A NEW CONSTRUCTION OF THE CASIMIR OPERATORS FOR THE FANTAPPIÉ-de SITTER GROUP

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ABSTRACT We obtain the Casimir operators associated to the Fantappié-de Sitter Group (isomorphic to the 5-dimensional pseudo-rotation group) which is the group of motions admited by the de Sitter cosmological spacetime, using the generalized derivative operators.

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Abstract

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1. INTRODUCTION

The de Sitter space is the curved spacetime which has been most studied by quantum field theoristics because togheter with the anti-de Sitter space are the uniques maximally symmetric curved spacetimes[1].

The symmetry group of de Sitter space is the ten parameters group $SO(4,1)$ of homogeneous Lorentz transformations in the 5-dimensional embedding space known as the de Sitter group[2].

There are many possible coordinatizations for the study the de Sitter space time, for exemple the steady-state universe (parametrization) of Bondi & Gold[3] and Hoyle[4] which covers the half of the de Sitter manifold and the static system which also only covers the half of the de Sitter manifold[5]. Tagirov[6] studies the Einstein universe using a conformal time where the coordinates cover the whole of the de Sitter manifold.

The theory of hyperspherical models of universe (tied to the integer numbers) proposed by Fantappié[7] and perfected by Arcidiacono[8,9] based on group theory is a original way to study the cosmological problem. In this theory it is necessary to distinguish the absolute spacetime (with constant curvature) effective seat of the physics events from the infinite relative space time (tangents) where each observer localize and see the phenomena. Then we use a flat representation of the de Sitter universe on their tangent spaces. Among the infinite representations we use the Beltrami[10] geodesic representation where the geodesics of the hyperspherical spacetime corresponds to the straight lines of the flat tangent space time of the observer's location.

It follows that the group of motions in itself of the de Sitter universe is represented by the so called Fantappié-de Sitter Group (Isomorphic to the 5-dimensional pseudo-rotation group.) i.e. by the projectivities of the tangent space, which change in itself the Cayley-Klein absolute of equation

$$R^2 A^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 + (x_0)^2 + R^2 = 0$$

where $x_0 = ict$. (For the definition of A see eq (2.2))

This paper, which is the first of three, is organized as follow: in section two we present how to pass from the de Sitter formulation to the orthogonal coordinates using the Beltrami geodesic representation and we obtain the formulas with relates the derivatives; in section three we discuss the Fantappié-de Sitter group and we obtain the explicit formulas for the invariant associated operators (Casimir operators); in section four, using spherical coordinates we present the commutation relations and construct explicity the Casimir invariant operators and finally we present ours comments.

In the following papers we solve the equations obtained from the second and fourth orders Casimir invariant operators.

2. ORTHOGONAL COORDINATES AND DERIVATIVES

In this section we consider how to pass from the Beltrami representation $x_\mu(\mu = 0, 1, 2, 3)$ of the de Sitter Universe to the homogeneous coordinates $\xi_A(A = 0, 1, 2, 3, 4)$ of the embedded space $R_{4,1}$. They are related by (see Apendice).

$$x_\mu = R\,\frac{\xi_\mu}{\xi_4} \tag{2.1}$$

satisfying the relation of normalization $\xi_A \xi_A = R^2$ where R is the radius of the de Sitter universe.

Introducing the following notation

$$A^2 = 1 + \alpha^2 - \gamma^2 = 1 + \alpha_\mu \alpha_\mu \tag{2.2}$$

where

$$\alpha_\mu = \frac{1}{R} x_\mu \qquad \text{and} \qquad \gamma = \frac{1}{R} ct = \frac{t}{t_0}$$

we can remove the ξ_4 coordinate, then we have the following relations

$$\xi_4 = \frac{R}{A} \qquad \text{and} \qquad \xi_\mu = \frac{x_\mu}{A}.$$

To obtain the relation for the partial derivates we consider a function $\varphi(\xi_A)$ being an homogeneous function of degree N in all five variables ξ_A, and using Euler's theorem for homogeneous functions, we have

$$\xi_A \partial_A \varphi(\xi_A) = N\varphi(\xi_A) \tag{2.3}$$

where we have put $\partial_A = \partial/\partial\xi_A$. Using the definition of homogeneous function we can write

$$\varphi\left(R\frac{\xi_4}{\xi_4},\ R\frac{\xi_1}{\xi_4},\dots\right) = \left(\frac{R}{\xi_4}\right)^N \varphi(\xi_A) \tag{2.4}$$

and finally we get the following relation

$$R^N\varphi(\xi_A) = (\xi_4)^N\varphi(R,\ x_\mu) \tag{2.5}$$

where the function in the right hand side is a function obtained from $\varphi(\xi_A)$ with the substitutions, $\xi_4 \to R$ and $\xi_\mu \to x_\mu$.

Deriving eq. (2.7) firstly in relation to ξ_4 and secondly in relation ξ_μ we obtain, respectively

$$R\frac{\partial}{\partial\xi_4}\varphi(\xi_A) = A^{1-N}(N - x_\mu\partial_\mu)\varphi(R,\ x_\mu) \tag{2.8a}$$

and

$$\frac{\partial}{\partial\xi_\mu}\varphi(\xi_A) = A^{1-N}\partial_\mu\varphi(R,\ x_\mu) \tag{2.8b}$$

where A is given by eq. (2.2) and we have put $\partial_\mu \equiv \partial/\partial x_\mu$.

Introducing a function $\psi(x_\mu)$ defined by

$$\psi(x_\mu) = A^{-N}\varphi(R,\ x_\mu) \tag{2.9}$$

in the above equations we can finally write the derivates, respectively, as follows

$$R\frac{\partial}{\partial\xi_4}\varphi(\xi_A) = \left(\frac{N}{A} - Ax_\mu\partial_\mu\right)\psi(x_\mu) \tag{2.10a}$$

and

$$\frac{\partial}{\partial\xi_\mu}\varphi(\xi_A) = \left(A\partial_\mu + \frac{N}{AR^2}x_\mu\right)\psi(x_\mu) \tag{2.10b}$$

Then, we have solved the problem to pass of the 5-dimensional formulation, ξ_A, to spacetime formulation, x_μ, i.e. in orthogonal cartesian coordinates. The relations eq. (2.10a) and eq. (2.10b) are the link between the two formulations.

3. THE FANTAPPIÉ-de SITTER GROUP

In this section we present the Fantappié- de Sitter Group and write its invariant operators.

The Fantappié-de Sitter Group - isomorphic to the 5-dimensional pseudo rotation group - is the group of motions admited by a cosmological space with line element given by

$$-ds^2 = A^2 dx_\mu dx_\mu = A^2[(dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_0)^2]$$

where $x_0 = ict$, and $R^2 A^2 = R^2 + \rho^2 - x_0^2$ and $\rho^2 = (x_1)^2 + (x_2)^2 + (x_3)^2$.

This space can be embedded in a flat 5-dimensional space time, being the x_μ, the Beltrami projection from the "sfere" with equation

$$\sum_{A=0}^{4} \xi_A \xi_A = (\xi_1)^2 + (\xi_2)^2 + (\xi_3)^2 + (\xi_4)^2 - (\xi_0)^2 = R^2 \tag{3.1}$$

The coordinates are related by the following expressions

$$x_\mu = R\frac{\xi_\mu}{\xi_4} \qquad \xi_\mu = \frac{1}{A}x_\mu \qquad \xi_4 = \frac{1}{A}R$$

where $\mu = 0,\ 1,\ 2,\ 3$ and for the differential operator we have

$$\frac{\partial}{\partial \xi_\mu} = A\partial_\mu + \frac{N}{AR^2}x_\mu \tag{3.3a}$$

and

$$\frac{\partial}{\partial \xi_4} = \frac{1}{R}\left(\frac{N}{A} - Ax_\mu \partial_\mu\right) \tag{3.3b}$$

where N is a parameter, R is the radius of the de Sitter universe and A is given by eq. (2.2).

The generators of the 5 dimensional pseudo rotation group satisfy[2],

$$-i[J_{k\lambda},\ J_{\mu\nu}] = \delta_{k\nu}J_{\lambda\mu} - \delta_{k\mu}J_{\lambda\nu} + \delta_{\lambda\mu}J_{k\nu} - \delta_{\lambda\nu}J_{k\mu}$$
$$-i[\pi_\lambda,\ J_{\mu\nu}] = \delta_{\lambda\mu}\pi_\nu - \delta_{\lambda\nu}\pi_\mu$$
$$-i[\pi_\mu,\ \pi_\nu] = -\frac{1}{R^2}J_{\mu\nu}$$

where $\pi_\mu = \frac{1}{R}J_{0\mu}$. We note that to $R \to \infty$ we have

$$\pi_\mu \to p_\mu$$

which is the four dimensional operator associate with the translations of the Minkowski space time, when $R \to \infty$ we obtain the Lie Algebra of the non homogeneous Lorentz group.

Introducing

$$p_\mu \to -i\partial_\mu$$

we have a representation for the Fantappié-de Sitter group which is given by the 5-dimensional angular momentum operators

$$J_{AB} = -i\hbar\left(\xi_A \frac{\partial}{\partial \xi_A} - \xi_B \frac{\partial}{\partial \xi_A}\right) \equiv L_{AB}$$

where $A,\ B = 0,\ 1,\ 2,\ 3,\ 4$ which in terms of the Beltrami coordinates are given by

$$L_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu \tag{3.4a}$$

and

$$\pi_\lambda \equiv \frac{1}{R} L_{0\lambda} = A^2 p_\lambda + \frac{1}{R^2} x_\mu L_{\lambda\mu} \tag{3.4b}$$

where $\mu,\ \nu,\ \lambda = 0,\ 1,\ 2,\ 3$.

We note that in the above equations (where π_μ are the analogous of the momentum operators in the Minkowski space) that the linear momentum and the angular momentum mix in a unique tensor. This mixing is due to the fact that transformation of displacents are the analogous of the translations and therefore the energy-momentum operators are not conserved in relation to the Fantappié-de Sitter group.

Now, we consider the explicit form to the ten operators. Introducing the T_0-operator, representing the temporal translations, defined by

$$L_{40} \equiv \frac{i}{c} R T_0 = -i\hbar\left(\xi_4 \frac{\partial}{\partial \xi_0} - \xi_0 \frac{\partial}{\partial \xi_4}\right)$$

we have

$$T_0 = -\hbar c(\partial_0 + \frac{1}{R^2} x_0 x_\mu \partial_\mu) \tag{3.5}$$

where $\partial_\mu = \partial/\partial x_\mu$ and $\mu = 0,\ 1,\ 2,\ 3$.

The T_μ-operators, representing the "spatial translations", are defined by

$$L_{\mu 4} \equiv RT_\mu = -i\hbar\left(\xi_\mu \frac{\partial}{\partial \xi_4} - \xi_4 \frac{\partial}{\partial \xi_\mu}\right)$$

and we obtain

$$T_\mu = \frac{i\hbar}{R^2}(R^2 \partial_\mu + x^\mu x_\nu \partial_\nu) \tag{3.6}$$

where $\mu = 1,\ 2,\ 3$ and $\nu = 0,\ 1,\ 2,\ 3$.

Introducing the V_μ operators which are related to the center of mass inertia momentum, given by

$$L_{0\mu} \equiv icV_\mu = -i\hbar\Big(\xi_0\frac{\partial}{\partial\xi_\mu} - \xi_\mu\frac{\partial}{\partial\xi_0}\Big)$$

we have

$$V_\mu = -\frac{\hbar}{c}(x_0\partial_\mu - x_\mu\partial_0) \tag{3.7}$$

where $\mu = 1,\ 2,\ 3$.

Finally, we introduce L_λ-operators, representing the spatial rotations, defined by

$$L_{\mu\nu} \equiv L_\lambda = -i\hbar\Big(\xi_\mu\frac{\partial}{\partial\xi_\nu} - \xi_\nu\frac{\partial}{\partial\xi_\mu}\Big)$$

and we obtain

$$L_\lambda = -i\hbar(x_\mu\partial_\nu - x_\nu\partial_\mu) \tag{3.8}$$

where $\mu,\ \nu,\ \lambda = 1,\ 2,\ 3$ and in the above expressions $\hbar$ and c have the usual meanings.

Now, we can write the two invariant operators of the Fantappié-de Sitter group (Casimir operators) using T_0, T_μ, V_μ and L_μ as follow

$$I_2 = (T^2 - \frac{1}{c^2}T_0^2) + \frac{1}{R^2}(L^2 - c^2V^2) = M^2 \tag{3.9a}$$

and

$$I_4 = (\vec{L}\cdot\vec{T})^2 - \frac{1}{c^2}(T_0\,\vec{L} + c^2\,\vec{T}\times\vec{V})^2 - \frac{c^2}{R^2}(\vec{L}\cdot\vec{V})^2 = N^2 \tag{3.9b}$$

where M^2 and N^2 are constants.

We note that, in the limit $R \to \infty$ we obtain

$$I_2 \to m^2 \qquad \text{and} \qquad I_4 \to m^2 s(s+1)$$

where m and s are, respectively, the rest mass and the spin which caracterize the representations of the Poincaré Group[2]. Then, the representations of the Fantappié-de Sitter group are labeled by eigenvalues of I_2 and I_4 which generalizes the usual mass and spin. Yet, a particle in a Fantappié-de Sitter universe has not a well defined mass and a spin but eigenvalues of the I_2 and I_4 invariant operators.

4. COMUTATION RELATIONS AND CASIMIR OPERATORS

In this section we introduce a spherical coordinate sistem $(r,\ \theta,\ \phi)$ and we obtain the explicit comutation relations and the explicit form of Casimir operators, in these coordinates. This result is important for the following papers[14,15].

The relativistic spherical coordinates are given by $x_0 = t$, $x_3 = r\cos\theta$, $x_2 = r\sin\theta\sin\phi$ and $x_1 = r\sin\theta\cos\phi$, and we obtain ten differential operators in the explicit forms given by

$$T_0 = -\hbar c\Big[\Big(1+\frac{t^2}{R^2}\Big)\frac{\partial}{\partial t} + \frac{tr}{R^2}\frac{\partial}{\partial r}\Big]$$

$$T_1 = -\frac{i\hbar}{R^2}\Big[(r^2+R^2)\sin\theta\cos\phi\frac{\partial}{\partial r} + \frac{R^2}{r}\Big(\cos\theta\cos\phi\frac{\partial}{\partial\theta} - \frac{\sin\phi}{\sin\theta}\frac{\partial}{\partial\phi}\Big) + rt\sin\theta\cos\phi\frac{\partial}{\partial t}\Big]$$

$$T_2 = -\frac{i\hbar}{R^2}\Big[(r^2+R^2)\sin\theta\sin\phi\frac{\partial}{\partial r} + \frac{R^2}{r}\Big(\cos\theta\sin\phi\frac{\partial}{\partial\theta} + \frac{\cos\phi}{\sin\theta}\frac{\partial}{\partial\phi}\Big) + rt\sin\theta\sin\phi\frac{\partial}{\partial t}\Big]$$

$$T_3 = -\frac{i\hbar}{R^2}\Big[(r^2+R^2)\cos\theta\frac{\partial}{\partial r} - \frac{R^2}{r}\sin\theta\frac{\partial}{\partial\theta} + rt\cos\theta\frac{\partial}{\partial t}\Big]$$

$$V_1 = \frac{\hbar}{c}\Big[t(\sin\theta\cos\phi\frac{\partial}{\partial r} + \frac{1}{r}\cos\phi\cos\theta\frac{\partial}{\partial\theta} - \frac{1}{r}\frac{\sin\phi}{\sin\theta}\frac{\partial}{\partial\phi}\Big) - r\sin\theta\cos\phi\frac{\partial}{\partial t}\Big]$$

$$V_2 = \frac{\hbar}{c}\Big[t(\sin\theta\sin\phi\frac{\partial}{\partial r} + \frac{1}{r}\cos\theta\sin\phi\frac{\partial}{\partial\theta} + \frac{1}{r}\frac{\cos\phi}{\sin\theta}\frac{\partial}{\partial\phi}\Big) - r\sin\theta\sin\phi\frac{\partial}{\partial t}\Big]$$

$$V_3 = \frac{\hbar}{c}\Big[t(\cos\theta\frac{\partial}{\partial r} - \frac{1}{r}\sin\theta\frac{\partial}{\partial\theta}) - r\cos\theta\frac{\partial}{\partial t}\Big]$$

$$L_1 = i\hbar(-\sin\phi\frac{\partial}{\partial\theta} - \cot\theta\cos\phi\frac{\partial}{\partial\phi})$$

$$L_2 = i\hbar(-\cos\phi\frac{\partial}{\partial\theta} + \cot\theta\sin\phi\frac{\partial}{\partial\phi})$$

$$L_3 = i\hbar\frac{\partial}{\partial\phi}$$

Now, considering a ciclic permutation of the index μ, ν and λ we obtain the following comutation relations for the differential operators,

$$[T_0,\ T_\mu] = -i\hbar\frac{c^2}{R^2}V_\mu \qquad [T_0,\ V_\mu] = -i\hbar T_\mu \qquad [T_0,\ R_\mu] = 0$$

$$[T_\mu,\ T_\nu] = -\frac{i\hbar}{R^2}L_\lambda \qquad [T_\mu,\ V_\nu] = -\frac{i\hbar}{c^2}\delta_{\mu\nu}T_0 \qquad [T_\mu,\ L_\nu] = i\hbar T_\lambda$$

$$[V_\mu,\ V_\nu] = \frac{i\hbar}{c^2}L_\lambda \qquad [L_\mu,\ V_\nu] = -i\hbar V_\lambda \qquad [L_\mu,\ L_\nu] = -i\hbar L_\lambda$$

where μ, ν, $\lambda = 1, 2, 3$.

Finally, we obtain the explicit forms for the Casimir operators, introducing the differential operators given above in eq. (3.9a) and (3.9b).

The Casimir operator of second order is given by

$$I_2 = -\hbar^2 A^2 \Big\{ \Big(1+\frac{r^2}{R^2}\Big)\frac{\partial^2}{\partial r^2} + 2rt\,\frac{\partial^2}{\partial r\partial t} - \Big(1-\frac{t^2c^2}{R^2}\Big)\frac{\partial^2}{\partial(ct)^2} +$$

$$+\frac{2}{r}\Big(1+\frac{r^2}{R^2}\Big)\frac{\partial}{\partial r} + \frac{2t}{R^2}\frac{\partial}{\partial t} + \frac{1}{r^2}\mathcal{L}^2\Big\}$$

where we have put $t \to ict$ and the $\mathcal{L}^2$ operator is

$$\mathcal{L}^2 = \frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}.$$

We note that when $R \to \infty$ the above equation reduces to, the D'Alembert wave operator, i.e.,

$$\lim_{R\to\infty} I_2 \equiv \Box = \hbar^2\Big(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Big)$$

where Δ is the Laplacian operators writen in spherical coordinates.

For the fourth order Casimir differential operator we have

$$I_4 = 4\frac{c^2\hbar^2}{R^4}\mathcal{L}^2\mathcal{H}^2$$

where

$$\mathcal{H}^2 = t^2r^2\frac{\partial^2}{\partial r^2} + (R^2-t^2)\frac{\partial^2}{\partial t^2} - 2rt(R^2-t^2)\frac{\partial^2}{\partial r\partial t} + r(2t^2-R^2)\frac{\partial}{\partial r} - 2t(R^2-t^2)\frac{\partial}{\partial t}$$

and $\mathcal{L}^2$ is given above.

5. COMMENTS

In this paper we have discussed an alternative way to obtain the Casimir invariant operators of the Fantappié-de Sitter Group which is isomorphic to 5 dimensional pseudo rotation group.

It is clear the dependence in the two Casimir invariant operators in both spatial and temporal parts, given by the T_0, T_μ, V_μ and L_μ operators. In consequence, a particle in a Fantappié-de Sitter universe has not a well defined mass and a spin but has constant eigenvalues of the I_2 and I_4 Casimir invariant operators.

The next point is solve the generalized Klein-Gordon wave equation for the scalar field[11, 12, 13] obtained from the second order Casimir invariant operator and the equation obtained from the fourth order Casimir invariant operator, which must generalizes the concept of mass and spin[14, 15]. These topics are presented in another paper.

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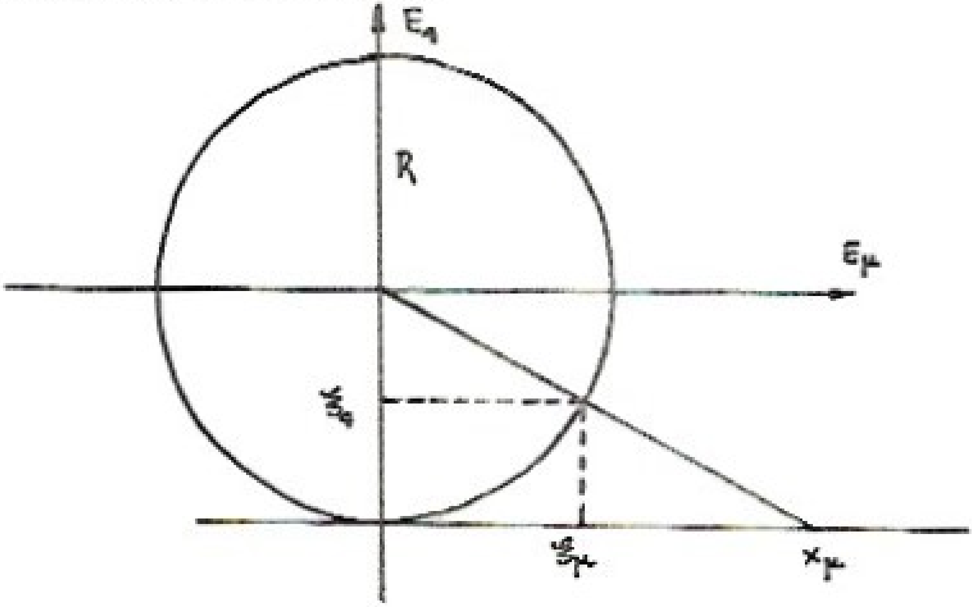
APENDICE

The de Sitter space can be represented as the surface of a four-dimensional pseudosphere (of a hyperbolic character in one direction) embedded in a five-dimensional space. It is described by five coordinates ξ_1, ξ_2, ξ_3, ξ_4, ξ_0 connected by the relation of normalization condition

$$\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 - \xi_0^2 = R^2$$

R being the radius of the "sphere".

To see how to pass from the five dimensional formulation to the four-dimensional orthogonal coordinates $x_\mu (\mu = 0, 1, 2, 3)$ we consider the Beltrami representation (geodesic representations) as in the figure:



where

$$x_\mu = R\frac{\xi_\mu}{\xi_4}$$

Introducing $\rho^2 = x^\mu x_\mu = -(x_0)^2 + (x_1)^2 + (x_2)^2 + (x_3)^2$ and using normalization condition we can write

$$\xi_4 = \frac{R}{(1 + \rho^2/R^2)^{1/2}}.$$

Then, the relations to pass from the pentadimensional formulation to the four-dimensional formulation are given by

$$\xi_4 = \frac{R}{A} \qquad \text{and} \qquad \xi_\mu = \frac{x_\mu}{A}$$

where $A^2 = 1 + \rho^2/R^2$

REFERENCES

(1) S. Weinberg (1972) *Gravitation and Cosmology: Principles and Applications of the General theory of Relativity* (New York: Wiley)

(2) F. Gürsey (1963) Introduction to Group Theory in Relativity, Groups and Cosmology edited by Dewitt and Dewitt (Gordon Breach)

(3) H. Bondi and T. Gold, Mon. Not. R. Astron. Soc. **108**, 252 (1948)

(4) F. Hoyle, Mon. Not. R. Astron. Soc. **108**, 372 (1948)

(5) G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977)

(6) E. A. Tagirov, Ann. Phys **76**, 561 (1973)

(7) L. Fantappié, *Opere Scelte*, (1973) Ed. Unione Matematica Italiana (Bologna, Italia)

(8) G. Arcidiacono, Gen. Rel. Grav. **7**, 885(1976), **8**, 865(1977) and **9**, 949(1978)

(9) G. Arcidiacono (1986) *Projective, Relativity, Cosmology and Gravitation*, Hadronic Press Inc., Massachusetts, USA and (1987) *Relativitá e Cosmologia*, Veschi, Roma

(10) E. Beltrami, Ann. di Matem. **8**, 135 (1865)

(11) G. Arcidiacono and E. Capelas de Oliveira, Hadronic Journal **14**, 347 (1991)

(12) G. Arcidiacono and E. Capelas de Oliveira, Hadronic Journal **14**, 353 (1991)

(13) E. Capelas de Oliveira, "On the solution of homogeneous generalized wave equation" to appear in J. Math. Phys (1992)

(14) E. A. Notte Cuello and E. Capelas de Oliveira "D'Alembert equation in the de-Sitter Universe" to be published (1993)

(15) E. A. Notte Cuello and E. Capelas de Oliveira "Dirac equation in the de Sitter Universe" to be published (1993).

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