Quantum computations on macroscopical automata

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Abstract

Unitary operations in Hilbert space of spin one half system for one qubit and two qubit systems are realized in terms of vertices of graphs of macroscopical automata realizing quantum logic.Examples of simple logical operations are analysed.

1 Introduction

Quantum computers and quantum computations became much popular topic in modern theoretical physics. Theoretical works show new advantages of quantum computers, such as Shor's algorithm for solving the factorization problem and etc.[5]. The first experimental evidence of the realization of a quantum computer using nuclear magnetic resonance spectrocopy on organic molecule appeared in[1]. In all these examples one must have *hardware*, made of quantum particles, described by quantum mechanics, so that its *software*, differently from *classical* computers, works according to *quantum logic*. This makes possible totally new kind of computations. Nevertheless, one can ask the following questions: Is microscopic quantum *hardware* necessary to obtain *quantum logical software*? Can one construct macroscopical automata with *classical hardware* but with *quantum software*? Experimentally, if the answer is positive, this can lead to new possibilities for quantum computation, because differently from microparticle *hardware* one doesn't need to struggle seriously with all kinds of noise, destroying coherence of the quantum entagled states used in quantum computation. Other advantage can be some new understanding of brain processes, when quantum logic can work even for the macroscopic system. Indeed, in papers [3,4] following the idea of [2] examples of macroscopic systems—macroscopic automata—were presented, where due to the special rules of their work quantum logic arises ! Using correspondence between lattices and graphs of automata and *negative logic* for identification of the states of automata one obtains nondistributive quantum logical lattice as description of its work. This means that the behaviour of these macroscopic objects can be described by some Hilbert space with projectors in this space as some observables— *yes-no questions*. In terms of graphs of automata to different wave functions correspond different *weights*. Analogue of the Heisenberg's uncertainty relations was constructed and breaking of Bell's inequalities was demonstrated in terms of these *weights*. Due to quantum logic, if properties of this system are unknown, the stochasticity will be described not by the classical probability measure but by the wave function-probability amplitude! Spin one half and spin one particle were modelled for one particle and two particle quantum systems by such automata. In a sense the whole construction can be understood as some version of the *hidden variables* theory when *hardware* is macroscopic and is described by classical physics and Boolean logic but the *software,* due to the impossibility to observe some aspects of the system (disjunction a or b is true if "a " is true or "b" is true but not "only if "!) behaves according to nonBoolean quantum logic. So we get a positive answer on our question concerning the possibility of having *quantum software* with classical *hardware*! Question arises about the technical realization of such automata as quantum computers. Can one use the standard computer but with some special *quantum programm* for it? The answer seems to be negative. Surely one can solve Schrödinger equation by use of the standard computer. But the advantages of the quantum computer with *microscopic hardware* are:

(i) use of *qubits* instead of *bits*,

(ii) existence of special *measurement processes* due to wave packet collapse.

In our version of the *macroscopic quantum computer* one also has *qubits* because of the quantum logical structure of its software. In order to simulate quantum rules for getting probabilities due to wave packet collapse one must simulate the *observer* by putting some new *command*, identifying the *state of the automaton* and some stochasticity concerning this state. But this is just what hidden variables theorists do in order to obtain quantum mechanical formalism for classical systems introducing unobservable interaction with the measuring apparatus. In this paper we continue the investigation, made in[3,4], by demonstrating how some quantum computations, described by unitary operators in Hilbert space for microscopic quantum computers can be realized by some operations on weights on graphs for our macroscopic quantum computers. First we analyze the case of a simple one spin one- half particle and show how simple logical operations look in terms of our graph. Then the two particles spin one- half system is analyzed—classical automata described by the same quantum logical lattice are investigated and logical operations in terms of transformation of weights of vertices of its graph are realized. A simple example of a system called by us *discrete quantum computer* for one-qubit and two-qubits is investigated.

2 Weights on the graph of the spin one half particle and qubit automata

In papers [3,4] it was shown how in some cases quantum logic arises as description of properties of macroscopic automata, so that if it's states are random, the description will be not in terms of the standard probability measure but in terms of the probability function. Remind the definition of the automaton. Normalized automaton is defined by a nonoriented graph ,satisfying the following conditions: (i) the set of input symbols and the set of interior states of the automaton coincide with the set of vertices of the graph; (ii) the transition function (i.e., the rule of operation of the automaton) is such that if it is initially in "i "and the input symbol is "j "then if the vertices of the graph are adjacent the new state will be "j "and if not connected it stops. 'To vertices of the graph it was proposed in[3,4] to give some weights, having one to one connection with the wave function. So our problem will be: how to describe unitary operations with wave functions (logical gates) in terms of weights of the graph of the automata?

Following[3,4] let us take spin one half particle with observables

$$S_x = \frac{h}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ S_y = \frac{h}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},\ S_z = \frac{h}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{1}$$

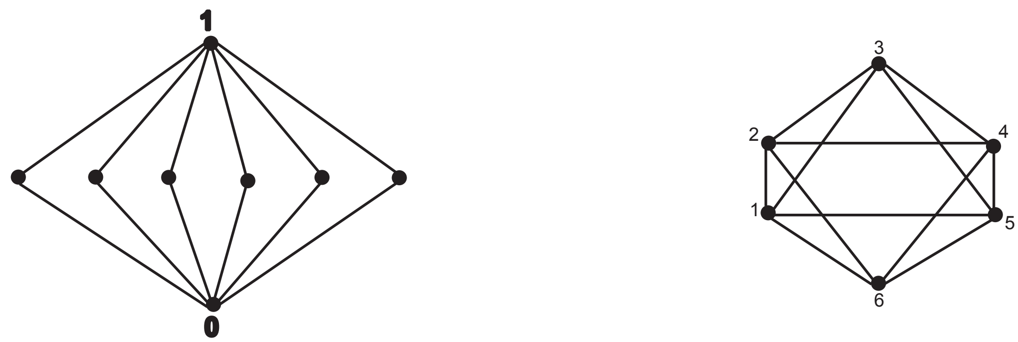

Figure 1: Hasse diagram and the graph of the automaton.

For this system we have the Hasse diagramm and graph of the automaton corresponding to it (see Figure 1). The Hasse diagramm on the left represents the quantum logical lattice, to atoms of which correspond the following yes-no questions: 1. $S_x = \frac{1}{2}$?, 2. $S_y = \frac{1}{2}$?, 3. $S_z = \frac{1}{2}$?, 4. $S_x = -\frac{1}{2}$?, 5. $S_y = -\frac{1}{2}$?, 6. $S_z = -\frac{1}{2}$?.

In Hilbert space of the spin one half system to these questions correspond projectors on the following state vectors:

$$\begin{aligned} |e_1 &> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, |e_2>= \frac{1}{\sqrt{2}}\begin{bmatrix} i \\ -1 \end{bmatrix}, |e_3>= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ |e_4 &> = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}, |e_5>= \frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}, |\ e_6>= \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{aligned}$$

Take the computational basis $|0\rangle = |e_3\rangle$, $|1\rangle = |e_6\rangle$. Then to the qubit. $|\Psi\rangle = C_0|0> +C_1|1\rangle$ correspond weights of the graph on the right of Figure1. We have, $w_\alpha = ||P_\alpha\Psi\rangle|^2 = |\langle e_\alpha|\Psi\rangle|^2, \alpha = 1,2,3,4,5,6$. Writting $S_\alpha = |\langle e_\alpha|0\rangle|^2$, $t_\alpha = |\langle e_\alpha|1\rangle|^2 = 1 - S_\alpha$, $p_\alpha = \langle 0|e_\alpha\rangle\langle e_\alpha|1\rangle$ one obtains $w_\alpha = S_\alpha|C_0|^2 + t_\alpha|C_1|^2 + \mathrm{Re}(2p_\alpha C_0^+ C_1)$. One can easily obtain for $S_\alpha, t_\alpha, p_\alpha$ the following values

$$
\begin{array}{cccc}
 & S_\alpha & t_\alpha & 2p_\alpha \\
\langle e_1| & \frac{1}{2} & \frac{1}{2} & 1 \\
\langle e_4| & \frac{1}{2} & \frac{1}{2} & -1 \\
\langle e_2| & \frac{1}{2} & \frac{1}{2} & i \\
\langle e_5| & \frac{1}{2} & \frac{1}{2} & -i \\
\langle e_3| & 1 & 0 & 0 \\
\langle e_6| & 0 & 1 & 0
\end{array} \tag{2}
$$

Putting $C_0 = c_0 \exp i\varphi_0$, $C_1 = c_1 \exp i\varphi_1$, $c_0^2 + c_1^2 = 1$, $\varphi = \varphi_1 - \varphi_0$, one obtains the weights $w_\alpha = S_\alpha c_0^2 + t_\alpha c_1^2 + c_0 c_1 \,\mathrm{Re}(2p_\alpha \exp i\varphi)$, so that, $w_1 = \frac{1}{2} + c_0 c_1 \cos\varphi$, $w_2 = \frac{1}{2} + c_0 c_1 \sin\varphi$, $w_3 = c_0^2$, $w_4 = \frac{1}{2} - c_0 c_1 \cos\varphi$, $w_5 = \frac{1}{2} - c_0 c_1 \sin\varphi$, $w_6 = c_1^2$. It is evident that $w_1 + w_4 = w_2 + w_5 = w_3 + w_6 = 1$.

Let us describe the *space* of weights of one qubit.

1. $0 \leq w_\alpha \leq 1$.

2. All weights depend on two parameters $c_0, c_1, \varphi, c_0^2 + c_1^2 = 1$.

3. One can easily see that $(w_1 - \frac{1}{2})^2 + (w_2 - \frac{1}{2})^2 + (w_3 - \frac{1}{2})^2 = (\frac{1}{2})^2$, i.e., different values of weights correspond to points on the sphere, so that change of these values can be described by rotation of the diametre of the sphere.

And now let us describe unitary operations on the qubit automata. Let us take U$\in U(2)$, i.e., $UU^+ = U^+U = 1$,

$$
U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix},
$$

with $|u_{11}|^2 + |u_{12}|^2 = |u_{21}|^2 + |u_{22}|^2 = 1$, $u_{11}u_{21}^+ + u_{12}u_{22}^+ = u_{11}u_{12}^+ + u_{21}u_{22}^+ = 0$.

One can parametrize $U(2)$ matrices using angles α, θ, β.

$$
U = \exp i\delta \begin{bmatrix} \exp i\frac{\alpha}{2} & 0 \\ 0 & \exp -i\frac{\alpha}{2} \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} \exp i\frac{\beta}{2} & 0 \\ 0 & \exp -i\frac{\beta}{2} \end{bmatrix}. \tag{3}
$$

Denoting the first matrix as $R_z(\alpha)$, the second as $R_y(\theta)$ one can say that any unitary U on the weights of qubit can be represented if one can represent these $R_z(\alpha)$, $R_y(\theta)$ matrices. Knowing how they act on qubit $|\Psi\rangle$, i.e., $R_z(\alpha)|\Psi\rangle = \exp(i\frac{\alpha}{2}c_0)|0\rangle + \exp(-i\frac{\alpha}{2}c_1)|1\rangle$, one comes to the following formulas for weights:

$$R_{z(\alpha)}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \cos\alpha & \sin\alpha & 1 & -\cos\alpha & -\sin\alpha & 1 \\ -\sin\alpha & \cos\alpha & 1 & \sin\alpha & -\cos\alpha & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -\cos\alpha & -\sin\alpha & 1 & \cos\alpha & \sin\alpha & 1 \\ \sin\alpha & -\cos\alpha & 1 & -\sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}. \tag{4}$$

One also has

$$R_y(\theta)\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \cos\theta & 1 & -\sin\theta & -\cos\theta & 1 & \sin\theta \\ 0 & 1 & 0 & 0 & 1 & 0 \\ \sin\theta & 1 & \cos\theta & -\sin\theta & 1 & -\cos\theta \\ -\cos\theta & 1 & \sin\theta & \cos\theta & 1 & -\sin\theta \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -\sin\theta & 1 & -\cos\theta & \sin\theta & 1 & \cos\theta \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}. \tag{5}$$

Since the *space of weights* is the set of points of the sphere, these operations are just some rotations. On one qubit system one can perform the following logical operations:

(i) identity $I = |0><0| + |1><1|$;

(ii) operator NOT as $X = |0><1| + |1><0|$;

(iii) Hadamard transformation $H = \frac{1}{\sqrt{2}}[(|0> + |1>) < 0| + (|0> - |1>) < 1|$.

To these operations correspond the following matrices in the Hilbert space of the spin one half system:

$$\begin{aligned} I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ Y = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \\ H &= YX. \end{aligned}$$

which are easily obtained from the general form of the unitary matrices for special values of angles and to these correspond formulas for weights for the same angles. Real advantages for quantum computations need investigation of the two or more qubit systems. So now we shall investigate two qubit system.

3 Two qubit system

To two qubit system correspond two spin one half particle state, which can be either the product of one particle states or some entangled state. In[4] we constructed the automaton with the graph corresponding to this microparticle quantum system. So our task is again to look for transformations of weights of the graph when unitary transformations act on the wave function.

The basis is: $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|1\rangle|0\rangle$, $|1\rangle|1\rangle$. So, any state vector of our quantum system is

$$\mid \Psi\rangle = C_0|0\rangle|0\rangle + C_1|0\rangle|1\rangle + C_2|1\rangle|0\rangle + C_3|1\rangle|1\rangle, \ |C_0|^2+|C_1|^2+|C_2|^2+|C_3|^2 = 1 \tag{6}$$

So differently from the classical physics of a two particles system one gets not four parameters but six parameters to fix the state. This is just due to the possibility of entanglement, leading to a new possibility for computation. If one takes the graph without what was called in [4] *nonlocal questions* , i.e., without questions about entangled states one has 36 *yes-no* questions $|e_\alpha > |e_\beta >, \alpha, \beta = 1,..6$ with 36weights $w_{\alpha,\beta} = |\langle e_\alpha|\langle e_\beta|\Psi\rangle|^2$. For factorised states $|\Psi\rangle = |\Phi\rangle|\Lambda\rangle$, $|\Phi\rangle = B_0|0\rangle + B_1|1\rangle$, $|\Lambda\rangle = A_0|0\rangle + A_1|1\rangle$ one has only 4 free parameters and the weights are obtained from one qubit system as $w_{\alpha\beta} = w_\alpha w_\beta, w_\alpha = |\langle e_\alpha|\Phi\rangle|^2, w_\beta = |\langle e_\beta|\Lambda\rangle|^2$. But, for the general case one has

$$\begin{aligned}
|\langle e_\alpha|\langle e_\beta|\Psi\rangle|^2 &= |C_0|^2 S_\alpha S_\beta + C_0 C_1^+ S_\alpha p_\beta^+ \\
&+ C_0 C_2^+ p_\alpha^+ S_\beta + C_0 C_3^+ (p_\alpha p_\beta)^+ + C_1 C_0^+ S_\alpha p_\beta \\
&+ |C_1|^2 S_\alpha t_\beta + C_1 C_2^+ p_\alpha^+ p_\beta + C_1 C_3^+ p_\alpha^+ t_\beta \\
&+ C_2 C_0^+ p_\alpha S_\beta + C_2 C_1^+ p_\alpha p_\beta^+ + |C_2|^2 t_\alpha S_\beta \\
&+ C_2 C_3^+ t_\alpha p_\beta^+ + C_3 C_0^+ p_\alpha p_\beta + C_3 C_1^+ p_\alpha t_\beta \\
&+ C_3 C_2^+ t_\alpha p_\beta + |C_3|^2 t_\alpha t_\beta, \\
S_\alpha &= |\langle e_\alpha|0\rangle|^2, \ t_\alpha = |\langle e_\alpha|1\rangle|^2, \ p_\alpha = \langle 0|e_\alpha\rangle\langle e_\alpha|1\rangle
\end{aligned} \tag{7}$$

.

So one has Hermitean forms $W_{\alpha\beta}(x,y) = \sum_{i,j=0}^{3} B_{ij}^{(\alpha\beta)} x_i y_j^+$, where

$$||W_{\alpha\beta}|| = \{B_{ij}^{(\alpha\beta)}\} = \begin{bmatrix} S_\alpha S_\beta & S_\alpha p_\beta^+ & p_\alpha^+ S_\beta & (p_\alpha p_\beta)^+ \\ S_\alpha p_\beta & S_\alpha t_\beta & p_\alpha^+ p_\beta & p_\alpha^+ t_\beta \\ p_\alpha S_\beta & p_\alpha p_\beta^+ & t_\alpha S_\beta & t_\alpha p_\beta^+ \\ p_\alpha p_\beta & p_\alpha t_\beta & t_\alpha p_\beta & t_\alpha t_\beta \end{bmatrix}$$

are some Hermitean matrices.Weights are positive values $w_{\alpha\beta} = W_{\alpha\beta}(\vec{C}, \vec{C})$. So, if vector of the coefficients is transformed unitarily $\vec{C} \mapsto U\vec{C}$one obtains new weights $w'_{\alpha\beta} = W_{\alpha\beta}(U\vec{C}, U\vec{C})$, which can be understood as some transformation of the matrix $||W_{\alpha\beta}|| \mapsto ||W'_{\alpha\beta}||$.

4 Discrete quantum computer

Here we 'll investigate these transformations for the simplified case,called by us *discrete quantum computer* . Instead of the general case take $C_i \in \{-1, 0, 1\}$, i.e., the space of coefficients is some 4-dimensional discrete space.So we'll describe only those operations which don't evolve from this space as if one has *arithmetics* on the field $P_3 = \{-e, 0, e\}$, where $e + e = -e$. For this case new discrete qubit corresponds to: $|0>, |1>, |0> + |1>, |0> - |1>$ .Operations X, H form a complete set of operations on the *discrete qubit,* this meaning that any transformation of qubit can be obtained by subsequent using of these operations

$$
\begin{aligned}
X &: |0\rangle \mapsto |1\rangle, |1\rangle \mapsto |0\rangle, \\
H &: |0\rangle \mapsto |0\rangle + |1\rangle, |1\rangle \mapsto |0\rangle - |1\rangle.
\end{aligned}
$$

They are some reflections $X^2 = H^2 = I$. Other operations are $R = HX$, $R^2 = HXHX$, $R^3 = (R^{-1}) = XH$, $R^4 = I$, $OH = XHX$, $OH^2 = I$, $OX = HXH$, $OX^2 = I$. One can see that the set of operations on such a *discrete qubit* forms the group of symmetries of the quadrangle. Let us call this qubit- *quadrit* .

Now, let us investigate the two qubit system. One qubit operations are obtained as the following tensor products: $X \otimes I$, $I \otimes X$, $H \otimes I$, $I \otimes H$. Introduce controllable $CNOT$, which we denote for our special case as $\vec{X}, \overleftarrow{X}$ being $CNOT$ for the first and second qubit:

$$\vec{X} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X, \tag{8}$$

$$\overleftarrow{X} = I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1| \tag{9}$$

Also introduce the unitary operation S ($Swap$) :

$$\begin{aligned} |00\rangle &\mapsto |00\rangle \\ |01\rangle &\mapsto |10\rangle \\ |10\rangle &\mapsto |01\rangle \\ |11\rangle &\mapsto |11\rangle \end{aligned} \tag{10}$$

So one comes to the following system of commands (operations): $\{I$, $X \otimes I$, $H \otimes I$, $\vec{X}$, $S\}$.

Our quantum computer is the set of *quadrits* with the properties:

(i) each *quadrit* can be prepared in the state $|0\rangle$;

(ii) on each set of *quadrits* one can do logical operations to which correspond unitary operations;

(iii) it is possible to do measurements for "quadrit "in the computational basis formed by $\{|0\rangle, |1\rangle\}$.

Our *quadrit* corresponds to one spin one half particle with two observables S_x, S_z, described by the corresponding graph and Hasse diagramm[3,4]. All its possible states and weights can be easily enumerated, using notations h_0, h_1, showing the possibility to obtain them by operation H from the corresponding states of the basis:

i	$\|\Psi^i\rangle$	$\vec{w}^i = (w_0^i, w_{h_0}^i, w_1^i, w_{h_1}^i)$
0	$\|0\rangle$	$(1, \frac{1}{2}, 0, \frac{1}{2})$
h_0	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	$(\frac{1}{2}, 1, \frac{1}{2}, 0)$
1	$\|1\rangle$	$(0, \frac{1}{2}, 1, \frac{1}{2})$
h_1	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	$(\frac{1}{2}, 0, \frac{1}{2}, 1)$

To logical operations X, H correspond matrix transformations of weights:

$$X \mapsto \begin{bmatrix} w_0 \\ w_{h_0} \\ w_1 \\ w_{h_1} \end{bmatrix} \mapsto \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_0 \\ w_{h_0} \\ w_1 \\ w_{h_1} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_{h_0} \\ w_0 \\ w_{h_1} \end{bmatrix} \tag{11}$$

$$H \mapsto \begin{bmatrix} w_0 \\ w_{h_0} \\ w_1 \\ w_{h_1} \end{bmatrix} \mapsto \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_0 \\ w_{h_0} \\ w_1 \\ w_{h_1} \end{bmatrix} = \begin{bmatrix} w_{h_0} \\ w_0 \\ w_{h_1} \\ w_1 \end{bmatrix} \tag{12}$$

To unitary operations correspond permutations of indexes of the weight vector.

Now consider *biquadrit*-two particle system. Here for our discrete case there are the following states:

1. $|0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|1\rangle|0\rangle$, $|1\rangle|1\rangle$— 4 factorised states F_1;

2. $|0\rangle(|0\rangle \pm |1\rangle)$, $|1\rangle(|0\rangle \pm |1\rangle)$— 4 factorised states E_1;

$(|0\rangle \pm |1\rangle)|0\rangle$, $(|0\rangle \pm |1\rangle)|1\rangle$— 4 factorised states F ;

$|0\rangle|0\rangle \pm |1\rangle|1\rangle$, $|0\rangle|1\rangle \pm |1\rangle|0\rangle$— 4 entangled states E_2.

3. Entangled states $|0\rangle|0\rangle \pm |0\rangle|1\rangle \pm |1\rangle|0\rangle$, $|0\rangle|0\rangle \pm |0\rangle|1\rangle \pm |1\rangle|1\rangle$, $|0\rangle|0\rangle \pm |1\rangle|0\rangle \pm |1\rangle|1\rangle$,

$|0\rangle|1\rangle \pm |1\rangle|0\rangle \pm |1\rangle|1\rangle$.

4. there are also 8 states with 4 terms, forming two sets $F_{4,}$ E_4.

States of the type 3 can't be obtained from states of type 1 by using the operations X, H, $CNOT$. We shall be interested only in those states which can be obtained from $|0\rangle|0\rangle$ by our operations enlarged by new operations which cannot be obtained from one qubit operations,making permutations of qubits ($SWAP$ operation), which we introduce later. One observes that X, H don't lead to $F \mapsto E$, $CNOT$ acts as $F_2 \mapsto E_{2,}$ $F_4 \mapsto E_4$. It is interesting to mention here that due to existence of new 8 entangled states there is a difference between a quantum computer and a classical one: for one qubit system there are 4 states, but for the two qubit system one has 24 state instead of 16 as it is the case for a classical computer. The graph of the two particles spin one- half system is the same as in the paper [4]. It consists of 16 vertices and *nonlocal* vertices, one of them corresponding to the antisymmetrised state $|0\rangle|1\rangle - |1\rangle|0\rangle$ which we'll denote by q^-or as rx, meaning that it can be obtained as

$\vec{X}$ $(R \otimes X)|0\rangle|0\rangle$. It is easy to see that for the 16 states obtained as $|e_\alpha\rangle|e_\beta\rangle$, $\alpha, \beta = 1...4$, $w_{\alpha\beta} = |\langle e_\alpha|\langle e_\beta|\Psi\rangle|^2$, the new weights after unitary operations $U|\Psi\rangle$ are obtained by permutation of weights in the weight vector.

One qubit operations are generalised as tensor products:$gi = \hat{g} \otimes I$, $ig = I \otimes \hat{g}$. Here $\hat{g}$ is some one qubit operation. But for a two qubit system one can also introduce new operations: $\vec{g} = |0\rangle\langle 0| \otimes I + |1\rangle < 1| \otimes \hat{g}$-controlled g by the first qubit, and $\overleftarrow{g}= I \otimes |0\rangle\langle 0| + \hat{g} \otimes |1\rangle\langle 1|$— controlled g by the second qubit.

In paper [4]only one *nonlocal yes-no question* corresponding to the antisymmetrised state was considered. The vertex of the graph $|q^-\rangle$ is not connected in the graph by the arc with vertices 11, 22, 33, 44 and is connected with all others.By unitary operations for one qubit and $\vec{g}$, $\overleftarrow{g}$ one can obtain from the $|q^-\rangle$ seven weights corresponding to entangled states of our discrete two qubit system.These entangled states are

$$\begin{aligned}
|q^-\rangle &= |01\rangle - |10\rangle, |q_s^+\rangle = |00\rangle + |11\rangle, |q_s^-\rangle = |00\rangle - |11\rangle, |q^+ = |01\rangle + |10\rangle, \\
|q_{sa}^+\rangle &= (|00> +|11>) + (|01> -|10>), |q_{as}^+\rangle = (|00\rangle - |11\rangle) + (|01\rangle + |10\rangle), \\
|q_{sa}^-\rangle &= (|00\rangle + |11\rangle) - (|01\rangle - |10\rangle), |q_{as}^-\rangle = (|00\rangle - |11\rangle) - (|01\rangle + |10\rangle).
\end{aligned}$$

Normalization constants are supposed but not written here. Weights corresponding to these states can easily be obtained one from the other by automorphisms of the graph where besides $|q^-\rangle$ other seven vertices are drawn. The structure of the graph is the following. Differently from paper[4], for simplicity we shall not draw the (complete) picture of the graph— instead we are going to exhibit four by four tables putted inside bold brackets in which each place corresponds to a vertex (of the graph) and where we put zeros for those vertices which are not connected with the vertex corresponding to the entangled state question. For example, zeros on the left diagonal for $|q^-\rangle$ mean that the vertex $|q^-\rangle$ is not connected by arcs with diagonal vertices $11, 22, 33, 44$. So the graph has the structure:

$$|q-\rangle \mapsto \begin{pmatrix} 0 & * & * & * \\ * & 0 & * & * \\ * & * & 0 & * \\ * & * & * & 0 \end{pmatrix}, \ |q_s^+\rangle \mapsto \begin{pmatrix} * & * & 0 & * \\ * & * & * & 0 \\ 0 & * & * & * \\ * & 0 & * & * \end{pmatrix}, \ |q_s^-\rangle \mapsto \begin{pmatrix} * & * & 0 & * \\ * & 0 & * & * \\ 0 & * & * & * \\ * & * & * & 0 \end{pmatrix}$$

$$|q^+\rangle \mapsto \begin{pmatrix} 0 & * & * & * \\ * & * & * & 0 \\ * & * & 0 & * \\ * & 0 & * & * \end{pmatrix}, \ |q_{+sa}\rangle \mapsto \begin{pmatrix} * & * & * & 0 \\ 0 & * & * & * \\ * & 0 & * & * \\ * & * & 0 & * \end{pmatrix}, \ |q_{as}^+\rangle \mapsto \begin{pmatrix} * & * & * & 0 \\ * & * & 0 & * \\ * & 0 & * & * \\ 0 & * & * & * \end{pmatrix},$$

$$|q_{sa}^-\rangle \mapsto \begin{pmatrix} * & 0 & * & * \\ * & * & 0 & * \\ * & * & * & 0 \\ 0 & * & * & * \end{pmatrix}, \qquad |q_{as}^-\rangle \mapsto \begin{pmatrix} * & 0 & * & * \\ 0 & * & * & * \\ * & * & * & 0 \\ * & * & 0 & * \end{pmatrix}$$

Vertices of entangled questions are also connected by arcs with themselves due to the rule: they are located on the circle in the order $|q^-\rangle$, $|q_{as}^-\rangle$, $|q_s^+\rangle$, $|q_{as}^+\rangle$, $|q_s^-\rangle$, $|q_{sa}^-\rangle$, $|q^+\rangle$, $|q_{sa}^+\rangle$. After $|q^+\rangle$ again follows $|q^-\rangle$. For biquadrit one qubit operations and new operations $\vec{g}$, $\overleftarrow{g}$ are just automorphisms of the graph. This solves the problem of logical operations in terms of transformations of the graph and weights of its vertices.

5 Conclusion

In this paper we gave the rule for quantum computations described by unitary matrices in Hilbert space realized on macroscopic automata with quantum logic. This rule is very simple for the one qubit system, while for the two qubit system it can be easily formulated for the simplified case of the *discrete quantum computer*. To unitary operations correspond some transformations of the weights of vertices of the graph of the macroscopic automaton. Even for the simplified discrete case of the two qubit system typically quantum entangled states arise and logical operations can be made by use of them due to which new advantages of quantum computers occur. It seems that there is no principal objection for the generalization of the scheme for three qubit system where new logical operations (for example "and ") arise. So we hope in future to formulate the rule for transformation of weights as for the general (not the *discrete* one) case for the two qubit system as for the n- qubit system and to give examples of realization of Shor's algorithm and others for our *macroscopic quantum computer*.

Other interesting problem is to consider some interaction of our *macroscopi cquantum computer* with some microscopic quantum system (a photon, electron, etc.) described by a spin wave function so that some many- particles system will arise, one of them being our macroscopic sytem, now described also by the spin wave function. Then many interesting possibilities concerning entangled states, teleportation, etc., arise if standard quantum mechanics can be used for such a system.

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6 References

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