, that minimizes the kinetic energy.

Figure 2 : Sample 1 results: (a) Capillary Pressure curve obtained (full curve) and from a multi-speed centrifuge experiment ('*'). (b) The saturation distribution, $s(x)$, that minimizes the kinetic energy.

Figure 3 : Sample 2 results: (a) Capillary Pressure curve obtained (full curve) and from a multi-speed centrifuge experiment ('*'). (b) The saturation distribution, $s(x)$, that minimizes the kinetic energy.

Figure 4 : Sample 2 results: (a) Capillary Pressure curve obtained (full curve) and from a multi-speed centrifuge experiment ('*'). (b) The saturation distribution, $s(x)$, that minimizes the kinetic energy.

Figures 1 and 2

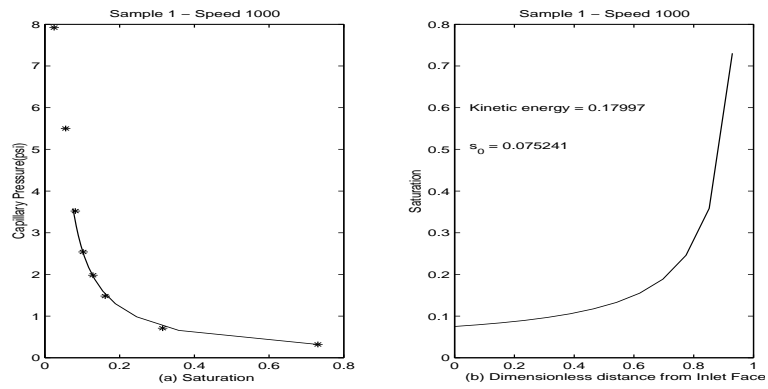


Figure 1: Sample 1 - Speed 1000 rpm

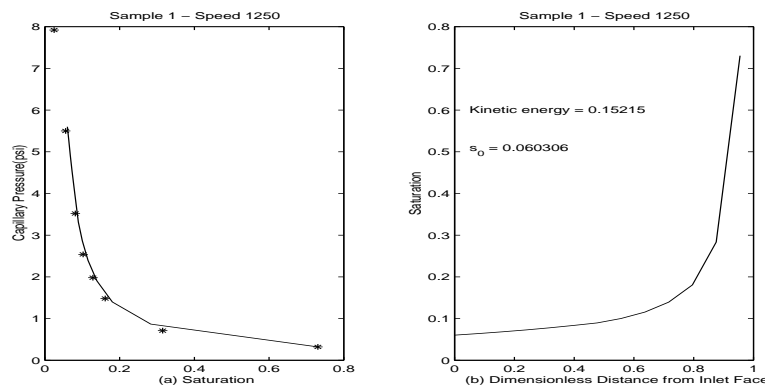


Figure 2: Sample 1 - Speed 1250 rpm

Figures 3 and 4

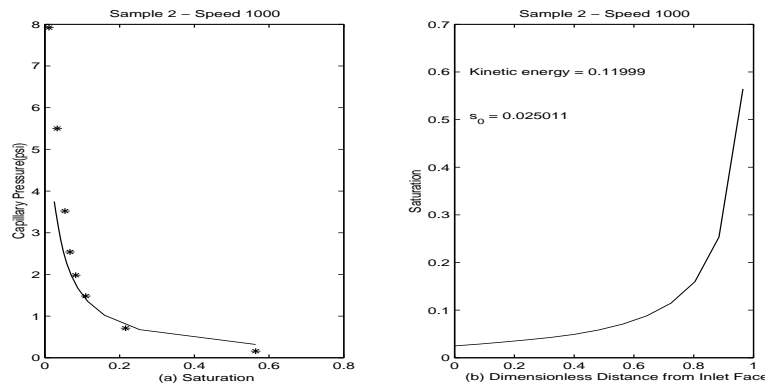


Figure 3: Sample 2 - Speed 1000 rpm

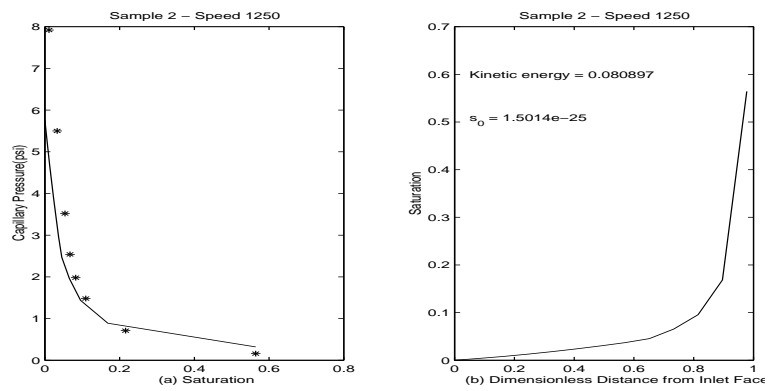


Figure 4: Sample 2 - Speed 1250 rpm

Caption for Table 1

Table 1 : Data for Samples 1 and 2 with different speeds

Table 1

Table 1: Data for Samples 1 and 2

	Speed	Length	Porous	Porosity	he	R	$\Delta\rho$	Production	s_{w_i}
	ω (rpm)	L (cm)	A (cc)	ϕ		(cm)	(gr/cm ³)	(cc)	
Sample 1	1000	4.07	11.31	0.259	0.9290	7.35	0.7534	5.945	0.27
Sample 1	1250	4.07	11.31	0.259	0.9548	7.35	0.7534	6.470	0.27
Sample 2	1000	3.96	10.92	0.251	0.9648	7.40	0.873	6.560	0.36
Sample 2	1250	3.96	10.92	0.251	0.9776	7.40	0.873	7.190	0.36