Ideals of Holomorphic Functions on Tsirelson's Space [1]

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Abstract

We show that if U is a convex, balanced, open subset of the Banach space X constructed by Tsirelson, then every proper, finitely generated ideal of $\mathcal{H}(U)$ has a common zero.

Introduction

Let $\mathcal{H}(U)$ denote the algebra of all holomorphic functions on an open subset U of a locally convex space E. $\mathcal{H}(U)$ will be always endowed with the compact-open topology.

A classical theorem of Cartan [8] asserts that *if U is a domain of holomorphy in $\mathbf{C}^n$, then every proper, finitely generated ideal of $\mathcal{H}(U)$ has a common zero.* Schottenloher [26] has shown that the same conclusion holds if U is a domain of holomorphy in a (DFC)-space, that is in a space of the form $E = F'_c$, with F Fréchet.

On the other hand we have proved in [19] that *if U is a polynomially convex open set in a Fréchet space with the approximation property, then every ideal of $\mathcal{H}(U)$ without common zeros is dense in $\mathcal{H}(U)$*. It follows from the results in [20] that the same conclusion holds if U is a polynomially convex open set in a quasi-complete locally convex space with the approximation property. Moreover Schottenloher [25] has shown that the same conclusion holds if U is a domain of holomorphy in a separable Fréchet space with the bounded approximation property.

In [19] we gave an example of a proper ideal of $\mathcal{H}(E)$ without common zeros, when E is any locally convex space of strictly positive dimension. Thus the conclusion of Cartan's theorem is false in general if the ideal under consideration is not finitely generated. But the following question, raised in [19], still remains open: *If U is a domain of holomorphy in a Fréchet space, does every proper, finitely genetated ideal of $\mathcal{H}(U)$ have a common zero*? Not only does this problem still remain open, but until now no example was known of a domain U in an infinite dimensional Fréchet space with the property that every proper, finitely generated ideal of $\mathcal{H}(U)$ has a common zero.

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In this paper we remedy this situation in part by proving that *if U is a convex, balanced, open set in the Banach space X constructed by Tsirelson [27], then every proper, finitely generated ideal of $\mathcal{H}(U)$ has a common zero.*

In Section 1 we obtain a similar result for the Fréchet algebra $\mathcal{H}_b(U)$ of all holomorphic functions of bounded type on U. A key ingredient in the proof is a theorem of Arens [4] for Fréchet algebras.

In Section 2 we obtain our main result. Since $\mathcal{H}(U)$ is not a Fréchet algebra, we reduce the study of the problem in $\mathcal{H}(U)$ to the study of a similar problem in a suitable Fréchet subalgebra of $\mathcal{H}(U)$.

We refer to the books of Dineen [10] or the author [21] for background information from infinite dimensional complex analysis.

1. Holomorphic Function of Bounded Type on Tsirelson's Space

If E and F are complex Banach spaces, then $\mathcal{P}(^mE;F)$ denotes the Banach space of all continuous m-homogeneous polynomials from E into F. $\mathcal{P}_f(^mE;F)$ denotes the subspace of $\mathcal{P}(^mE;F)$ generated by all polynomials of the form $P(x) = (\phi(x))^m b$, with $\phi \in E'$ and $b \in F$. And $\mathcal{P}(E;F)$ denotes the vector space of all continuous polynomials from E into F. When $F = \mathbf{C}$ we write $\mathcal{P}(^mE), \mathcal{P}_f(^mE)$ and $\mathcal{P}(E)$ instead of $\mathcal{P}(^mE;\mathbf{C}), \mathcal{P}_f(^mE;\mathbf{C})$ and $\mathcal{P}(E;\mathbf{C})$, respectively.

If U is an open subset of E, then a set $A \subset U$ is said to be *U-bounded* if A is bounded and is bounded away from the boundary of U. $\mathcal{H}_b(U;F)$ denotes the vector space of all holomorphic mappings $f : U \to F$ which are bounded on U-bounded sets. $\mathcal{H}_b(U;F)$ is a Fréchet space for the topology of uniform convergence on U-bounded sets. When $F = \mathbf{C}$ we write $\mathcal{H}_b(U)$ instead of $\mathcal{H}_b(U;\mathbf{C})$. Thus $\mathcal{H}_b(U)$ is a Fréchet algebra.

It is well known, and easy to see, that $\mathcal{P}(^mE;F)$ is a complemented subspace of $\mathcal{H}_b(U;F)$. Let $d_U(x)$ denote the distance from x to the boundary of U for each $x \in U$, and let $d_U(A) = \inf_{x\in A} d_U(x)$ for each $A \subset U$. Then each of the sets

$$U_n = \{x \in U : ||x|| < n \quad \text{and} \quad d_U(x) > 2^{-n}\}$$

is U-bounded, and each U-bounded set is contained in some U_n. One can readily see that $d_{U_{n+1}}(U_n) \geq 2^{-(n+1)}$, and hence it follows that $\rho_n U_n \subset U_{n+1}$, when $\rho_n = 1 + 1/n2^{n+1}$.

If U is balanced, then one can readily see that each U_n is *circular*, that is $e^{i\theta}U_n \subset U_n$ for every $\theta \in \mathbf{R}$. Then it follows from the Cauchy inequalities that the Taylor series of each $f \in \mathcal{H}_b(U;F)$ at the origin converges uniformly on each U_n. In particular $\mathcal{P}(E;F)$ is dense in $\mathcal{H}_b(U;F)$.

If U is convex, then one can readily see than each U_n is convex as well. Thus if U is convex and balanced, then U_n is convex and balanced as soon as $0 \in U_n$.

If $a \in U$, then δ_a denotes the homomorphism $f \in \mathcal{H}_b(U) \to f(a) \in \mathbf{C}$. The homomorphisms δ_a, with $a \in U$, are called *evaluations.*

1.1. Theorem. *Let E be a reflexive Banach space such that $\mathcal{P}_f({}^mE)$ is dense in $\mathcal{P}({}^mE)$ for every $m \in \boldsymbol{N}$. Let U be a convex, balanced, open subset of E. Then each continuous homomorphism $T : \mathcal{H}_b(U) \to \boldsymbol{C}$ is an evaluation.*

Proof. The technique of the proof goes back to a paper of Isidro [17, Props. 3 and 4], or even to an earlier paper of Dineen [9, Prop. 24], and has been the model for the proofs of results of this kind. Since T is continuous, there are $n \in \mathbf{N}$ and $c > 0$ such that

$$(1) \qquad |T(f)| \leq c||f||_{U_n}$$

for every $f \in \mathcal{H}_b(U)$. In particular $T|\mathcal{P}({}^mE)$ is continuous for every $m \in \mathbf{N}$. Since E is reflexive, there is a unique $a \in E$ such that $T(f) = f(a)$ for every $f \in E'$, and therefore for every $f \in \mathcal{P}_f({}^mE)$ and $m \in \mathbf{N}$. Since $\mathcal{P}_f({}^mE)$ is dense in $\mathcal{P}({}^mE)$, it follows that $T(f) = f(a)$ for every $f \in \mathcal{P}(E)$. A routine argument shows that we may assume that $c = 1$ in (1). Then it follows that $a \in U_n^{\circ\circ} = \overline{U_n} \subset U$. Since $\mathcal{P}(E)$ is dense in $\mathcal{H}_b(U)$, it follows that $T(f) = f(a)$ for every $f \in \mathcal{H}_b(U)$, as asserted.

We should mention that the first result of this kind was obtained by Michael in 1968. That result is stated without proof in the introduction to the second printing of his celebrated memoir [18].

Theorem 1.1 has the following converse.

1.2. Theorem. *Let U be a convex, balanced, open subset of a Banach space E. If each continuous homomorphism $T : \mathcal{H}_b(U) \to \boldsymbol{C}$ is an evaluation, then E is reflexive and every $P \in \mathcal{P}(E)$ is weakly continuous on bounded sets.*

Proof. We follows an argument of Aron, Cole and Gamelin [5, Th. 7.2]. Let A_n denote the normed algebra $(\mathcal{H}_b(U), ||\cdot||_{\overline{U_n}})$, and let $S(A_n)$ denote the spectrum of A_n. It follows from the hypothesis that

$$S(A_n) \subset S(\mathcal{H}_b(U)) = \{\delta_a : a \in U\}.$$

If $T = \delta_a \in S(A_n)$, then $|f(a)| = |T(f)| \leq ||f||_{\overline{U_n}}$ for every $f \in \mathcal{H}_b(U)$, and therefore $a \in \overline{U}_n^{\circ\circ} = \overline{U_n}$. Thus

$$S(A_n) = \{\delta_a : a \in \overline{U_n}\}.$$

Since $S(A_n)$ is compact for the Gelfand topology, we see that $\overline{U_n}$ is compact for the weak topology $\sigma(U, \mathcal{H}_b(U))$. Hence the weak topologies $\sigma(U, \mathcal{H}_b(U))$ and $\sigma(E, E')$ induce the same topology on each $\overline{U_n}$. Thus each $\overline{U_n}$ is weakly compact, and each $f \in \mathcal{H}_b(U)$ is weakly continuous on each $\overline{U_n}$. Since $B(0; 2^{-n}) \subset U_n$ for n large enough, we conclude that E is reflexive, and each $P \in \mathcal{P}(E)$ is weakly continuous on bounded sets.

When $U = E$, Theorems 1.1 and 1.2 are due to Aron, Cole and Gamelin [5, Th. 7.2].

1.3. Remark. If E is a reflexive Banach space with the approximation property, then the following conditions are equivalent:

(a) $\mathcal{P}_f(^mE)$ is dense in $\mathcal{P}(^mE)$ for every $m \in \mathbf{N}$.

(b) Every $P \in \mathcal{P}(E)$ is weakly continuous on bounded sets.

(c) $\mathcal{P}(^mE)$ is reflexive for every $m \in \mathbf{N}$.

The equivalence (a) $\Leftrightarrow$ (b) follows from [6, Th. 2.9] and [7, Prop. 2.7]. The equivalence (a) $\Leftrightarrow$ (c) follows from [1, Th. 7]. In [22, Prop. 5.3] Ryan had claimed that conditions (b) and (c) are always equivalent, without any assumption on the approximation property. But his proof of the implication $(c) \Rightarrow (b)$ is incomplete, and until now it seems to be unknown whether this implication is true in general.

Following Farmer [10] we will say that a Banach space E is *polynomially reflexive* if $\mathcal{P}(^mE)$ is reflexive for every $m \in \mathbf{N}$.

1.4. Examples. (a) Tsirelson [27] constructed a reflexive Banach space X, with an unconditional basis, which contains no subspace isomorphic to any ℓ^p. Alencar, Aron and Dineen [2] proved that X is polynomially reflexive.

(b) $\widetilde{\otimes}^n_\pi X$ is a polynomially reflexive Banach space with a basis. Indeed, since X has a basis, $\widetilde{\otimes}^n_\pi X$ has a basis as well, by [14]. And $\mathcal{P}(^m\widetilde{\otimes}^n_\pi X)$ is reflexive, since

$$\mathcal{L}(^m\widetilde{\otimes}^n_\pi X) = (\widetilde{\otimes}^m_\pi(\widetilde{\otimes}^n_\pi X)))' = (\widetilde{\otimes}^{mn}_\pi X)' = \mathcal{L}(^{mn}X),$$

and $\mathcal{L}(^{mn}X)$ is reflexive, by [3]. Moreover Alencar, Aron and Fricke [3] have shown that $\widetilde{\otimes}^n_\pi X$ is not isomorphic to X when $n > 1$.

Let A be a commutative Fréchet algebra with a unit element, and let $x_1, \ldots, x_p \in A$. The *joint spectrum* of $x_1, \ldots, x_p$ is the set $\sigma(x_1, \ldots, x_p)$ of all $(\lambda_1, \ldots, \lambda_p) \in \mathbf{C}^p$ such that the elements $x_1 - \lambda_1, \ldots, x_p - \lambda_p$ generate a proper ideal. A theorem of Arens (see [4] or [21, Prop. 32.13]) asserts that

$$\sigma(x_1, \ldots, x_p) = \{(T(x_1), \ldots, T(x_p)) : T \in S(A)\}.$$

By combining Theorem 1.1 and Arens' theorem we get the following.

1.5 Theorem. *Let E be a reflexive Banach space such that $\mathcal{P}_f(^mE)$ is dense in $\mathcal{P}(^mE)$ for every $m \in \mathbf{N}$. Let U be a convex, balanced, open subset of E. Then given $f_1, \ldots, f_p \in \mathcal{H}_b(U)$, without common zeros, we can find $g_1, \ldots, g_p \in \mathcal{H}_b(U)$ such that $\sum_{j=1}^p f_j g_j = 1$.*

Proof. It follows from Theorem 1.1 that

$$(T(f_1), \ldots, T(f_p)) \neq (0, \ldots, 0)$$

for every $T \in S(\mathcal{H}_b(U))$. By Arens' theorem the point $(0, \ldots, 0)$ does not lie in the joint spectrum of $f_1, \ldots, f_p$. Hence the functions $f_1, \ldots, f_p$ generate the improper ideal.

As a further application of Theorem 1.1 we obtain the following.

1.6. Theorem. *Let E be a reflexive Banach space such that $\mathcal{P}_f(^mE)$ is dense in $\mathcal{P}(^mE)$ for every $m \in \boldsymbol{N}$. Let U be a convex, balanced, open subset of E, and let V be an open subset of a Banach space F. Then for each continuous homomorphism $T : \mathcal{H}_b(U) \to \mathcal{H}_b(V)$ there exists a unique mapping $H : V \to U$ such that $T(f) = f \circ H$ for every $f \in \mathcal{H}_b(U)$. The mapping H belongs to $\mathcal{H}_b(V; E)$.*

Proof. If $y \in V$, then the mapping $\delta_y \circ T : \mathcal{H}_b(U) \to \mathbf{C}$ is a continuous homomorphism. Thus by Theorem 1.1 there exists a unique point $H(y) \in U$ such that $\delta_y \circ T(f) = f(H(y))$ for every $f \in \mathcal{H}_b(U)$. Thus we have found a mapping $H : V \to U$ with the property that

$$(2) \qquad T(f) = f \circ H$$

for every $f \in \mathcal{H}_b(U)$, and in particular for every $f \in E'$. Uniqueness of H follows at once from (2). It follows also from (2) that $f \circ H$ is holomorphic for every $f \in E'$. Thus H is holomorphic, by [21, Theorem 8.12]. Finally if B is a V-bounded set, then it follows also from (2) that $H(B)$ is weakly bounded, and therefore bounded. Thus $H \in \mathcal{H}_b(V; E)$, as asserted.

The mappings of the form $f \in \mathcal{H}_b(U) \to f \circ H \in \mathcal{H}_b(V)$ are called *composition operators*. They have been systematically studied in [12], [13], [15] and [16].

2. Holomorphic Functions on Tsirelson's Space

Theorem 1.5 was derived from Theorem 1.1 with the aid of Arens' theorem. To obtain a similar result for $\mathcal{H}(U)$ we have to proceed differently, for $\mathcal{H}(U)$ is not a Fréchet algebra. In order to apply Arens' theorem we will introduce certain Fréchet subalgebras of $\mathcal{H}(U)$, which are akin to $\mathcal{H}_b(U)$.

Let U be an open subset of a Banach space E, and let $\mathcal{U} = (U_n)_{n=1}^{\infty}$ be an increasing sequence of bounded, open subsets of E such that $U = \cup_{n=1}^{\infty} U_n$ and $d_{U_{n+1}}(U_n) > 0$ for every $n \in \mathbf{N}$. Then we can easily find $(\rho_n)_{n=1}^{\infty}$, such that $\rho_n > 1$ and $\rho_n U_n \subset U_{n+1}$ for every $n \in \mathbf{N}$. Let $\mathcal{H}^{\infty}(\mathcal{U})$ denote the algebra of all $f \in \mathcal{H}(U)$ which are bounded on each U_n. $\mathcal{H}^{\infty}(\mathcal{U})$ is a Fréchet algebra for the topology of uniform convergence on the sets U_n. As in the case of $\mathcal{H}_b(U)$ one can readily see that $\mathcal{P}(^mE)$ is a complemented subspace of $\mathcal{H}^{\infty}(\mathcal{U})$ for every $m \in \mathbf{N}$.

Assume that U is balanced and each U_n is circular. Then it follows again from the Cauchy inequalities that the Taylor series of each $f \in \mathcal{H}^{\infty}(\mathcal{U})$ at the origin converges uniformly on each U_n. In particular $\mathcal{P}(E)$ is dense in $\mathcal{H}^{\infty}(\mathcal{U})$. Then the proofs of Theorems 1.1 and 1.5 can be modified to yield the following analogous result.

2.1. Lemma. *Let E be a reflexive Banach space such that $\mathcal{P}_f(^mE)$ is dense in $\mathcal{P}(^mE)$ for every $m \in \boldsymbol{N}$. Let U be a convex, balanced, open subset of E. Let $\mathcal{U} = (U_n)_{n=1}^{\infty}$, be an increasing sequence of circular, bounded, open subsets of E such that $U = \cup_{n=1}^{\infty} U_n$ and $d_{U_{n+1}}(U_n) > 0$ for every $n \in \boldsymbol{N}$. Then:*

(a) Each continuous homomorphism $T : \mathcal{H}^{\infty}(\mathcal{U}) \to \boldsymbol{C}$ is an evaluation.

(b) Given $f_1, \ldots, f_p \in \mathcal{H}^{\infty}(\mathcal{U})$, without common zeros, we can find $g_1, \ldots, g_p \in \mathcal{H}^{\infty}(\mathcal{U})$ such that $\sum_{j=1}^{p} f_j g_j = 1$.

The increasing open covers $\mathcal{U} = (U_n)_{n=1}^{\infty}$ such that $d_{U_{n+1}}(U_n) > 0$ for every $n \in \mathbf{N}$, are called *regular covers* in [21]. They have been systematically utilized by Schottenloher in [23] and [24].

The following lemma is essentially known (see [23, 2.7]), but we include it here for the convenience of the reader.

2.2. Lemma. *Let U be a balanced, open subset of a Banach space E. Then each bounded subset $\mathcal{F}$ of $\mathcal{H}(U)$ is contained and bounded in $\mathcal{H}^{\infty}(\mathcal{U})$, for a suitable increasing sequence $\mathcal{U} = (U_n)_{n=1}^{\infty}$ of circular, bounded, open subsets of E such that $U = \cup_{n=1}^{\infty} U_n$ and $d_{U_{n+1}}(U_n) > 0$ for every $n \in \boldsymbol{N}$.*

Proof. Consider the sets

$$V_n = \text{int}\{x \in U : ||x|| < n \quad \text{and} \quad |f(\lambda x)| < n \quad \text{for} \quad f \in \mathcal{F} \quad \text{and} \quad |\lambda| \leq 1\},$$

$$U_n = \{x \in V_n : d_{V_n}(x) > 2^{-n}\}.$$

Since every bounded subset of $\mathcal{H}(U)$ is locally bounded, it follows that $U = \cup_{n=1}^{\infty} V_n$, and therefore $U = \cup_{n=1}^{\infty} U_n$. Clearly $d_{U_{n+1}}(U_n) \geq 2^{-(n+1)}$ for every $n \in \mathbf{N}$. Since U is balanced, each V_n is balanced as well, and hence it follows that each U_n is circular.

2.3. Theorem. *Let E be a reflexive Banach space such that $\mathcal{P}_f(^mE)$ is dense in $\mathcal{P}(^mE)$ for every $m \in \boldsymbol{N}$. Let U be a convex, balanced, open subset of E. Then given $f_1, \ldots, f_p \in \mathcal{H}(U)$, without common zeros, we can find $g_1, \ldots, g_p \in \mathcal{H}(U)$ such that $\sum_{j=1}^{p} f_j g_j = 1$.*

Proof. By Lemma 2.2 $f_1, \ldots, f_p \in \mathcal{H}^{\infty}(\mathcal{U})$ for a suitable increasing sequence $\mathcal{U} = (U_n)_{n=1}^{\infty}$ of circular, bounded, open subsets of E such that $U = \cup_{n=1}^{\infty} U_n$ and $d_{U_{n+1}}(U_n) > 0$ for every $n \in \mathbf{N}$. By Lemma 2.1 we can find $g_1, \ldots, g_p \in \mathcal{H}^{\infty}(\mathcal{U}) \subset \mathcal{H}(U)$ such that $\sum_{j=1}^{p} f_j g_j = 1$.

We have established our main results for convex, balanced, open sets, and we do not know whether the results are valid for more general domains, for instance for polynomially convex open sets. This would require some strong form of polynomial approximation to replace Taylor series expansions.

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