A new combinatorial interpretation for the Rogers-Ramanujan identities.

Paulo Mondek

and

José Plínio O. Santos

Abstract

In this paper we present a combinatorial interpretation for the Rogers-Ramanujan identities that are distinct from the ones already known that are given, for instance, in MacMahon [5] and Andrews [3].

1 Introduction

In this paper we use the standard notation

$$\begin{aligned}
(a;q)_0 &= 1\\
(a;q)_n &= (1-a)(1-aq)\cdots(1-aq^{n-1})\\
(a;q)_\infty &= \lim_{n\to\infty}(a;q)_n,\ \ |q|<1.
\end{aligned}$$

In order to get the new combinatorial interpretation for equations (1.1) and (1.2) we are going to describe how to build sets from certain known identities.

Let be

$$\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n}=\prod_{n=1}^{\infty}\frac{1}{(1-q^{5n-1})(1-q^{5n-4})} \tag{1.1}$$

$$\sum_{n=0}^{\infty}\frac{q^{n^2+n}}{(q;q)_n}=\prod_{n=1}^{\infty}\frac{1}{(1-q^{5n-2})(1-q^{5n-3})} \tag{1.2}$$

We observe that this method can be applied to others identities of the Rogers-Ramanujan type.

We consider the identities (16) and (20) of Slater [8].

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{q^{n^2+2n}}{(q^4;q^4)_n} &= \frac{(q;q^5)_\infty(q^4;q^5)_\infty(q^5;q^5)_\infty(-q;q^2)_\infty}{(q^2;q^2)_\infty}\\ &= \frac{(q;q^5)_\infty(q^4;q^5)_\infty(q^5;q^5)_\infty}{(q;q)_\infty}.\frac{(-q;q^2)_\infty}{(-q;q)_\infty}\\ &= \frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty}.\frac{1}{(-q^2;q^2)_\infty} \qquad (1.3)\end{aligned}$$

and

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q^4;q^4)_n} &= \frac{(q^2;q^5)_\infty(q^3;q^5)_\infty(q^5;q^5)_\infty(-q;q^2)_\infty}{(q^2;q^2)_\infty}\\ &= \frac{(q^2;q^5)_\infty(q^3;q^5)_\infty(q^5;q^5)_\infty}{(q;q)_\infty}.\frac{(-q;q^2)_\infty}{(-q;q)_\infty}\\ &= \frac{1}{(q;q^5)_\infty(q^4;q^5)_\infty}.\frac{1}{(-q^2;q^2)_\infty} \qquad (1.4)\end{aligned}$$

Hence we see that these two identities are related to the Rogers-Ramanujan identities by the equations:

$$(-q^2;q^2)_\infty\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q^4;q^4)_n} = \sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n} \qquad (1.5)$$

$$(-q^2;q^2)_\infty\sum_{n=0}^{\infty}\frac{q^{n^2+2n}}{(q^4;q^4)_n} = \sum_{n=0}^{\infty}\frac{q^{n^2+n}}{(q;q)_n} \qquad (1.6)$$

We define, associated with identities (1.3) and (1.4), the following two functions:

$$F_i(z) := \sum_{n=0}^{\infty}\frac{z^nq^{n^2+(4-2i)n}}{(q^4;q^4)_n}, i = 1,2. \qquad (1.7)$$

It is easy to verify that the functions given in (1.7) satisfy:

$$\begin{aligned}F_1(z)-F_1(zq^4) &= \sum_{n=0}^{\infty}\frac{z^nq^{n^2+2n}}{(q^4;q^4)_n}-\sum_{n=0}^{\infty}\frac{(zq^4)^nq^{n^2+2n}}{(q^4;q^4)_n}\\ &= \sum_{n=0}^{\infty}\frac{z^nq^{n^2+2n}}{(q^4;q^4)_n}(1-q^{4n}) = \sum_{n=1}^{\infty}\frac{z^nq^{n^2+2n}}{(q^4;q^4)_n}(1-q^{4n})\\ &= \sum_{n=1}^{\infty}\frac{z^nq^{n^2+2n}}{(q^4;q^4)_{n-1}} = \sum_{n=0}^{\infty}\frac{z^{n+1}q^{n^2+4n+3}}{(q^4;q^4)_n}\\ &= zq^3\sum_{n=0}^{\infty}\frac{(zq^2)^nq^{n^2+2n}}{(q^4;q^4)_n}\end{aligned}$$

i.e.,

$$F_1(z) = zq^3F_1(zq^2) + F_1(zq^4) \tag{1.8}$$

and

$$F_1(z) = F_2(zq^2). \tag{1.9}$$

We are going to look at the expressions

$$\frac{z^nq^{n^2+(4-2i)n}}{(q^4;q^4)_n}$$

that are in (1.7) as the generating functions for partitions of m in n parts. The expansion of these expressions will give us terms of the type z^nq^m where the exponent of z is the number of parts where the parts verify certain "properties". In order to find these "properties" we describe, next, the construction of two sets, A and B, using (1.8) and (1.9). These sets are associated, respectively, to the functions $F_1(z)$ and $F_2(z)$. They are formed by the partitions that are determinated by the interpretation of (1.7) and the functional equations (1.8) and (1.9).

We start with three important observations:

Observation (1): equation (1.9) tell us that if we know one of the sets A and B the other one is automatically determinated, since that $F_1(z) = F_2(zq^2)$ indicate that the partitions in A can be obtained from the partitions in B by the addition of 2 to each part of the later.

Observation (2): equation (1.8) tell us that set A can be partitioned into two disjoint sets $A = A_1 \uplus A_2$ where in the partitions that are in A_1 there exist a part equal to 3 and the remaining parts can be obtained from the partitions of set A, itself, by adding 2 to each part and A_2 has the partitions that can be obtained by just adding 4 to each part of the partitions that are in A.

Observation (3): given an element $\pi = a_1 + \cdots + a_n \in A$, by the previous observation there are certain operations that we can performed in set A:
If $\pi \in A_2$ then we may add or subtract 4 from each part of π and the new element will be an element of A. If $\pi \in A_1$ then by adding 4 to each part of π the new element is still an element of A. Being, in this case, $\pi = a_1 + \cdots + a_{n-1} + 3$, a partition obtained by dropping the part 3 and subtracting 2 from each of the remaining parts, i.e.: $(a_1 - 2) + (a_2 - 2) + \cdots + (a_{n-1} - 2)$ is an element of A.

2 Construction of Sets

In what follows we present the construction of set A.

Let $\pi = a_1 + a_2 + \cdots + a_n$ be a partition in A. By Observation (1) we have that $a_j \geq 2$ for $1 \leq j \leq n$. Observation (2) implies that either $a_n = 3$ and $a_{n-1} \geq 4$, once $F_1(zq^2) = F_2(zq^4)$, or $a_n \geq 6$ since $F_1(zq^4) = F_2(zq^6)$. Now with these informations we have $a_j \geq 3$ for $1 \leq j \leq n$. Therefore if $\pi \in A_1$ then $a_n = 3$ and $a_{n-1} \geq 5$ or if $\pi \in A_2$ then $a_n \geq 7$. To finish the construction of A we need the following three lemmas:

Lemma 1. If $\pi = a_1 + a_2 + \cdots + a_n \in A$ then $a_j \equiv 1 (mod\ 2)$ for $1 \leq j \leq n$.

Proof. Lets assume that there exists a j_0 such that a_{j_0} is even. By Observation (3) after a finite number of subtractions of $4's$ from each part, we will have $j_0 = n$ or $a_n = 3$ and $j_0 = n - 1$.

In the first case a_n is even and after a finite number of subtractions of $4's$ from each part we would have that either $a_1' + a_2' + \cdots + 8$ or $a_1" + a_2" + \cdots + 10$ been an element of A and this is an absurd because, then, either $(a_1' - 4) + (a_2' - 4) + \cdots + 4$ or $(a_1" - 4) + (a_2" - 4) + \cdots + 6 \in A$ and these are not in $A_1 \uplus A_2$.

In the second case $\pi \in A_1$ and then $(a_1 - 2) + \cdots + (a_{n-1} - 2) \in A$ with $(a_{n-1} - 2)$ even.

Now we can use the same argument of the first case to finish the proof.

□

In our next lemma we have a caracterization of the congruence class, modulo 4, to which the smallest part of a partition in A belongs to.

Lemma 2. If $\pi = a_1 + a_2 + \cdots + a_n \in A$ then $a_n \equiv 3 (mod\ 4)$.

Proof. From Lemma 1 we know that the parts of a partition in A, been odd, are $\equiv 1$ or $3 (mod\ 4)$. Lets suppose $a_n \equiv 1 (mod\ 4)$.

By Observation (3) this element $a_1 + a_2 + \cdots + a_n \in A_2$, and so, after a finite number of subtractions of $4's$, from each part, we have that $a_1' + a_2' + \cdots + 9 \in A$ which is impossible since this would imply $(a_1' - 4) + (a_2' - 4) + \cdots + 5 \in A$ and, as we have seen, this is not an element of A.

□

In our next lemma we show that the consecutive parts of a partition in A are incongruent modulo 4.

Lemma 3. If $\pi = a_1 + a_2 + \cdots + a_n \in A$ then $a_j \not\equiv a_{j+1}(mod\ 4), 1 \leq j < n$.

Proof. Lets assume that there exists j such that $a_j \equiv a_{j+1}(mod 4)$. Let j_0 be the greatest subscript for which $a_{j_0} \equiv a_{j_0+1}(mod\ 4)$.

By Lemma 2 and Observation (3) we may assume, after a finite number of subtractions of $4's$ from each part, if necessary, that

$$\pi = a_1 + a_2 + \cdots + a_{j_0} + a_{j_0+1} + \cdots + a_{n-1} + 3.$$

If $j_0 = n-1$ then $a_{n-1} = 3 + 4k$ and by Observation (2) we have that $(a_1 - 2) + (a_2 - 2) + \cdots + (1 + 4k) \in A$ which is in contradiction with Lemma 2. If $j_0 < n-1$ then by Observation (3) $\pi' = (a_1 - 2) + \cdots + (a_{j_0} - 2) + (a_{j_0+1} - 2) + \cdots + (a_{n-1} - 2) \in A$. For this π' also we may assume, as we did for π, that

$$\pi' = a'_1 + a'_2 + \cdots + a'_{j_0} + a'_{j_0+1} + \cdots + 3.$$

Hence, by repeting the previous argument we may assume, after a finite number of subtractions of $2's$ or $4's$ from each part that

$$\pi' = a'_1 + a'_2 + \cdots + (3 + 4\ell) + 3,$$

i.e., $j_0 = n' - 1$ and, as we saw above, this implies in a contradiction which completes the proof.

$\square$

Since the properties obtained from these three lemmas determined the class, modulo 4, to which each part of a partition belongs to, the only restriction that we still have to verify is related to the difference between parts. But, from Observation (3) each partition of the form

$$(2n + 1 + 4k_1) + (2n - 1 + 4k_2) + \cdots + (5 + 4k_{n-1}) + (3 + 4k_n), \quad \text{where} \quad k_j \geq k_{j+1}$$

is an element of A independendly of the values of $k_{j's}$. Therefore the results from these lemmas define, univocally, the set A. This is stated in the next theorem.

Theorem 1. If a partition π of an integer $m, \pi = a_1 + a_2 + \cdots + a_n \in A$ then

$$\begin{aligned} &a_j \equiv 1(mod\ 2) \quad \text{for} \quad 1 \leq j \leq n \\ &a_n \equiv 3(mod\ 4) \quad \text{and} \\ &a_j \not\equiv a_{j+1}(mod\ 4) \quad \text{for} \quad 1 \leq j < n. \end{aligned} \tag{1.10}$$

By Observation (1) the elements of set B can be knew since, as we have seen, $a_1 + \cdots + a_s \in A$ if, and only if, $(a_1 - 2) + \cdots + (a_s - 2) \in B$. From this the following theorem is true.

Theorem 2. If a partition π of an integer $m, \pi = a_1 + a_2 + \cdots + a_n \in B$ then

$$\begin{aligned} &a_j \equiv 1 (mod\ 2) \quad \text{for} \quad 1 \leq j \leq n \\ &a_n \equiv 1 (mod\ 4) \quad \text{and} \\ &a_j \not\equiv a_{j+1} (mod\ 4) \quad \text{for} \quad 1 \leq j < n. \end{aligned} \tag{1.11}$$

We list below all the partitions of $m = 35$ and $m = 40$ that are in A.

35	$37 + 3$
$27 + 5 + 3$	$33 + 7$
$23 + 9 + 3$	$29 + 11$
$19 + 13 + 3$	$25 + 15$
$19 + 9 + 7$	$21 + 19$
$15 + 13 + 7$	$25 + 7 + 5 + 3$
$11 + 9 + 7 + 5 + 3$	$21 + 11 + 5 + 3$
	$17 + 15 + 5 + 3$
	$17 + 11 + 9 + 3$
	$13 + 11 + 9 + 7$

Observing the expansion in power series of

$$\sum_{n=0}^{\infty} \frac{q^{n(n+2)}}{(q^4; q^4)_n}$$

it was possible to make a conjecture about the integers having representation in A. The proof for that is in the next theorem where we denote by $p_A(m)$ the number of partitions of m belonging to A.

Theorem 3. Let m be a positive integer. Then $p_A(m) = 0$ if, and only, if $m = 4$ or $m \equiv 1, 2 (mod\ 4)$.

Proof. Let $\pi = a_1 + a_2 + \cdots a_n \in A$ be a partition of m. We consider two cases, n even and n odd.

Case 1. $n = 2t$. Since by Theorem 1 $a_{2t} \equiv 3 (mod\ 4), a_k$ is odd for $1 \leq k \leq n$ and $a_k \not\equiv a_{k+1} (mod\ 4)$ we have that

$$a_{2j-1} = 1 + 4k_{2j-1}, \quad k_{2j-1} > 0$$

and

$$a_{2j} = 3 + 4k_{2j},\ k_{2j} \geq 0,\ 1 \leq j \leq t$$

Therefore

$$\begin{aligned} m &= a_1 + a_2 + \cdots + a_{2t} \\ &= t.1 + 4k' + t.3 + 4.k'' \\ &= t(1+3) + 4(k'+k'') \\ &\equiv 0(mod\ 4) \end{aligned}$$

where $k' = \sum_{j=1}^{t} k_{2j-1}$ and $k'' = \sum_{j=1}^{t} k_{2j}$.

Case 2. $n = 2t+1$. In this case

$$\begin{aligned} a_{2j+1} &= 3 + 4k_{2j-1},\ k_{2j-1} \geq 0,\ 0 \leq j \leq t, \\ a_{2j} &= 1 + 4k_{2j}, k_{2j} > 0, 1 \leq j \leq t \end{aligned}$$

and, by doing what we did in case 1, we get that

$$m \equiv 3(mod\ 4).$$

Of course that $p_A(4) = 0$.

Now we have to show that $p_A(m) \neq 0$ when $m \equiv 0$ or $3(mod\ 4)$ and $m \neq 4$.

For $m \equiv 3(mod\ 4)$ the partition $\pi = m \in A$ and if $m \equiv 0(mod\ 4), m > 4$ then

$$\begin{aligned} m &= 4k,\ k \geq 2 \\ &= (4k-3) + 3 \in A, \end{aligned}$$

since $4k - 3 \geq 5$ and $4k - 3 \equiv 1(mod\ 4)$. So the proof is completed.

□

For set B we have, also, a similar result.

Theorem 4. Let m be a positive integer. Then $p_B(m) = 0$ if, and only if, $m \equiv 2, 3(mod\ 4)$.

We omit the proof since it can be obtained by the same argument used in Theorem 3.

3 Some Combinatorial Results

Using Theorems 1 and 2 we prove the following combinatorial results:

Theorem 5. The number of partitions of $m = a_1 + \cdots + a_s$ with $a_j \equiv (s+2)(mod\ 4)$ and $a_j \geq s+2,\ 1 \leq j \leq s$ is equal to the number of partitions of m satisfying (1.10).

Proof. Let $m = a_1 + \cdots + a_s$ be a partition of m as described in the first part of the theorem. Subtracting $s+2$ from each part we are left with a partition of $m - s(s+2)$ in at most s parts $\equiv 0(mod\ 4)$ and these are the ones generated by

$$\frac{q^{s(s+2)}}{(q^4; q^4)_s}.$$

Therefore the generated function for the partitions we are considering is

$$\sum_{s=0}^{\infty} \frac{q^{s(s+2)}}{(q^4; q^4)_s}.$$

On the other hand, by Theorem 1, in the expansion of

$$F_1(z) = \sum_{n=0}^{\infty} \frac{z^n q^{n(n+2)}}{(q^4; q^4)_n}$$

the term

$$\frac{z^n q^{n(n+2)}}{(q^4; q^4)_n} = z^n(\cdots + q^{a_1+a_2+\cdots+a_n} + \cdots)$$

has, in the exponent of q, all the elements of A with exactly n parts.

From this we see that the coefficient of q^m in $F_1(1)$ is the total number of partitions of m that are in A, and so, $F_1(1)$ is the generating function for the partitions satisfying (1.10) which completes the proof.

$\square$

The table is an illustration of Theorem 5 for $m = 36$. In the right column are the partitions satisfying (1.10).

32 + 4	33+3
28 + 8	29+7
24 + 12	25+11
20 + 16	21 + 15
18 + 6 + 6 + 6	21 + 7 + 5 + 3
14 + 10 + 6 + 6	17 + 11 + 5 + 3
10 + 10 + 10 + 6	13 + 11 + 9 + 3

Next we state a theorem that can be proved using Theorem 2 in a way that is similar to the proof of Theorem 5.

Theorem 6. The number of partitions of $m = a_1 + \cdots + a_s$ with $a_j \equiv s(mod\ 4)$ and $a_j \geq s, 1 \leq j \leq s$ is equal to the number of partitions of m satisfying (1.11).

Now we rewrite the identities (16) and (20) of Slater [8] that are given in (1.3) and (1.4). The right hand sides of (1.3) and (1.4) can be written, respectively, as

$$\frac{(-q^3;q^{10})_\infty(-q^7;q^{10})_\infty}{(q^8;q^{20})_\infty(q^{12};q^{20})_\infty}.(q^{10};q^{20})_\infty$$

and

$$\frac{(-q;q^{10})_\infty(-q^8;q^{10})_\infty}{(q^4;q^{20})_\infty(q^{16};q^{20})_\infty}.(q^{10};q^{20})_\infty$$

Therefore we have the identities:

$$\frac{1}{(q^{10};q^{20})_\infty}\sum_{n=0}^{\infty}\frac{q^{n^2+2n}}{(q^4;q^4)_n} = \frac{(-q^3;q^{10})_\infty(-q^7;q^{10})_\infty}{(q^8;q^{20})_\infty(q^{12};q^{20})_\infty} \tag{1.12}$$

and

$$\frac{1}{(q^{10};q^{20})_\infty}\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q^4;q^4)_n} = \frac{(-q;q^{10})_\infty(-q^9;q^{10})_\infty}{(q^4;q^{20})_\infty(q^{16};q^{20})_\infty} \tag{1.13}$$

If we consider the following sets

$C_1 := \{b_1 + \cdots + b_r|$ the even parts are $\equiv 10(mod\ 20)$ and the odd verify (1.10)$\}$ and

$C_2 := \{b_1 + \cdots + b_r |$ the even parts are $\equiv 10(mod\ 20)$ and the odd verify (1.11)$\}$

then, from equations (1.12) and (1.13) we have, immediately, the following two theorems:

Theorem 7. The number of partitions of m in C_1 is equal to the number of partitions of m where the odd parts are distinct and $\equiv \pm 3(mod\ 10)$ and the even $\equiv \pm 8(mod 20)$.

An illustration for $n = 31$ is

31	28+3
25+5+3	23+8
20 + 11	13+8+7+3
19+9+3	12+12+7
15+13+3	12+8+8+1
15+9+7	8+8+8+7

Theorem 8. The number of partitions of m in C_2 is equal to the number of partitions of m in parts that are distinct odd $\equiv \pm 1(mod\ 10)$ or even $\equiv \pm 4(mod\ 20)$.

An illustration for $n = 25$ is

25	24+1
21+3+1	21+4
20+5	11+9+4
17+7+1	16+9
13+11+1	16+4+4+1
13+7+5	9+4+4+4+4
9+7+5+3+1	4+4+4+4+4+4+1

4 The Rogers-Ramanujan Identities

Now we are going to show how it is possible to get a new combinatorial interpretation for the Rogers-Ramanujan identities from Theorems 1 and 2.

As we have seen the following equations are true:

$$(-q^2;q^2)_\infty \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^4;q^4)_n} = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} \tag{1.14}$$

$$(-q^2;q^2)_\infty \sum_{n=0}^{\infty} \frac{q^{n^2+2n}}{(q^4;q^4)_n} = \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q;q)_n} \tag{1.15}$$

Defining

$X_1 : \{b_1 + \cdots + b_r|$ the parts are distinct and the odd verify $(1.11)\}$
$X_2 : \{b_1 + \cdots + b_r|$ the parts are distinct and the odd verify $(1.10)\}$

we make the following observation for equation (1.14) (similar observations can be made for (1.15)): the caracterization of set B given by Theorem 2 tell us that the coefficient of q^m in the expansion of

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^4;q^4)_n} = 1 + q + q^{3+1} + q^5 + \cdots$$

is the number of partitions of m satisfying (1.11). Hence the coefficient of $q^{m'}(m' = m + \sum$ (distinct even)) in the expansion of the left hand side of (1.14) is the number of partitions of m' that are in X_1.

From these simple observations we have that the number of partitions of m that are em X_λ is equal to the number of partitions of m in parts $\equiv \pm\lambda(mod\ 5), \lambda = 1, 2$.

In this way we have obtained a new combinatorial interpretation for the Rogers-Ramanujan identities that we state in our next theorem.

Theorem 9. The number of partitions of m in distinct parts of the form $m = (a_1 + a_2 + \cdots + a_n) + (b_1 + b_2 + \cdots + b_s)$ where a_j is odd and b_i even with $a_j \not\equiv a_{j+1}(mod\ 4), a_n \equiv (2\lambda - 1)(mod\ 4)$ equals to the number of partitions of m in parts $\equiv \pm\lambda(mod\ 5), \lambda = 1, 2$.

We know for the first of the Rogers-Ramanujan identities the following interpretations:

R-R1a The number of partitions of n in parts $\equiv \pm 1(mod\ 5)$

R-R1b The number of partitions of n in parts differing by at least 2.

R-R1c The number of partitions of n such that the smallest part is greater than or equal to the number of parts

R-R1d The number of partitions of n into distinct parts such that r of them are odd and s are even and the smallest even part is $> 2r$.

R-R1e The number of partitions of n into distinct parts such that r of them are odd and s are even and the smallest odd part is $> 2s$.

The table below has the partitions of $n = 11$ for the first identity where the last column has the partitions as described in Theorem 9.

R-R1a	R-R1b	R-R1c
11	11	11
$9+1+1$	$10+1$	$9+2$
$6+4+1$	$9+2$	$8+3$
$6+1+\cdots+1$	$8+3$	$7+4$
$4+4+1+1+1$	$7+4$	$6+5$
$4+1+\cdots+1$	$7+3+1$	$5+3+3$
$1+1+\cdots+1$	$6+4+1$	$4+4+3$

R-R1d	R-R1e	Theorem 9
11	11	$10+1$
$10+1$	$9+2$	$9+2$
$8+3$	$8+3$	$8+2+1$
$7+4$	$7+4$	$6+5$
$6+5$	$6+5$	$6+4+1$
$7+3+1$	$7+3+1$	$5+4+2$
$6+4+1$	$5+4+2$	$5+3+2+1$

For the second identity we know the following interpretations:

R-R2a The number of partitions of n in parts $\equiv \pm 2 (mod\ 5)$.

R-R2b The number of partitions of n in parts differing by at least 2 and each part > 1.

R-R2c The number of partitions of n such that the smallest part is larger than the number of parts.

R-R2d The number of partitions of n into distinct parts wherein each odd part is larger than 2 plus twice the number of even parts.

R-R2e The number of partitions of n into distinct parts greater than 1 wherein each even part is larger than twice the number of odd parts.

In the following table are the partitions of 13 where in the last column are the partitions as described in Theorem 9.

R-R2a	R-R2b	R-R2c
13	13	13
$8+3+2$	$11+2$	$10+3$
$7+3+3$	$10+3$	$9+4$
$7+2+2+2$	$9+4$	$8+5$
$3+3+3+2+2$	$8+5$	$7+6$
$3+2+\cdots+2$	$7+4+2$	$5+4+4$
R-R2d	R-R2e	Theorem 9
13	13	$11+2$
$11+2$	$10+3$	$10+3$
$9+4$	$9+4$	$8+3+2$
$8+5$	$8+5$	$7+6$
$7+6$	$7+6$	$7+4+2$
$7+4+2$	$6+4+3$	$6+4+3$

We observe that the descriptions given in R-R2d and R-R2e are particular cases of a general theorem in Santos and Mondek [6].

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CCET - UFMS Cx.P. 549
79070-900 - Campo Grande - MS - Brasil
email:mondek@hilbert.dmt.ufms.br

IMECC-UNICAMP Cx.P. 6065
13081-970 - Campinas - SP - Brasil
email:josepli@ime.unicamp.br