# RELATÓRIO DE PESQUISA 1994

## ESTIMATING LEAF AREA INDEX FOR CANOPIES USING COVERAGE PROCESSES

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**ABSTRACT** - One measure of canopies structure is the *leaf area index* (LAI). Direct measurements are hard to get and indirect measures are based on the relationship between radiation penetration and foliage area. In practice, a certain model has been used and this model is correct when the canopy is homogeneous. For non-homogeneous canopies, the model used subestimates LAI.

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# Estimating Leaf Area Index for Canopies using Coverage Processes

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**Abstract**

One measure of canopies structure is the *leaf area index* (LAI). Direct measurements are hard to get and indirect measures are based on the relationship between radiation penetration and foliage area. In practice, a certain model has been used and this model is correct when the canopy is homogeneous. For non-homogeneous canopies, the model used subestimates LAI.
Key words: Leaf area index, coverage process.

## 1 Introduction

The canopy structure represents a fundamental role in a great number of ecologic processes and plays important part in the description between vegetation and environment. A very important measure is *leaf area index* (LAI, leaf area by unit ground area). Direct measurements of canopy structure are tedious and hard to get in small canopies and virtually impossible in large forests and crops. Indirect techniques are based on the relationship between penetration of solar radiation and canopy structure. The fraction of sky seen through the canopy at several different angles is a mesure widely used and there are commercialized instruments built for this purpose, Wells and Norman (1991) analyse the performance of one of these instruments (model LAI-2000).

When radiation goes through the canopy, there is a certain chance of being intercepted by the foliage. This probability is proportional to the length of the path, foliage density and foliage orientation. Wells and Norman (1991) gives the following model: If the leafs are randomly distributed in the region crossed by the sun ray and their individual size is small compared to the canopy size, then a ray from a direction described by zenith angle $\theta$ and azimuth angle $\phi$ has a probability of non-interception $T(\theta,\phi)$ given by:

$$T(\theta,\phi) = \exp[-G(\theta,\phi)\,\mu\, S(\theta,\phi)] \tag{1}$$

where $G(\theta,\phi)$ is the fraction of foliage projected toward direction $(\theta,\phi)$, $\mu$ is foliage density (square meter foliage per cubic meter canopy) and $S(\theta,\phi)$ is the path length (meter) through the canopy. The model LAI-2000, under study by Wells and Norman (1991), used for indirect measurements averages over azimuth and the model utilized is:

$$G(\theta)\,\mu = \frac{-\ln(T(\theta))}{S(\theta)} \tag{2}$$

here all quatities described are azimuthal averages. In a full cover, homogeneous canopy, foliage density is related to leaf area index $L$ by canopy height $z$ and path length $S$ by

$$L = \mu z$$

$$S(\theta) = \frac{z}{\cos\theta}$$

and the final model is

$$T(\theta) = \exp[-K(\theta)L/\cos\theta] \tag{3}$$

where $K(\theta)$ is the contact frequency, related to the shape and orientation of the leafs.

We can see it is not always that (1) and (2) are equivalent. The objective of this work is to construct a probabilistic model for this indirect measurement and verify under which conditions model (3) is adequate.



## 2 Modelling through a Coverage Process

We can think of the indirect measurement of the canopy of trees as a coverage process $\mathcal{C}$ sectioned by a cone. Suppose $\xi$ is a nonhomogeneous Poisson process on $\mathbf{R}^2 \times [0,\infty)$ with intensity $\lambda(\mathbf{y},h)$. That is,

(a) $\xi$ is a pontual process on $\mathbf{R}^2 \times [0,\infty)$;
(b) The number of points in disjoint regions are independent random variables:
(c) If $A \subset \mathbf{R}^2 \times [0,\infty)$, then $\xi(A)$ = number of points in $A$ is a Poisson random variable with parameter $\int_A \lambda(\mathbf{y},h)\,dy\,dh$.

Let $\xi_1,\xi_2,\ldots$ be the points of the process $\xi$ and consider the coverage process

$$\mathcal{C} \equiv \{\xi_i + S_i, i \geq 1\} \tag{4}$$

where $S_i$ are the random shapes of the leafs (for the general theory of coverage processes see Hall (1988)). Assume that the leafs are isotropic (that is randomly and uniformily oriented) sets in $\mathbf{R}^3$. Consider the coverage process $\mathcal{C}$ sectioned by a line $L(\mathbf{x},\theta,\phi)$ that intercepts the point $(\mathbf{x},0)$ with zenith angle $\theta$ and azimuthal angle $\phi$, and define

$$T(\mathbf{x},\theta,\phi) = \begin{cases} 1, & \text{if } L(\mathbf{x},\theta,\phi)\cap\mathcal{C} = \emptyset \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Define $\pi(\mathbf{x},\theta,\phi)$ the perpendicular plane to $L(\mathbf{x},\theta,\phi)$ that passes through $(\mathbf{x},0)$. For each set $S \subset \mathbf{R}^3$ consider $S(\mathbf{x},\theta,\phi)$ as the projection of $S$ onto $\pi(\mathbf{x},\theta,\phi)$ and let $S^*(\mathbf{x},\theta,\phi) = \{\mathbf{z} \in \pi(\mathbf{x},\theta,\phi); \mathbf{x} \in \mathbf{z}+S(\mathbf{x},\theta,\phi)\}$, with the abuse of natation that $\mathbf{z} \in \pi(\mathbf{x},\theta,\phi)$ if $(\mathbf{z},h) \in \pi(\mathbf{x},\theta,\phi)$ for some $h$. Note that for each $S$, $S^*(\mathbf{x},\theta,\phi)$ is uniquely determined. Let $\mathcal{C}(\mathbf{x},\theta,\phi)$ be the coverage process obtained by projecting $\mathcal{C}$ onto $\pi(\mathbf{x},\theta,\phi)$.

$$\mathcal{C}(\mathbf{x},\theta,\phi) \equiv \{\xi_i(\mathbf{x},\theta,\phi) + S_i(\mathbf{x},\theta,\phi), i \geq 1\} \tag{6}$$

and $T(\mathbf{x},\theta,\phi) = 1$ if, and only if, $(\mathbf{x},0) \in \mathcal{C}(\mathbf{x},\theta,\phi)$. By the properties of the Poisson process, the projected process $\{\xi_i(\mathbf{x},\theta,\phi), i \geq 1\}$ in $\pi(\mathbf{x},\theta,\phi)$ is a non-homogeneous Poisson process with intensity

$$\gamma(\mathbf{z}) = \int_0^\infty \lambda(\mathbf{z} + u(\sin\cos\phi, \sin\theta\sin\phi), u\cos\theta)\,du \tag{7}$$

for $\mathbf{z} \in \pi(\mathbf{x},\theta,\phi)$. Moreover,

$$\begin{aligned} \mathbf{E}[T(\mathbf{x},\theta,\phi)] &= \mathbf{P}\{(\mathbf{x},0)\text{not covered by}\mathcal{C}(\mathbf{x},\theta,\phi)\} \\ &= \mathbf{P}\{\text{for all } i; (\mathbf{x},0) \notin \xi_i(\mathbf{x},\theta,\phi) + S_i(\mathbf{x},\theta,\phi)\} \end{aligned} \tag{8}$$

The expected number of sets of $\mathcal{C}(\mathbf{x},\theta,\phi)$ that contain $(\mathbf{x},0)$ is

$$\mathbf{E}(N) = \mathbf{E}\{\sum_i I[(\mathbf{x},0) \in \xi_i(\mathbf{x},\theta,\phi) + S_i(\mathbf{x},\theta,\phi)]\} \tag{9}$$

By the properties fo Poisson processes,

$$\mu = \mathbf{E}(N) = \mathbf{E}[\int_{S^*(\mathbf{x},\theta,\phi)} \gamma(\mathbf{z})\,d\mathbf{z} \tag{10}$$

Since $\{\xi_i(\mathbf{x},\theta,\phi), i \geq 1\}$ is Poisson process, then $N$ has Poisson distribution and

$$\mathbf{E}[T(\mathbf{x},\theta,\phi)] = \mathbf{E}(N) = \exp\{-\mathbf{E}[\int_{S^*(\mathbf{x},\theta,\phi)} \gamma(\mathbf{z})\,d\mathbf{z}]\} \tag{11}$$

Case 1: $\lambda(\mathbf{x},h) = \lambda(h)$, that is, the process is homogeneous in the "ground component".

$$\gamma(\mathbf{z}) = \int_0^\infty \lambda(u\cos\theta)\,du = \frac{1}{\cos\theta}\int_0^\infty \lambda(u)\,du \tag{12}$$

and

$$\begin{aligned}\mathbf{E}[T(\mathbf{x},\theta,\phi)] &= \exp\{-\mathbf{E}[\text{area}(S^*(\mathbf{x},\theta,\phi))\int_0^\infty \lambda(u)\,du/\cos\theta]\} \\ &= \exp\{-\mathbf{E}[\text{area}(S(\mathbf{x},\theta,\phi))]\int_0^\infty \lambda(u)\,du/\cos\theta\} \end{aligned} \tag{13}$$

Since we are assuming the sets $S$ to be randomly and uniformly oriented we have

$$\mathbf{E}[\text{area}(S(\mathbf{x},\theta,\phi))] = K\mathbf{E}[\text{area}(S)] \tag{14}$$

where $K$ does not depend on $(\theta,\phi)$. Since, $\int_0^\infty \lambda(u)\,du$ is the expected number of leafs per unit ground area, we have (13) to become

$$\mathbf{E}[T(\mathbf{x},\theta,\phi)] = e^{-KL/\cos\theta} \tag{15}$$

However, what is measured by LAI-2000 is

$$T(\mathbf{x},\theta) = \frac{1}{2\pi}\int_0^\pi T(\mathbf{x},\theta,\phi)d\phi \tag{16}$$

and since (15) does not depend on $\phi$ we have

$$\begin{aligned}\mathbf{E}[T(\mathbf{x},\theta)] &= \frac{1}{2\pi}\int_0^\pi \mathbf{E}[T(\mathbf{x},\theta,\phi)]d\phi \\ &= e^{-KL/\cos\theta}\end{aligned} \tag{17}$$

and the model is adequate if we consider a homogeneous canopy and leafs that are isotropic.

Case 2: $\lambda(\mathbf{y},h)$ depens on both coordinates, then supposing we can approximate:

$$\begin{aligned}\mathbf{E}[T(\mathbf{x},\theta,\phi)] &= \exp\{-\mathbf{E}[\int_{S^*(\mathbf{x},\theta,\phi)}\int_0^\infty \lambda(\mathbf{z}+u(\sin\theta\cos\phi,\sin\theta\sin\phi),u\cos\theta))du\,d\mathbf{z}\} \\ &\approx \exp\{-K\mathbf{E}(\text{area}(S)]\int_0^\infty \lambda(\mathbf{x}+u(\sin\theta\cos\phi,\sin\theta\sin\phi),u\cos\theta))du\}\end{aligned} \tag{18}$$



Therefore, the measured value $T(\mathbf{x},\theta)$ given by (16) has expected value:

$$\mathbf{E}[T(\mathbf{x},\theta)] \approx \frac{1}{2\pi}\int_0^{2\pi}\exp\{-K\mathbf{E}(\text{area}(S)]\int_0^\infty \lambda(\mathbf{x}+u(\sin\theta\cos\phi,\sin\theta\sin\phi),u\cos\theta))du\} \tag{19}$$

and by Jensen's inequality $L$ has its valeu subestimated when using the model $\mathbf{E}[T(\mathbf{x},\theta)] = e^{-KL/\cos\theta}$ and

$$L = \mathbf{E}(\text{area}(S)]\frac{1}{2\pi}\int_0^{2\pi}\int_0^\infty \lambda(\mathbf{x}+u(\tan\theta\cos\phi,\tan\theta\sin\phi),u))du\,d\phi.$$

## 3 Conclusion

Note that the mathematical model (1) proposed in the literature is correct if we take into account both angles (zenith and azimuth), however when taking azimuthal averages, model (3) is valid only for canopies that are homogeneous in the "ground" coordinate. It would be desirable to have a correction factor for non-homogeneous canopies, however, this factor depends heavily in the non-homegeneity factor and this would be hard to get.

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