The Electric Field Outside a Stationary Resistive Wire Carrying a Constant Current

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Abstract

We present the opinion of some authors who believed there was no force between a stationary charge and a stationary resistive wire carrying a constant current. We show that this force is different from zero and present its main components: The force due to the charges induced in the wire by the test charge and a force proportional to the current in the resistive wire. We also discuss briefly a component of the force proportional to the square of the current which should exist according to some models and another component due to the acceleration of the conduction electrons in a curved wire carrying a dc current (centripetal acceleration). Finally we analyse experiments showing the existence of the electric field proportional to the current in resistive wires.

Key Words: Force of induction, surface charges, motional electric field.

PACS: 41.20.-q (Electric, magnetic, and electromagnetic fields), 72.15.-v (Electronic conduction in metals and alloys)

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1 Introduction

Consider a circuit like that of Figure 1, where a stationary resistive wire connected to a battery carries a constant current I. Will it exert a force on a stationary charge q located nearby?

One force which will be there regardless of the value of the current is that due to the induced charges in the wire. That is, the point particle q induces a distribution of charges in the conducting wire and the net result will be an attraction between the wire and q. Most authors know about this fact, although forgetting to mention it. Moreover, they do not consider it in detail nor give its order of magnitude.

Is there any other force between the wire and the stationary charge? Many physicists believe the answer to this question is no, and this opinion has been held for a long time. There are three main ideas leading to this belief. We analyze each one of them here.

(A) The first idea is related to the supposition that a stationary resistive wire carrying a constant current is essentially neutral in its interior and along its surface. And this leads to the idea that a resistive current carrying wire generates only a magnetic field outside it. For more than a century scientists have been used to believe in this statement. Clausius, for instance, based all his electrodynamics on this belief. In 1877 he wrote: "We accept as criterion the experimental result that a closed constant current in a stationary conductor exerts no force on stationary electricity" (quoted in [1, page 589]). Although he affirmed that this is an experimental result, he didn't cite any experiments which tried to find this force. His electrodynamics led to this prediction: "The law formulated by me leads to the result that a constant stationary closed circuit exercises no force on a stationary charge" (Clausius statement in 1880 as quoted in [1, page 589]). As we will see, he based his electrodynamics in a wrong principle as there is a force between a stationary charge and a stationary wire carrying a constant current. This force has been shown by Jefimenko's experiment, [2, pages 299-319 and 509-511], and confirmed by our calculations.

Even in electromagnetic textbooks we can find statements like this. As we will see, the electric field inside and outside a resistive wire carrying a constant current is due to surface charges distributed along the wire. On the other hand, Reitz and Milford, for instance, seem to say that no steady surface charges can exist in resistive wires ([3], pp. 128-129): "Consider a conducting specimen obeying Ohm's law, in the shape of a straight wire of uniform cross section whose ends are maintained at a constant potential difference $\triangle U$. The wire is assumed to be homogeneous and characterized by the constant conductivity g. Under these conditions an electric field will exist in the wire, the field being related to $\triangle U$ by the relation $\triangle U = \int \vec{E} \cdot d\vec{\ell}$. It is evident that there can be no component of electric field at right angles to the axis of the wire, since by $\vec{J} = g\vec{E}$ this would produce a charging of the wire's surface. As was mentioned earlier, excess charge is dissipated rapidly in a conductor, and because of the low potential energy sink for charge carriers at one end of the wire, not even a surface charge can be tolerated." In this paper we consider exactly this situation and show not only the existence of these surface charges which they are rejecting, but that they are the charges which generate the electric field inside the resistive wire. The role of the battery is to keep the longitudinal distribution of surface charges constant in time for dc currents and to avoid the neutralization of the wire. But it is not the chemical battery itself which creates the electric field at all points in space. This electric field is created by the charges distributed along the surface of the wire. These surface charges will generate the longitudinal electric field inside the resistive wire and also an electric field outside the wire, with longitudinal and radial components. Moreover, when considering the radial Hall effect, we also show that there will be a radial component of the electric field inside the resistive current carrying wire, contrary to their statement. This radial electric field is due to the fact that the wire is negatively charged in its interior.

In Jackson's book, [4], there is the following statement in exercise 14.13, page 697: "As an idealization of steady-state currents flowing in a circuit, consider a system of N identical charges q moving with constant *speed* v (but subject to acceleration) in an arbitrary closed path. Successive charges are separated by a constant small interval $\triangle$. Starting with the Liénard-Wiechert fields for each particle, and making no assumptions concerning the speed v relative to the velocity of light show that, in the limit $N \to \infty$, $q \to 0$, and $\triangle \to 0$, but Nq = constant and $q/\triangle$ = constant, no radiation is emitted by the system and the electric and magnetic fields of the system are the usual static value. (Note that for a real circuit the stationary positive ions in the conductors will produce an electric field which just cancels that due to the moving charges.)" Any person reading this statement, especially the sentence in parenthesis, will conclude that Clausius was right. However, we will see here that there is a net electric field different from zero outside a stationary resistive wire carrying a steady current. Despite the words in this exercise, it must be stressed that Jackson himself is aware of this electric field outside wires carrying steady currents, see [5].

Here are the words of Edwards, Kenyon and Lemon, [6], related to first order terms, that is, to forces proportional to v_d/c or to the drifting velocity of the moving charges in the wire divided by c: "It has long been known that the zero- and first- order forces on a charged object near a charge- neutral, current-carrying conductor at rest in the laboratory are zero in magnitude." Jefimenko's experiment and our calculations show that a normal resistive wire carrying a constant current cannot be charge neutral. Moreover, it will generate zero-order and first-order forces on a charged object at rest near it, namely: the induction force F_0 and the first order force F_1 (see below).

One of us also assumed in previous works that a conducting wire is essentially neutral at all points, see: [7], [8] and [9, pp. 85 and 161]. Here we show in details that this is not valid for normal resistive wires carrying constant currents.

(B) The second idea leading to the conclusion that a normal resistive current carrying wire generates no electric field outside it arises from the supposition that magnetism is a relativistic effect. A typical representative of this position can be found in *Feynman's Lectures on Physics*, [10, Section 13-6: The relativity of magnetic and electric fields, p. 13-7], our emphasys: "We return to our atomic description of a wire carrying a current. **In a normal conductor, like copper**, the electric currents come from the motion of some of the negative electrons - called the conduction electrons - while the positive nuclear charges and the remainder of the electrons stay fixed in the body of the material. We let the density of the conduction electrons be ρ_- and their velocity in S be $\mathbf{v}$. The density of the charges at rest in S is ρ_+, which must be equal to the negative of ρ_-, since we are considering an uncharged wire. **There is thus no electric field outside the wire**, and the force on the moving particle is just $\mathbf{F} = q\mathbf{v_o} \times \mathbf{B}$."

In Purcell's *Electricity and Magnetism* we can find the same ideas. In Section 5.9 of this book, which considers magnetism as a relativistic phenomenon, he models a current carrying wire by by two strings of charges, positive and negative, moving relative to one another. He then consider two current carrying metallic wires at rest in the frame of the laboratory and says (p. 178): "In a metal, however, only the positive charges remain fixed in the crystal lattice. Two such wires carrying currents in opposite directions are seen in the lab frame in Fig. 5.23a. The wires being neutral, there is no electric force from the opposite wire on the positive ions which are stationary in the lab frame." That is, he believes there will be no electric field generated by the stationary current carrying resistive wire in any point outside itself.

Other books present similar statements, so that we will not quote them here.

(C) The third kind of idea related to this widespread belief is related to Weber's electrodynamics. As we shall see, even if a resistive current carrying wire were neutral at all points in its interior and along its surface, Weber's electrodynamics predicts that it would exert a net force on a point charge at rest outside it. This force is proportional to v_d^2/c^2, where v_d is the drifting velocity of the conduction electrons and $c = 3 \times 10^8 ms^{-1}$. Based on the wrong belief (see below) that this wire exerts no force on a stationary charge nearby, unware even of the larger first order electric field proportional to v_d, many authors condemned Weber's law as experimentally invalidated.

This goes back at least to Maxwell's *Treatise on Electricity and Magnetism*. He was considering the force between a conducting wire carrying a constant current and another wire which carries no current, both of them at rest in the laboratory. He then said, see [11, Volume 2, Article 848, page 482] (between square brackets are our words): "Now we know that by charging the second conducting wire as a whole, we can make $e' + e'_1$ [net charge on the wire without current] either positive or negative. Such a charged wire, even without a current, according to this formula [based on Weber's electrodynamics], would act on the first wire carrying a current in which $v^2 e + v_1^2 e_1$ [sum of the positive and negative charges of the current carrying wire by the square of their drifting velocities] has a value different from zero. Such an action has never been observed." As with Clausius comment, Maxwell did not quote any experiments which tried to observe this force (and which failed to find the effect), the upper limit of this effect etc.

Writing in 1951 Whittaker criticized Weber's electrodynamics along the same lines ([12, page 205], our emphasys): "The assumption that positive and negative charges move with equal and opposite velocities relative to the matter of the conductor is one to which, for various reasons which will appear later, objection may be taken; but it is an integral part of Weber's theory, and cannot be excised from it. In fact, if this condition were not satisfied, and if the law of force were Weber's, electric currents **would exert** forces on electrostatic charges at rest (...)". Obviously he is here expressing the view that there are no such forces. By consequence, Weber's electrodynamics must be wrong according to Whittaker's view, because we now know that only the negative electrons move in metallic wires. And applying Weber's electrodynamics to this situation (in which a current in a metallic conductor is due to the motion of conduction electrons, while the positive charges of the lattice remain stationary) implies that a conducting wire should exert force on a stationary electric charge nearby. Whittaker seems to be unware of the experimental fact that *electric currents exert forces on electrostatic charges at rest*, see the experiments by Jefimenko discussed below.

Other examples of this widespread belief: In 1969 Skinner said, relative to Figure 2 in which the stationary closed circuit carries a constant current and there is a stationary charge at P ([13, page 163]): "According to Weber's force law, the current of Figure 2.39 [Figure 2] would exert a force on an electric charge at rest at the point P. (...) And yet a charge at P does not experience any force." As with Clausius's and Maxwell's generic statements, Skinner did not quote any specific experiment which tried to find this force.

Pearson and Kilambi, in a paper discussing the analogies between Weber's electrodynamics and nuclear forces, made the same kind of criticisms in a Section called "Invalidity of Weber's electrodynamics," [14]. They consider a straight wire carrying a constant current. They calculate the force on a stationary charge nearby due to this wire with classical electromagnetism and with Weber's law, supposing the wire to be electrically neutral at all points. According to his calculations, classical electromagnetism does not yield any force on the test charge and he interprets this as (our emphasis): "The vanishing of the force on the stationary charge q corresponds simply to the **fact** that a steady current does not give rise to any induced electric field." With Weber's law he finds a second order force and interprets this as meaning (our emphasis): "that Weber's electrodynamics give rise to **spurious** induction effects. This is probably the most obvious defect of the theory, and the only way of avoiding it is to suppose that the positive charges in the wire move with an equal velocity in the opposite direction, which of course they do not." As we will see, the **fact** is that a steady current give rise to an induced electric field, as shown by Jefimenko's experiment.

In this work we argue that all of these statements were misleading. That is, we show the existence of a force on the stationary charge proportional to the current in a resistive stationary wire carrying a constant current. We also compare our calculations with Jefimenko's experiment, see below, which proved the existence of this force.

2 Geometry of the Problem

In this work the frame of reference will always be the laboratory. The situation considered here is that of a cylindrical conducting resistive wire of length ℓ and radius $a \ll \ell$, Figure 3. The axis of the wire coincides with the z direction, with $z = 0$ at the center of the wire. A battery maintains constant potentials at the extremities $z = -\ell/2$ and $z = +\ell/2$ of the wire given by ϕ_L and ϕ_R, respectively. The wire carries a constant current I, has a finite conductivity g and is at rest relative to the laboratory. There is air or vacuum outside the wire. At a distance r to the axis of the wire there is a stationary point charge q. We want to know the force exerted by the wire on q in the following approximation:

$$\ell \gg r > a \text{ and } \ell \gg |z| \ , \tag{1}$$

where z is the longitudinal component of the vector position of q. We utilize throughout this paper cylindrical coordinates (r, φ, z) with $r = \sqrt{x^2 + y^2}$ and unit vectors $\hat{r}$, $\hat{\varphi}$ and $\hat{z}$.

This wire must be closed somewhere. The calculations presented here with this approximation should be valid for the circuit of Figure 4 (square circuit of side ℓ with a wire of radius $a \ll \ell$, with a point charge close to the middle of one of its sides and far from the battery). That is, the three other sides will not contribute significantly to the potential and field near the center of the fourth side. Alternatively, it should also give approximate results for a circular loop of larger radius $R = \ell/2\pi$ and smaller radius $a \ll R$ (a ring) if the point charge is at a distance $R + r$ to the center of the wire, such that $a < r \ll R$. It might even be utilized as a first gross approximation for the force on the point charge of Figure 1 considering a generic circuit of large length and small curvatures (that is, with radii of curvature much larger than the diameter of the wire and also much larger than the distance of the point charge to the wire).

We consider separetely three components of the force exerted by the wire on q: That due to the charges induced in the wire by q, that due to the surface charges which exist in resistive current carrying wires (proportional to the current or to the drifting velocity v_d of the electrons) and that due to v_d^2/c^2.

3 Induction Force

Consider a neutral conductor carrying no current. If we put a point particle q nearby, it will induce a distribution of charges in the conductor such that the potential anywhere inside it will reach a constant value in equilibrium. The net effect of these induced charges is an attraction between q and the conductor.

We can estimate its value for the situation of Figure 3 in the case $\ell \gg r \gg a$ without any calculation. This situation is equivalent to the force between a point charge at a distance r to an infinite conducting line. As there is only one charge and one distance involved in this problem, dimensional analysis requires the force between the point charge and the infinite conducting line to be given by

$$\vec{F}_0 = -\alpha_L \frac{q^2}{4\pi\varepsilon_o} \frac{\hat{r}}{r^2} \ , \ 0 < \alpha_L < 1 \tag{2}$$

where $\hat{r}$ is the unit vector pointing away from the line to the charge q and α_L is a positive dimensionless constant of the order of unity. It would be one if all the induced charge were located at the origin, that is, at a distance r to q. As part of the induced charge will be distributed along the wire with a linear charge density $\lambda(z)$, which means at a distance to q greater than r, we conclude that α_L must be smaller than one. Although we don't know the exact value of α_L, we know the order of magnitude of the induction force.

An analogous analysis might be performed for the induction force between a point charge q at a distance r from an infinite plane. As before, there is only one charge and one distance involved in this problem, so that the force must be given by Eq. (2) with a dimensionless constant α_P replacing α_L (as we now have an infinite plane instead of an infinite line, the dimensionless constant does not need to be the same). But in this case we can easily solve exactly the problem by the method of images. The final solution yields in this case an image charge $-q$ at the other side of the plane, also at a distance r to it. As the distance between q and $-q$ is $2r$, this yields $\alpha_P = 1/4$. This shows that our reasoning without performing any calculation was correct.

Suppose now we have the case of Figure 3, but now with r being of the same order of magnitude as a. As there is only one charge and two distances involved in the problem (considering ℓ going to infinity), the force must be given by $\vec{F}_0 = -h(r,a)q^2\hat{r}/4\pi\varepsilon_o$. Here $h(r,a)$ is a function of r and a such that if $r \gg a$ it will be proportional to $1/r^2$ and if $r \to a$ it diverges to infinity as this is the general behaviour of induction forces (if the charge approaches an infinite plane or the surface of a conducting sphere the induction force always goes to infinity).

We have then estimated the value of the induction force in the case of figure 3, for $\ell \gg r \gg a$, as given by Eq. (2). This estimative is ours, as we were unable to locate it anywhere in the literature. This force will be there irrespective of whether or not there is current in the wire. For an order of magnitude, suppose a charge generated by friction of $10^{-9}C$, at a distance of $10cm$ from a long thin wire. The induction force in this case should be of the order of $10^{-6}N$.

In the sequence we consider the influence of the current on the net force exerted by the wire on q.

4 Force Proportional to the Current

When a current flows in a resistive wire connected to a battery, the electric field driving the conduction electrons against the resistive friction of the wire is due to free charges distributed along the surface of the wire. We represent this surface charge density by $\sigma_f(a,\varphi,z)$. The battery creates and maintains this distribution of charges but does not generate the electric field along the circuit. This was first pointed out by Kirchhoff: [15], [16] and [17], with English translation in [18]. These surface charges generate not only the electric field inside the wire but also an electric field outside it.

However, most authors are not aware of these surface charges and related electric field outside the wire, as we can see from the quotations above. Fortunately this subject has been considered again in some important works: Heald, Jefimenko, Griffiths, Jackson and those quoted by them (see [19], [2, pages 299-319 and 509-511], [20, pages 279 and 336] and [5]). As none of them considered the geometry of Figure 3, we decided to analyse it here.

Our approach in this paper is the following: We consider the cylindrical wire carrying the constant current I and calculate the potential ϕ_1 and electric field $\vec{E}_1$ inside and outside the wire due to these surface charges in the absence of the test charge q. When we put the test charge at a distance r from the wire the force on it due to the surface charges will be then given by $\vec{F}_1 = q\vec{E}_1$, supposing that it is small enough such that it does not disturb the current nor the wire (except from the induction charges already considered above). We begin calculating the potential due to the surface charges.

As there is a constant current in the wire, the electric field inside it and driving the current must be constant over the cross section of the wire, neglecting the small radial Hall effect inside the wire due to the poloidal magnetic field generated by the current. This means that the potential and surface charge distribution must be a linear function of z, [21]. Due to the axial symmetry of the wire it cannot depend on the poloidal angle either. This means that $\sigma_f(a,\varphi,z) = \sigma_A z/\ell + \sigma_B$, where σ_A and σ_B are constants. Due to this axial symmetry we can calculate ϕ at $\varphi = 0$ and then generalize the solution to all φ. The potential inside or outside the wire is then given by

$$\phi_1(r,z) = \frac{1}{4\pi\varepsilon_o}\int_{\varphi_2=0}^{2\pi}\int_{z_2=-\ell/2}^{\ell/2}\frac{\sigma_f a d\varphi_2 dz_2}{\sqrt{r^2+a^2-2ra\cos\varphi_2+(z_2-z)^2}}$$

$$= \frac{1}{4\pi\varepsilon_o}\int_{\varphi_2=0}^{2\pi}\int_{z_2=-\ell/2}^{\ell/2}\frac{(\sigma_A z_2/\ell+\sigma_B)d\varphi_2 dz_2}{\sqrt{\left(1-2\frac{r}{a}\cos\varphi_2+\frac{r^2}{a^2}\right)+\left(\frac{z_2-z}{a}\right)^2}} \ . \tag{3}$$

Defining the dimensionless variables $s^2 \equiv 1-2(r/a)\cos\varphi_2+(r^2/a^2)$ and $u \equiv (z_2-z)/a$ we are then led to: $\phi_1(r,z) = (a/4\pi\varepsilon_o)[(\sigma_A a/\ell)I_1 + (\sigma_A z/\ell+\sigma_B)I_2]$, where

$$I_1 \equiv \int_{\varphi_2=0}^{2\pi}\int_{u=-(\ell/2a+z/a)}^{\ell/2a-z/a} u\frac{d\varphi_2 du}{\sqrt{s^2+u^2}} \ , \tag{4}$$

and

$$I_2 \equiv \int_{\varphi_2=0}^{2\pi}\int_{u=-(\ell/2a+z/a)}^{\ell/2a-z/a} \frac{d\varphi_2 du}{\sqrt{s^2+u^2}} \ . \tag{5}$$

These integrals can be solved with the approximation (1), where we now allow r to be smaller or greater than a, yielding (see Appendix):

$$\phi_1(r,\varphi,z) = \frac{a\sigma_f(z)}{\varepsilon_o}\ln\frac{\ell}{a} = \frac{a(\sigma_A z/\ell + \sigma_B)}{\varepsilon_o}\ln\frac{\ell}{a} \text{ if } r \leq a \ , \tag{6}$$

$$\phi_1(r,\varphi,z) = \frac{a\sigma_f(z)}{\varepsilon_o}\ln\frac{\ell}{r} = \frac{a(\sigma_A z/\ell + \sigma_B)}{\varepsilon_o}\ln\frac{\ell}{r} \text{ if } r \geq a \ . \tag{7}$$

The coulombian force on a test charge q located at (r,φ,z) is then given by: (with $\vec{F}_1 = -q\nabla\phi_1$):

$$\vec{F}_1 = -\frac{qa}{\varepsilon_o}\frac{\partial\sigma_f(z)}{\partial z}\left(\ln\frac{\ell}{a}\right)\hat{z} = -\frac{qa\sigma_A}{\ell\varepsilon_o}\left(\ln\frac{\ell}{a}\right)\hat{z} \text{ if } r < a \ , \tag{8}$$

$$\vec{F}_1 = \frac{qa\sigma_f(z)}{\varepsilon_o}\frac{\hat{r}}{r} - \frac{qa}{\varepsilon_o}\frac{\partial\sigma_f(z)}{\partial z}\left(\ln\frac{\ell}{r}\right)\hat{z} =$$

$$= \frac{qa(\sigma_A z/\ell + \sigma_B)}{\varepsilon_o}\frac{\hat{r}}{r} - \frac{qa\sigma_A}{\ell\varepsilon_o}\left(\ln\frac{\ell}{r}\right)\hat{z} \text{ if } r \geq a \ . \tag{9}$$

We can relate these expressions with the current I flowing in the wire. From Figure 3 and the fact that ϕ_1 is a linear function of z yields:

$$\phi_1(r \leq a, z) = \frac{\phi_R - \phi_L}{\ell}z + \frac{\phi_R + \phi_L}{2} \ . \tag{10}$$

Equating this with Eq. (6) and utilizing Ohm's law $\phi_L - \phi_R = RI$, where $R = \ell/g\pi a^2$ is the resistance of the wire, with g being its conductivity, yields $\sigma_A = -R\varepsilon_o I/a\ln(\ell/a)$ and $\sigma_B = \varepsilon_o(\phi_R + \phi_L)/2a\ln(\ell/a) = \varepsilon_o(RI + 2\phi_R)/2a\ln(\ell/a)$. The density of free charges along the surface of the wire can then be written as:

$$\sigma_f(a,\varphi,z) = -\frac{R\varepsilon_o I}{a\ln(\ell/a)}z + \frac{\varepsilon_o(\phi_R + \phi_L)}{2a\ln(\ell/a)} \ . \tag{11}$$

This means that the potential and the force on the test charge q are given by:

$$\phi_1 = -\frac{RI}{\ell}z + \frac{\phi_R + \phi_L}{2} \text{ if } r \leq a \ , \tag{12}$$

$$\phi_1 = -\frac{RI}{\ell}\frac{\ln(\ell/r)}{\ln(\ell/a)}z + \frac{\phi_R + \phi_L}{2}\frac{\ln(\ell/r)}{\ln(\ell/a)} \text{ if } r \geq a \ , \tag{13}$$

$$\vec{F}_1 = q\frac{RI}{\ell}\hat{z} \text{ if } r < a \ , \tag{14}$$

$$\vec{F}_1 = q\left[\frac{-1}{\ln(\ell/a)}\left(\frac{RI}{\ell}z - \frac{RI + 2\phi_R}{2}\right)\frac{\hat{r}}{r} + \frac{RI}{\ell}\frac{\ln(\ell/r)}{\ln(\ell/a)}\hat{z}\right] \text{ if } r \geq a \ . \tag{15}$$

Now that we have obtained the potential outside the wire we might also revert the argument. That is, we might solve Laplace's equation $\nabla^2\phi = 0$

in cylindrical coordinates inside and outside the wire (for $a \leq r \leq \ell$) by the method of separation of variables imposing the following boundary conditions: finite $\phi(0, \varphi, z)$, $\phi(a, \varphi, z) = (\phi_R - \phi_L)z/\ell + (\phi_R + \phi_L)/2$ and $\phi(\ell, \varphi, z) = 0$. This last condition is not a trivial one and was obtained only after we found the solution in the order presented in this work. The usual boundary condition that the potential goes to zero at infinity does not work in the case of a long cylinder carrying a *dc* current. By this reverse method we obtain the potential inside and outside the wire, then the electric field by $\vec{E} = -\nabla\phi$ and lastly the surface charge density by ε_o times the normal component of the electric field outside the wire in the limit in which $r \to a$. In this way we checked our calculations.

If we put $\phi_L = \phi_R = \phi_o$ or $I = 0$ in Eqs. (12) to (15) we recover the electrostatic solution (long wire carried uniformly with a constant charge density σ_B), namely:

$$\phi(r \leq a) = \phi_o = \frac{a\sigma_B}{\varepsilon_o} \ln \frac{\ell}{a} \ , \tag{16}$$

$$\phi(r \geq a) = \phi_o \frac{\ln(\ell/r)}{\ln(\ell/a)} = \frac{a\sigma_B}{\varepsilon_o} \ln \frac{\ell}{r} \ , \tag{17}$$

$$\vec{F}_1(r < a) = 0 \ , \tag{18}$$

$$\vec{F}_1(r \geq a) = \frac{q\phi_o}{\ln(\ell/a)} \frac{\hat{r}}{r} = \frac{qa\sigma_B}{\varepsilon_o} \frac{\hat{r}}{r} \ . \tag{19}$$

We can also obtain the capacitance per unit length of this long and thin cylindrical wire as $C/\ell = (Q_B/\phi(a))/\ell = 2\pi\varepsilon_o/\ln(\ell/a)$.

This is the first time in the literature the potentials (7) or (13) and the forces (9) or (15) outside a cylindrical wire are calculated. Kirchhoff had obtained Eq. (6) but did not consider the fields and forces outside the wire (see [16], especially the last equation of page 400). Recently Coombes and Laue analysed the same problem, [22]. Their paper is fine but they arrived at Eqs. (6) and (8) believing they would be valid inside and outside the wire. This is evident from their statements in the paragraph below their Eq. (8), our emphasis: "Thus we obtain to the questions asked at the beginning the surprising answer that an infinitely long wire in which a steady current is flowing has a vanishing surface charge density $cz/R = [-E\varepsilon_o/R\ln(L/R)]z$ and a uniform electric field $\vec{E} = -\nabla\phi = -(c/\varepsilon_o)(\ln L/R)\hat{z}$ **both inside and outside the wire**." The reason for the discrepancy can be seen in their Eq. (A7) which is correct and represents the potential, namely (replacing their L, R and c by our equivalent ℓ, a and σ_A, remembering that they are considering the particular case in which $\sigma_B = 0$):

$$\phi(r, z) = \frac{\sigma_A z}{\varepsilon_o} \left(\ln \frac{\ell}{a} + \frac{1}{4\pi} \int_0^{2\pi} d\theta \ln \frac{4}{\sin^2\theta + (\cos\theta - r/a)^2} - 1 \right) \ . \tag{20}$$

Just after this equation they wrote: "For sufficiently large ℓ, Eq. (A7) [this equation] is dominated by the first term on the right-hand side, and we obtain formula $\phi(r,z) = (\sigma_A/\varepsilon_o)(\ln \ell/a)z$, with $\ell \gg a$, $\ell \gg z, r$." Their only mistake was to disregard the integral of Eq. (20). Its correct value can be obtained utilizing Eqs. (33) and (34) of our Appendix. If they had taken this into account they would have arrived at our Eqs. (6) to (9).

These expressions show that this force is proportional to the current in the wire. Moreover, there will be not only a tangential component of the electric field outside the wire but also a radial one. In the symmetric case in which $\phi_L = -\phi_R = RI/2$ the ratio of the radial component of $\vec{F}_1$ to the tangential component is given by $z/(r\ln(\ell/r))$. For a wire of $1m$ length and $z = r = 10cm$ we have this ratio as 0.4, indicating that these two components are of the same order of magnitude.

Schaeffer (reference in [5]), Sommerfeld, Marcus, Griffiths and Jackson considered the electric field due to a long coaxial cable of length ℓ carrying a constant current along the inner wire of resistivity g and radius a, returning along a hollow cylinder with inner radius b such that $\ell \gg b > a$: [23, pp. 125-130, Eq. (8)], [24], [20, pp. 336-337] and [5, Eq. (A17)]. In Sommerfeld's case the return conductor had finite conductivity and an external radius tending to infinity, while in Marcus, Griffiths and Jackson's case the return conductor was a cylindrical shell of radius b and zero resistivity. For all these authors the potential and electric field went to zero for $r > b$. Their solution in the region $a < r < b$ and considering the zero of the potential at $z = 0$ is given by:

$$\phi_{coaxial} = -\frac{I}{g\pi a^2}\frac{\ln(b/r)}{\ln(b/a)}z \ . \tag{21}$$

We now compare this solution with our Eq. (13) in this particular case in which $\phi_R + \phi_L = 0$. The main difference is the appearance in our case of $\ln(\ell/r)/\ln(\ell/a)$ instead of $\ln(b/r)/\ln(b/a)$. That is, while the potential and electric field outside the resistive current carrying wire (and also the force exerted by this wire on a point charge) depend on the length of the long wire, the same does not happen in the interior region of the coaxial cable near $z = 0$. If we keep a, g and I constant (and also b for the coaxial cable) and double the length of the wire (coaxial cable), the potential outside the wire will change, but not inside the coaxial cable. The two solutions will only coincide if we fix $b = \ell$. As this is not the general case, the two solutions are not equivalent to one another in all situations.

In the sequence we consider a force due to the square of the current.

5 Force Proportional to the Square of the Current

Up to now we have only considered the induction force and the force of the surface charges on the stationary test charge. We have not yet taken into account the force of the stationary lattice and mobile conduction electrons on the stationary test charge. We consider it here in this Section, analysing two different theoretical models. We first consider Lorentz's law or Liénard-Schwarzschild's force. In this case there are also components of the force exerted by a charge q_2 belonging to the current carrying circuit on q which depend on the square of the velocity of q_2, v_d^2, and on its acceleration. If we have a constant current, the acceleration of q_2 will be its centripetal acceleration due to any curvature in the wire, proportional to v_d^2/r_c, where r_c is the radius of curvature of the wire in each point. This might lead to a force proportional to v_d^2 or to I^2. However, it has been shown that if we have a closed circuit carrying a constant current, there is no net effect of the sum of all these terms on a stationary charge outside the wire. For a proof see [4, page 697, exercise 14.13] or [6].

We now consider Weber's electrodynamics, [9]. As stated above, we are neglecting the small radial Hall effect inside the wire due to the poloidal magnetic field generated by the current. This means that the interior of the wire can be considered essentially neutral. Despite this fact Weber's electrodynamics predicts a force exerted by this neutral part of the wire in a stationary charge nearby. The reason for this effect is that the force exerted by the mobile electrons on the stationary test charge is different from the force exerted by the stationary positive ions of the lattice on the test charge. One of us have already performed these calculations in related situations, see for instance [9, Section 6.6, pages 161-168], so that we present here only the final result. Once more we assume (1). For the situation of Figure 3, with a uniform current density $\vec{J} = (I/\pi a^2)\hat{z}$, the force on the test charge is given by:

$$\vec{F}_2 = -q\frac{Iv_d}{4\pi\varepsilon_o c^2}\frac{\hat{r}}{r} = -\frac{\mu_o}{4\pi^2}\frac{qI^2}{a^2en}\frac{\hat{r}}{r} \text{ if } r > a, \tag{22}$$

where v_d is the drifting velocity of the electrons. We also utilized $\mu_o = 4\pi \times 10^{-7}\ kgmC^{-2}$, $c^2 = 1/\varepsilon_o\mu_o$ and $v_d = I/\pi a^2 en$, where $e = 1.6 \times 10^{-19}C$ is the elementary charge and n is the number of free electrons per unit volume.

This force is proportional to the square of the current. The electric field $\vec{E}_2 = \vec{F}_2/q$ points towards the current, as if the wire had become negatively charged. Sometimes this second order field is called motional electric field.

If we have a bent wire carrying a constant current, Weber's electrodynamics predicts another component of the force exerted by this current on a stationary charge outside it, proportional to the acceleration of the conduction electrons. As we are supposing a constant current, the relevant acceleration here is the centripetal one proportional to v_d^2/r_c, where r_c is the radius of curvature of the wire at that location. This means that also this component of the force will be proportional to v_d^2 or to I^2. The order of magnitude is the same as the previous one.

6 Radial Hall Effect

Another simple question which might be asked is the following: Is a stationary resistive wire carrying a constant current electrically neutral in its interior and along its surface?

Most authors quoted in the Introduction would answer positively to this question as this was their reason for believing this wire would not generate any electric field outside itself. However, we already showed that there will be a longitudinal distribution of surface charges which will give rise to the longitudinal electric field inside the wire and also to an electric field outside it. Here we show that there will also be a radial electric field inside the wire due to the fact that its interior is negatively charged. As we saw in the Introduction, Reitz and Milford rejected explicitly this charge. But they were not alone in this. See, for instance, Griffiths statements in [20, p. 273]: "Within a material of uniform conductivity, $\nabla \cdot \mathbf{E} = (\nabla \cdot \mathbf{J})/\sigma = 0$ for steady currents (equation $\nabla \cdot \mathbf{J} = 0$), and therefore the charge density is zero. Any unbalanced charge resides on the *surface*."

We here consider the radial Hall effect due to the poloidal magnetic field inside the wire. As is usually considered, [12, p. 90], we will suppose the constant total current I to flow uniformly over the cross section of the cylindrical wire with a current density $J = I/\pi a^2$. With the magnetic circuital law $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_o I_C$, where C is the circuit of integration and I_C is the current passing through the surface enclosed by C, we obtain that the magnetic field inside and outside the wire is given by:

$$\vec{B}(r \leq a) = \frac{\mu_o I r}{2\pi a^2}\hat{\varphi} \ , \tag{23}$$

$$\vec{B}(r \geq a) = \frac{\mu_o I}{2\pi r}\hat{\varphi} \ . \tag{24}$$

The magnetic force on a conduction electron of charge $q = -e$ inside the wire, at a distance $r < a$ from the center and moving with drifting velocity $\vec{v} = -|v_d|\hat{z}$ is given by:

$$\vec{F} = q\vec{v} \times \vec{B} = -\frac{|\mu_o e v_d I r|}{2\pi a^2}\hat{r} \ , \tag{25}$$

This radial force pointing inwards will create a concentration of negative charges in the body of the conductor. In equilibrium there will be a radial force generated by these charges which will balance the magnetic force: $qE = qvB$. That is, there will be inside the wire, beyond the longitudinal electric field E_1 driving the current, a radial electric field pointing inwards given by:

$$\vec{E}_r(r \leq a) = -\frac{|\mu_o v_d I r|}{2\pi a^2}\hat{r} \ . \tag{26}$$

The longitudinal electric field inside the wire driving the current is given by $E_1 = RI/\ell$. In order to compare it with the magnitude of the radial electric field E_r due to the Hall effect we consider the maximum value of this last field very close to the surface of the wire, at $r \to a$: $E_r \to |\mu_o v_d I|/2\pi a$. This means that (with $R = \ell/g\pi a^2$):

$$\frac{|E_r|}{|E_1|} = \frac{|\mu_o v_d g a|}{2} \ . \tag{27}$$

For a typical copper wire ($v_d \approx 4 \times 10^{-3} ms^{-1}$ and $g = 5.7 \times 10^7 \Omega m$) with $1mm$ diameter this yields: $E_r/E_1 \approx 7 \times 10^{-5}$. This shows that the radial electric field is negligible compared to the longitudinal one.

By Gauss's law $\nabla \cdot E = \rho/\varepsilon_o$ we obtain that inside the wire there will be a constant negative charge density ρ_- given by: $\rho_- = -|Iv_d|/\pi a^2 c^2$. The total charge inside the wire is compensated by a positive charge spread over the surface of the wire with a constant surface density $\sigma_+ = |\rho_- a/2| = |Iv_d|/2\pi a c^2$. That is, the negative charge inside the wire in a small segment of length dz, $\rho_- \pi a^2 dz$, is equal and opposite to the positive charge along its surface, $\sigma_+ 2\pi a dz$. This means that the radial Hall effect will not generate any electric field outside the wire, only inside it. For this reason it is not relevant to the experiments discussed here. In any event it is important to clarify this effect.

In our analysis of the radial Hall effect we are not considering the motional electric field discussed above as it is not yet completely clear if it exists or not.

In conclusion we may say that the total surface charge density along the wire, not taking into account the motional electric field and the induction of charges in the conductor due to external charges, is given by the constant σ_+ added to the σ_f given by Eq. (11).

We now compare all three components of the electric field outside the wire with one another and discuss an important experiment related to this subject.

7 Discussion and Conclusions

Although many authors forget about the induction force when dealing with a current carrying wire interacting with an external charge, there is no doubt it exists. Comparing the three forces above, it is the only one which diverges as we approach the wire. If we are far away from the wire it falls as $1/r^2$, while the radial component of F_1 and F_2 fall as $1/r$.

We now compare the three components of this force in a particular example: A copper wire ($g = 5.7 \times 10^7 \Omega m$, $n = 8.5 \times 10^{28} m^{-3}$) with a length $\ell = 1m$ and diameter $1mm$ ($a = 5 \times 10^{-4} m$). The resistance of the wire is then given by $R = \ell/g\pi a^2 = 0.022\Omega$. With a potential difference between its extremities of $\phi_L - \phi_R = 1V$ this yields a current of $I = 44,8A$. The drifting velocity in this case amounts to $v_d = I/\pi a^2 en = 4 \times 10^{-3} ms^{-1}$. We will suppose moreover the symmetrical case in which $\phi_R = -\phi_L = -0.5V$. The test charge will be a typical one generated by friction, $q = 10^{-9}C$, at a distance of $r = 10cm = 0.1m$ to the wire. The magnitude of each one of the forces and their ratios are then given by (considering only the radial component of $\vec{F}_1$ and $z = r = 10cm$): $F_o \approx 10^{-6}N$, $F_1 \approx 10^{-10}N$, $F_2 \approx 10^{-16}N$ (in terms of electric field: $E_o \approx 10^3 N/C$,

$E_1 \approx 10^{-1} N/C$ and $E_2 \approx 10^{-7} N/C$), so that $F_o/F_1 \approx 10^4$, $F_o/F_2 \approx 10^{10}$ and $F_1/F_2 \approx 10^6$. This means that in this case $F_o \gg F_1 \gg F_2$ or $E_o \gg E_1 \gg E_2$.

Despite this fact the force $\vec{F}_1$ has already been observed in the laboratory by Jefimenko. He had an ingenious idea of utilizing grass seeds as test particles near current carrying wires. They are electrically neutral in normal state, so that they do not induce any charges in the conductor. On the other hand, they are easily polarized in the presence of an electric field, aligning themselves with it. The lines of electric field are then observed in analogy with iron fillings generating the lines of magnetic field. What we consider here is the result of his experiment as presented in Plate 6 of [2] (see also his Section 9-6: Electric field outside a current-carrying conductor, pages 299-305) and Figure 1 of [25]. The current was flowing in a circuit like that of our Figure 3, with symmetrical potentials: $\phi_R = -\phi_L$. He performed the experiment but did not make the calculations for this case. These calculations have been presented here. In order to compare our results with his experiments, we need to obtain the lines of electric field. We obtain this in the plane xz ($y = 0$). Any plane containing the z axis will yield a similar solution. We are looking for a function $\xi(r, z)$ such that

$$\nabla\xi(r, z) \cdot \nabla\phi(r, z) = 0 \ . \tag{28}$$

For $r < a$ we have ϕ as a linear function of z, such that ξ can be found proportional to r. We write it as $\xi(r < a, z) = -A\ell r$, with A as a constant. The equipotential lines $\phi(r, z) = constant$ can be written as $z_1(r) = K_1$, where K_1 is a constant (for each constant we have a different equipotential line). Analogously the lines of electric force will be given by $z_2(r) = K_2$, where K_2 is another constant (for each K_2 we have a different line of electric force). From Eq. (28) we get $dz_2/dr = -1/(dz_1/dr) = (\partial\phi/\partial z)/(\partial\phi/\partial r)$. Integrating this equation we can obtain $\xi(r, z)$. With Eq. (7) this yields the solution for $r > a$. We are then led to:

$$\xi(r, z) = -A\ell r \text{ if } r < a \ , \tag{29}$$

$$\xi(r, z) = Ar^2 \ln\frac{r}{\ell} - A\frac{r^2}{2} - Az^2 - 2Bz \text{ if } r > a \ , \tag{30}$$

where $A = (\phi_R - \phi_L)/\ell = -I/\pi g a^2$ and $B = (\phi_R + \phi_L)/2$. From these equations we can easily verify Eq. (28).

In order to compare these results with Jefimenko's experiment we need essentially the value of ℓ/a. From his plate 6 we get $\ell/a \approx 40/3$. The plot of the equipotentials between $z = -\ell/2$ and $\ell/2$ given by Eqs. (6) and (7) is given in Figure 5. A plot of the lines of electric force given by Eqs. (29) and (30) is given in Figure 6. This is extremely similar to Jefimenko's experiment (Plate 6 of [2] or Figure 1 of [25]), showing the correctness of our approach.

The example discussed here is important to show clearly the existence of the electric field outside a resistive wire carrying a constant current. It does not depend on a variable current (longitudinal acceleration of the electrons along the wire) nor on a centripetal acceleration of the electrons (due to any curvature in the wire). That is, this electric field will be there even if there were not any acceleration of the conduction electrons. In the case of a coaxial cable discussed by Sommerfeld and many others (see above), they have found an electric field only in the region between the cables, but not outside the return conductor. The reason for this is that they were considering a return conductor of infinite area (Sommerfeld) or of zero resistivity (Marcus, Griffiths and Jackson). For this reason it may not have been clear to many people that usually any current carrying resistive wire should generate an electric field outside it. We hope the calculations presented in this paper, coupled with Jefimenko's experiments, will make people aware of this electric field.

As regards those who consider magnetism as a relativistic effect, we have shown here that a resistive current carrying wire generates not only a magnetic field but also an electric field. As Jackson has shown, it is impossible to derive magnetic fields from Coulomb's law and the kinematics of special relativity without additional assumptions, [4, pp. 578-581] and [26].

It should also be mentioned that the magnetic field in this case is the usual poloidal field in the direction $\hat{\varphi}$, proportional to r for $r \leq a$ and to $1/r$ for $r \geq a$. It is orthogonal to $\vec{E}_1$ at all points in space. This means that Poynting's vector $\vec{S} = \vec{E} \times \vec{B}/\mu_o$ will follow the equipotential lines represented in Figure 5 when $\phi_R = -\phi_L$. This general behaviour of the lines of Poynting's vector was pointed out by Heald, [19]. As we can see from Figure 5, just outside the wire $\vec{S}$ is orthogonal to it only at $z = 0$. At all other points it is inclined relative to the z axis, at an angle θ with a tangent given by the ratio of the radial and longitudinal components of $\vec{E}_1$. As we have seen, just outside the wire this is given by: $\tan\theta = z/(a\ln(\ell/a))$. Many textbooks only consider an electric field outside the current carrying wire when discussing boundary conditions. As the longitudinal component of $\vec{E}$ is continuous at a boundary and must exist inside a resistive wire carrying a current, it must also exist just outside the wire. These authors then present Poynting's vector pointing radially inwards towards the wire (see, for instance, [27, pp. 180-181] and [10, p. 27-8]). There are two main things to comment here. In the first place, these drawings and statements suggest that this electric field should exist only close to the wire. In the second place, they indicate that these authors are not aware of the surface charges generating the field. As we have seen, it is only at one point that $\vec{S}$ will be orthogonal to the wire just outside it. This point is an exception and not the rule. The rule is that there will be a radial component which may be larger than the longitudinal one, pointing towards the wire or away from it. One of the effects of this radial component is that $\vec{S}$ will usually be inclined just outside the wire and not orthogonal to it.

The verification of the existence or not of the second order electric field is much more difficult due to its small order of magnitude (as compared with E_o and E_1). However, if the resistance of the wire goes to zero, σ_A also goes to zero. This means that in a superconductor there should not be the external electric field proportional to the current. Avoiding also the induction force, there remains in this case only the second order electric field. This was the approach utilized by Edwards, Kenyon and Lemon in their experiment, [6], which is the best one known to us analysing this effect. They found an electric field proportional to I^2, independent of the direction of the current, pointing towards the wire and with an order of magnitude compatible with that predicted by Weber's law. Despite this positive evidence more research is necessary before a final conclusion may be drawn related to this second order electric field, [9, Section 6.6, pages 161-168].

As we have seen, usually $F_0 \gg F_1 \gg F_2$. Moreover, F_0 and F_1 have been shown to exist experimentally. We can then disregard the criticisms of Maxwell, Whittaker and Skinner presented above against Weber's electrodynamics. That is, there is a force between the wire and q proportional to the current I, contrary to their statements. It is much more difficult to know if there is or not a second order component of this force proportional to v_d^2/c^2. Only future experiments taking all of these effects as F_0 and F_1 into account can decide the matter in this case.

In conclusion we may say that despite the widespread belief that a stationary resistive wire carrying a constant current exerts no force on a stationary charge, there will certainly be a component of this force due to the induced charges and another one proportional to the current in the wire, as comproved by these calculations and Jefimenko's experiment. The existence or not of a second order force still needs to be confirmed.

Acknowledgments:

The authors wish to thank Prof. Mark A. Heald for calling their attention to reference [22] and for suggestions in related problems. They thank also Prof. O. D. Jefimenko, Prof. J. D. Jackson, J. A. Hernandes and J. E. Lamesa for comments regarding the first version of this paper. A. J. M. wishes to thank FAPESP-Brazil for financial support.

Appendix

We now show how to calculate the integrals (4) and (5).

Applying approximation (1) in the limits of integration of I_1 and integrating it in u yields a zero value (as it is an odd function integrated between symmetric limits).

Integrating I_2 in u yields, applying (1) in its limits of integration:

$$I_2 = \int_0^{2\pi} d\varphi_2 \ln \frac{\sqrt{s^2 + (\ell/2a)^2} + (\ell/2a)}{\sqrt{s^2 + (\ell/2a)^2} - (\ell/2a)} \ . \tag{31}$$

Once more with approximation (1) this can be written as

$$I_2 = \int_0^{2\pi} d\varphi_2 \ln \frac{(\ell/a)^2}{s^2} = 4\pi \ln \frac{\ell}{a} - \int_0^{2\pi} \left[\ln \left(1 - 2\frac{r}{a} \cos\varphi_2 + \frac{r^2}{a^2} \right) \right] d\varphi_2 \ . \quad (32)$$

This last integral is equal to zero if $r \leq a$. If $r > a$ we can put r^2/a^2 in evidence and utilize once more this result to solve the last integral, namely:

$$\int_0^{2\pi} \left[\ln \left(1 - 2\frac{r}{a} \cos\varphi_2 + \frac{r^2}{a^2} \right) \right] d\varphi_2 = 0 \text{ if } r \leq a \ , \quad (33)$$

$$\int_0^{2\pi} \left[\ln \left(1 - 2\frac{r}{a} \cos\varphi_2 + \frac{r^2}{a^2} \right) \right] d\varphi_2 = 2\pi \ln \frac{r^2}{a^2} \text{ if } r \geq a \ . \quad (34)$$

This means that the final value of I_2 is found to be

$$I_2 = 4\pi \ln \frac{\ell}{a} \text{ if } r \leq a \ , \quad (35)$$

$$I_2 = 4\pi \ln \frac{\ell}{r} \text{ if } r \geq a \ . \quad (36)$$

Figure Captions

1. A resistive stationary wire connected to a battery and carrying a *dc* current I, with a stationary point charge q nearby.

2. A constant current flows in the closed wire and there is a point charge at P.

3. A cylindrical wire of length ℓ and radius $a \ll \ell$ carrying a constant current I. A point charge q is at a distance r to the axis of the wire, with a longitudinal component z relative to the center of the wire.

4. Square circuit of side ℓ made of a cylindrical wire of radius $a \ll \ell$, with a point charge close to the middle of one of its sides.

5. Equipotentials as given by Eqs. (6) and (7) with Jefimenko's value $\ell/a \approx 40/3$ and with $\sigma_B = 0$ (or $\phi_R = -\phi_L$).

6. Lines of electric force as given by Eqs. (29) and (30) with $\phi_R = -\phi_L$.

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