HOMOCLINIC ORBITS FOR A QUASILINEAR HAMILTONIAN SYSTEMS

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ABSTRACT – The aim of this paper is to prove existence of nontrivial homoclinic orbits for the following Quasilinear Hamiltonian System

$$(a(|u'|^p)|u'|^{p-2}u')' - b(|u|^p)|u|^{p-2}u + W'_u(t, u) = 0 \quad \text{in} \quad \mathbb{R}$$

that is, to prove existence of $u : \mathbb{R} \to \mathbb{R}^n$ satisfying the system above and

$$u(-\infty) = u'(-\infty) = u(+\infty) = u'(+\infty) = 0.$$

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Abstract

The aim of this paper is to prove existence of nontrivial homoclinic orbits for the following Quasilinear Hamiltonian System

$$(a(|u'|^p)|u'|^{p-2}u')' - b(|u|^p)|u|^{p-2}u + W_u'(t,\ u) = 0 \quad \text{in} \quad \mathbb{R}$$

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§1. Introduction.

In P. Rabinowitz [4] is studied the existence of homoclinic orbits for the second order Hamiltonian system:

$$u'' + V_u'(t,\ u) = 0$$

where $u : \mathbb{R} \to \mathbb{R}^n$ and V satisfies:

(V1) $V(t,\ u) = \frac{1}{2}L(t)u \cdot u + W(t,\ u)$, where L is continuous T-periodic matrix valued function and $W \in C^1(\mathbb{R} \times \mathbb{R}^n,\ \mathbb{R})$ is T-periodic in t,

(V2) $L(t)$ is positive definite symmetric for all $t \in [0,\ T]$

(V3) there is a constant $\mu > 2$ such that

$$0 < \mu W(t,\ u) \leq W_u'(t,\ u) \cdot u, \quad \text{for all} \quad u \in \mathbb{R}^n \setminus \{0\}$$

(V4) $W_u'(t,\ u) = 0(|u|)$ as $u \to 0$ uniformly for $t \in [0,\ T]$

For the conservative case see A. Ambrosetti and M. L. Bertotti [1].

In this paper, we will obtain an extension of this result in the case $L(t) = id_{\mathbb{R}^n}$, that is, we will prove the existence of a nontrivial homoclinic orbit for the following Quasilinear

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Hamiltonian System

$$(P) \qquad (a(|u'|^p)|u'|^{p-2}u')' - b(|u|^p)|u|^{p-2}u + W'_u(t,\ u) = 0$$

where $a,\ b \in C(\mathbb{R}^+,\ \mathbb{R})$ and $p > 1$. By a homoclinic we mean an $u : \mathbb{R} \to \mathbb{R}^n$ satisfying:

$u \in C^1(\mathbb{R},\ \mathbb{R}),\quad a(|u'|^p)|u'|^{p-2}u' \in C^1(\mathbb{R},\ \mathbb{R}),$
$u \not\equiv 0$, wich solves (P) and such that
$u(t) \to 0,\quad u'(t) \to 0$ as $t \to \pm\infty$.

The greatest difficulties in studying this System are due to the particular behaviour of a and the lack of compactness of the functional usually associated with the Problem (P). So we are interested in the homoclinic solutions of Problem (P) where the function a satisfies the following conditions:

(1.1) $a \in C(\mathbb{R}^+,\ \mathbb{R})$

(1.2) $a(t^p)t^{p-1}$ is strictly increasing as t increases,

(1.3) there exist constants $1 < q < p$, $a_i > 0$, $i = 1,\ 2,\ 3,\ 4$ such that

$$a_1 t^q + a_2 t^p \le a(t^p)t^p \le a_3 t^q + a_4 t^p, \qquad \text{for each} \quad t > 0,$$

and the function b satisfies

(1.4) $b \in C(\mathbb{R}^+,\ \mathbb{R})$

(1.5) there exist constants $b_i > 0$, $i = 1,\ 2,\ 3,\ 4$ such that

$$b_1 t^q + b_2 t^p \le b(t^p)t^p \le b_3 t^q + b_4 t^p, \quad \text{for each} \quad t > 0,$$

and the function $W \in C^1(\mathbb{R} \times \mathbb{R}^n,\ \mathbb{R})$ satisfies the following condition:

There exist positive constants μ_q, μ_p and α such that

(1.6) $\dfrac{a_4}{\mu_p} < \dfrac{a_2}{p},\ \dfrac{a_3}{\mu_q} < \dfrac{a_1}{q},\ \dfrac{b_4}{\mu_p} < \dfrac{b_2}{p}$ and $\dfrac{b_3}{\mu_q} < \dfrac{b_1}{q}$

(1.7) $W'_u(t,\ u) \cdot u \ge \mu_q W(t,\ u) > 0$, for each, $0 < |u| \le \alpha$

(1.8) $W'_u(t, u) \cdot u \geq \mu_p W(t, u) > 0$, for each, $|u| \geq \alpha$ and we suppose that

(1.9) $W(t, u)$ is T-periodic in t and

(1.10) $W'_u(t, u) \cdot u = 0(|u|^q)$

The following theorem is the main result in this paper

Theorem 1.1. Suppose that the function a satisfies (1.1)-(1.3), the function b satisfies (1.4)-(1.5) and W verifies (1.6)-(1.10). Assume that $\mu_q < p$. Then Problem (P) possesses a homoclinic orbit.

We will prove this theorem using the same method used by Rabinowitz in [4], that is, we apply the Mountain Pass Theorem (See [7]) to obtain a sequence of periodic solutions with large period and then let the period go to infinity.

We denote the space of the T-periodic functions $u : [0, T] \to \mathbb{R}^n$ by

$$W^p_{T,P} = \{u \in W^{1,p}(0, T) \ / \ u(0) = u(T)\}$$

where in $W^p_{T,P}$ the norm is defined by

$$||u||_{p,T} =: (\int_0^T |u'|^p + \int_0^T |u|^p)^{1/p} = ||u||_{W^{1,p}(0,T)}.$$

Remark 1.1. If in Theorem 1.1 we suppose that $p = q$, using the same method employed in the case $p > q$, we observe that the proof this Theorem is more diret.

Remark 1.2. There are several results on existence and multiplicity of solutions considering the same type of elliptic operator so in System (P), that is, operators in divergence form

$$L_p u = -\text{div}\,(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u).$$

See, for instance N. Hirano [3], M. Badiale and G. Citti [2] and P. Ubilla [5, 6].

§2. Existence of periodic orbits through Mountain Pass.

We consider the following Problem,

$$(P)_T \begin{cases} (a(|u'|^p)|u'|^{p-2}u')' - b(|u|^p)|u|^{p-2}u + W'_u(t,\, u) = 0 \\ u(0) - u(T) = u'(0) - u'(T) = 0 \end{cases}$$

and let us consider the functional associated to $(P)_T$ $I : W^p_{P,\,T} \to I\!R$, defined by

$$I(u) = \frac{1}{p}\Big(\int_0^T A(|u'|^p) + B(|u|^p)\Big) - \int_0^T W(t,\, u)$$

where $A(t) = \int_0^t a(\sigma)d\sigma$ and $B(t) = \int_0^t b(\sigma)d\sigma$. It is well known that the critical points of the functional I are the weak solutions of Problem $(P)_T$.

Theorem 2.1. Under hypotheses of Theorem 1.1, Problem $(P)_T$ possesses at least one non-trivial weak solution.

Proof. We will prove that I satisfies the hypothesis of Montain Pass Theorem, that is,

(a) There exists $\delta > 0$ and $\rho > 0$ such that

$$I(u) \geq \delta, \quad \text{for each} \quad u \in \partial B^p_\rho(0)$$

where $B^p_\rho = \{u \in W^p_{P,\,T} \;/\; ||u||_{p,\,T} < \rho\}$,

(b) $I(0) = 0$ and there exists $e \in W^p_{T,\,P} \setminus B^p_\rho(0)$ such that, $I(e) \leq 0$

(c) I satisfies Palais-Smale Condition.

We first analyse the geometric conditions (a) and (b). Using (1.7) and (1.8) we have that there exist constant $c_q,\; c_p > 0$ such that

(1.7)' $\qquad W(t,\, u) \leq c_q|u|^{\mu_q}$, for each $|u| \leq 1$, and

(1.8)' $\qquad W(t,\, u) \geq c_p|u|^{\mu_p}$, for each $|u| \geq 1$.

On the other hand, from the compact inclusion $W^p_{T,\,P} \hookrightarrow C[0,\; T]$, we have that there exists $\rho_1 > 0$ such that,

$$||u||_{p,\,T} \leq \rho_1 \Rightarrow ||u||_{L^\infty(0,\,T)} \leq 1,$$

thus, using (1.3), (1.5) and (1.7)' we obtain that for each $||u||_{p,\,T} \leq \rho_1$ there are **constants** $c_1,\ c_2 > 0$, such that

$$I(u) \geq c_1||u||^q_{q,\,T} + c_2||u||^p_{p,\,T} - c_q||u||^{\mu_q}_{L^{\mu_q}[0,\,T]}.$$

And hence from inclusion $W^q_{T,\,P} \hookrightarrow L^{\mu_q}[0,\,T]$ there exists a constant $c_3 > 0$ such that

$$I(u) \geq c_1||u||^q_{q,\,T} - c_3||u||^{\mu_q}_{q,\,T} + c_2||u||^p_{p,\,T}$$

Since $\mu_q > q$, there exist $\rho < \rho_1$, such that,

$$||u||_{p,\,T} = \rho \Rightarrow I(u) \geq c_2\rho^p =: \alpha$$

that is (a) holds. For (b) we first see that $I(0) = 0$. Next we take $v \in W^p_{T,\,P}$, $v \neq 0$ and obtain from (1.3), (1.5) and (1.8)' that for each $t \in I\!R$

$$I(tv) \leq c_1|t|^q||v||^q_{q,\,T} + c_2|t|^p||v||^p_{p,\,T} - c_p|t|^{\mu_p}||v||^{\mu_p}_{L^{\mu_p}[0,\,T]} + c_3$$

where $c_1,\ c_2$ and c_3 are positive constants. Since $\mu_p > p > q$ we have

$$\lim_{t\to\pm\infty} I(tv) = -\infty,$$

then by choosing $\tilde{t}$ large enough and defining $e = \tilde{t}v$ we obtain

$$I(e) \leq 0 \quad \text{and} \quad ||e||_{p,\,T} > \rho,$$

that is (b) holds. We still need to prove the Palais Smale Condition for I. Let $\{u_n\}$ be a sequence in $W^p_{T,\,P}$, such that there exists a constant C such that

$$\text{(2.1)} \qquad \begin{cases} |\varphi(u_n)| \leq C \\ \\ \varphi'(u_n) \to 0. \end{cases}$$

First we will prove that $\{u_n\}$ is bounded. In fact, from (2.1) we have

$$\text{(2.2)} \qquad \begin{cases} -C \ \leq \ \dfrac{1}{p}(\displaystyle\int_0^T A(|u_n'|^p) + B(|u_n|^p)) - \int_0^T W(t,\,u_n) \leq C, \quad \text{and} \\ \\ |\displaystyle\int_0^T a(|u_n'|^p)|u_n'|^{p-2}u_n' \cdot v' + \int_0^T b(|u_n|^p)|u_n|^{p-2}u_n \cdot v \\ \\ -\displaystyle\int_0^T W_u'(t,\,u_n)\cdot v| \leq \varepsilon_n||v||_{p,\,T} \end{cases}$$

for each $v \in W^p_{T,\,P}$ and $\varepsilon_n \to 0$. Hence that we have

$$\frac{\mu_p}{p}\Big(\int_0^T A(|u_n'|^p) + B(|u_n|^p)\Big) \; - \; \Big(\int_0^T a(|u_n'|^p)|u_n'|^p + b(|u_n|^p)|u_n|^p\Big)$$
$$+ \; \int_0^T W_u'(t,\, u_n)\cdot u_n - \mu_p W(t,\, u_n) \le \varepsilon_n ||u_n||_{p,\,T} + \mu_p C,$$

thus using (1.3), (1.5), (1.6), (1.7) and (1.8) we obtain

$$\left(\frac{\mu_p a_2}{p} - a_4\right)\int_0^T |u_n'|^p + \left(\frac{\mu_p b_2}{p} - b_4\right)\int_0^T |u_n|^p \le \varepsilon_n ||u_n||_{p,\,T} + \mu_p C,$$

that is, there exists $\tilde{C}$ such that

$$\tilde{C}||u_n||^p_{p,\,T} \le \varepsilon_n ||u_n||_{p,\,T} + \mu_p\, C,$$

hence $\{u_n\}$ is bounded in $W^p_{T,\,P}$. All we have to prove is that $\{u_n\}$ contains a subsequence wich converges in the norm of $W^p_{T,\,P}$. Since $\{u_n\}$ is bounded, there exists a subsequence $\{u_{nj}\}$ converging weakly in $W^p_{T,\,P}$ to some u_0.

On the other hand, using the second assertion in (2.2) with $v = u_{nj} - u_0$ and taking limits over the subsequence we obtain that

$$\int_0^T a(|u_{nj}'|^p)|u_{nj}'|^{p-2}u_{nj}' \cdot (u_{nj}' - u_0') \xrightarrow[n_j \to +\infty]{} 0.$$

Let $J : W^{1,\,p}(0,\,T) \to \mathbb{R}$ be the functional defined by

$$J(u) = \frac{1}{p}\int_0^T A(|u'|^p)\,.$$

The strong convergence of the subsequence $\{u_{nj}\}$ is consequence of the following lemma

Lemma 2.1. Suppose that the function a satisfies (1.1), (1.2) and assume that there exist constants $\alpha_1,\ \alpha_2 > 0$ such that

(2.3) $$|a(t^p)t^{p-1}| \le \alpha_1 + \alpha_2|t|^{p-1}, \quad \text{for each} \quad t \in \mathbb{R}^+, \quad \text{and}$$

(2.4) there exist $\beta_1,\ \beta_2 > 0$ such that

$$A(|t|^p) \ge \beta_1|t|^p - \beta_2, \quad \text{for each} \quad t \in \mathbb{R}.$$

Then J' belong to the class $(S)_+$. That is, for all sequences $\{u_n\} \subset W^{1,\,p}(0,\,T)$ such that

(2.5) $$\begin{cases} u_n \rightharpoonup u \\ \lim\limits_{n\to+\infty} \sup\langle J'(u_n),\ u_n - u\rangle \le 0 \end{cases}$$

it follows that $u_n \to u$.

Proof. Using (1.2) it is not difficult to verify that

$$u_n'(t) \to u'(t) \quad \text{a. e.},$$

we observe that

$$\langle J'(u_n),\, u_n - u\rangle = \int_0^T a(|u_n'|^p)|u_n'|^{p-2}u_n' \cdot (u_n' - u').$$

Denoting $S_n(t) =: |u_n'(t)|^{p-2}u_n'(t) \cdot (u_n'(t) - u'(t))$, we have

$$\langle J'(u_n),\, u_n - u\rangle = \int_0^T a(|u_n'|^p)S_n\chi_{A_n} + \int_0^T a(|u_n'|^p)S_n\chi_{A_n^c}$$

where $A_n = \{t \in [0,\, T] \;/\; |u_n'(t)| \leq M\}$ and M is a positive constant. We will prove that

$$\lim_{n\to+\infty} \int_0^T a(|u_n'|^p)S_n\chi_{A_n} \xrightarrow[n\to+\infty]{} 0. \tag{2.6}$$

In fact, we define

$$\eta_n(t) = a(|u_n'(t)|^p)S_n(t)\chi_{A_n}(t).$$

Then η_n satisfies

$$\begin{cases} \eta_n(t) \to 0 \quad \text{a.e. in} \quad [0,\, T] \\ \\ |\eta_n(t)| \leq a(M^p)M^{p-1}(M + |u'(t)|), \end{cases}$$

hence, using Lebesgue Dominated Convergence Theorem we have (2.6). Thus for each $M > 0$ we have

$$\lim_{n\to\infty} \sup\langle J'(u_n),\, u_n - u\rangle = \lim_{n\to\infty} \sup \int_0^T a(|u_n'|^p)S_n\chi_{A_n^c}. \tag{2.7}$$

We denote

$$\begin{aligned} S_n^+ &= \{t \in [0,\, T] \;/\; S_n(t) \geq 0\} \\ S_n^- &= \{t \in [0,\, T] \;/\; S_n(t) < 0\}, \end{aligned}$$

from (1.2) and (2.4) we have that

$$a(t^p)t^p \geq \frac{A(t^p)}{p} \geq \frac{\beta_1}{p}t^p - \frac{\beta_2}{p},$$

hence for M large enough

$$\int_0^T a(|u_n'|^p)S_n\chi_{A_n^c} = \int_0^T a(|u_n'|^p)S_n\chi_{A_n^c}\chi_{S_n^+} + \int_0^T a(|u_n'|^p)S_n\chi_{A_n^c}\chi_{S_n^-}$$

$$\geq \left(\frac{\beta_1}{p} - \delta_1\right) \int_0^T S_n \chi_{A_n^c} \chi_{S_n^+} + (\alpha_2 + \delta_2) \int_0^T S_n \chi_{A_n^c} \chi_{S_n^-}$$

where $0 < \delta_1 < \dfrac{\beta_1}{p}$ and $\delta_2 > 0$. Thus

$$(2.8) \int_0^T a(|u_n'|^p) S_n \chi_{A_n^c} \geq \left(\frac{\beta_1}{p} - \delta_1\right) \int_0^T S_n \chi_{A_n^c} + \left(\alpha_2 + \delta_2 - \frac{\beta_1}{p} + \delta_1\right) \cdot \int_0^T S_n \chi_{A_n^c} \chi_{S_n^-}.$$

Now we will prove that

$$(2.9) \qquad \lim_{n \to +\infty} \int_0^T S_n \chi_{A_n^c} \chi_{S_n^-} = 0,$$

Defining

$$\Gamma_n(t) = \chi_{A_n^c} \cdot S_n \cdot \chi_{S_n^-},$$

it is not difficult to verify that if $t \in S_n^-$, we have that $|u_n'(t)| \leq |u'(t)|$. Hence

$$|\Gamma_n(t)| \leq 2|u'(t)|^p, \quad \text{for each} \quad t \in [0,\ T]$$

But, $\Gamma_n(t) \xrightarrow[n \to +\infty]{} 0$ a.e. in $[0,\ T]$. Thus by Lebesgue Dominated Convergence Theorem we have proved (2.9). Hence using (2.7) and (2.8) we obtain

$$\left(\frac{\beta_1}{p} - \delta_1\right) \lim_{n \to +\infty} \sup \int_0^T S_n \chi_{A_n^c} \leq \lim_{n \to +\infty} \sup \langle J'(u_n),\ u_n - u\rangle \leq 0$$

Consequently, since $\lim\limits_{n \to +\infty} \int_0^T S_n \chi_{A_n} = 0$, we have

$$\lim_{n \to \infty} \sup \int_0^T |u_n'|^{p-2} u_n' \cdot (u_n' - u') \leq 0$$

that is, $\lim\limits_{n \to \infty} \sup \langle J_p'(u_n),\ u_n - u\rangle \leq 0$ where $J_p(u) = \int_0^T |u'|^p$, but it is well known that J_p' belongs to the class $(S)_+$. Observe that J_p' is the known p Laplacian. Thus we proved that

$$u_n \to u \quad (\text{strong}).$$

The proof of Lemma 2.1 is complete. Consequently by (a), (b) e (c) using Montain Pass Theorem there exists $u_0 \in W^p_{T,\,P}$, $u_0 \neq 0$ such that $I'(u_0) = 0$, hence there is a non-trivial solution of Problem (P_T). The proof of Theorem 2.1 is complete. Observe that the critical level is

$$c \equiv \inf_{g \in \Gamma} \sup_{t \in [0,\,1]} I(g(s))$$

where

$$\Gamma = \{g \in C([0,\ 1],\ W^p_{T,\,P})\ /\ g(0) = 0,\ g(1) = e\}$$

§3. Proof of Theorem 1.1

We consider the following problem for $k \in \mathbb{N}$

$$(P)_{2kT}\begin{cases} (a(|u'|^p)|u'|^{p-2}u')' - b(|u|^p)|u|^{p-2}u + W'_u(t,\, u) = 0 \\ u(0) - u(2kT) = u'(0) - u'(2kT) = 0. \end{cases}$$

From Theorem 2.1 there exists a non-trivial solution u_k $2kT$-periodica with critical value

$$c_k = \inf_{g \in \Gamma_k} \sup_{t \in [0,\,1]} I_k(g(s))$$

where

$$\Gamma_k = \{g \in C([0,\,1],\, W^p_{2kT,\,p}) \;/\; g(0) = 0,\;\; g(1) = e_k\}$$

and

$$I_k(u) = \frac{1}{p}\Big(\int_0^{2kT} A(|u'|^p) + B(|u|^p)\Big) - \int_0^{2kT} W(t,\, u)$$

It what follows we obtain an estimate for c_k and u_k independent of k. If we consider a function $e \in W^p_{2T,\,p}$ such that $e(0) = e(2T) = 0$ and $I_1(e) \leq 0$ (such an e exists according to reasoning in Theorem 2.1). Thus we can define $e_k \in W^p_{2kT,\,p}$ by

$$e_k(t) = \begin{cases} e(t) & \text{if} \quad t \in [0,\, 2T] \\ 0 & \text{if} \quad t \in [2T,\, 2kT] \end{cases}$$

hence, $I_k(e_k) = I_1(e) \leq 0$ and if we define

$$g_k(s) = s e_k$$

then $g_k \in \Gamma_k$ and

$$0 < c_k \leq \sup_{s \in [0,\,1]} I_k(g_k(t)) = \sup_{t \in [0,\,1]} I_1(g_1(t)) = M$$

that is, we obtained an estimate for c_k. Now we will find an estimate for u_k independent of k. In fact, we have that $I_k(u_k) = c_k$ and $I'_k(u_k)u_k = 0$, that is,

$$\text{(3.1)} \qquad \frac{1}{p}\Big(\int_0^{2kT} A(|u'_k|^p) + B(|u_k|^p)\Big) - \int_0^{2kt} W(t,\, u_k) = c_k, \quad \text{and}$$

$$\text{(3.2)} \qquad \int_0^{2kT} a(|u'_k|^p)|u'_k|^p + \int_0^{2kT} b(|u_k|^p)|u_k|^p = \int_0^{2kT} W'_u(t,\, u_k) \cdot u_k.$$

Let $\omega_p > \mu_q$ be, such that

$$\text{(3.3)} \qquad \frac{a_4}{\mu_p} < \frac{a_4}{\omega_p} < \frac{a_2}{p} \quad \text{and} \quad \frac{b_4}{\mu_p} < \frac{b_4}{\omega_p} < \frac{b_2}{p}$$

hence, from (3.1) and (3.3) using (1.3), (1.5) and (1.6) we have

$$(3.4) \qquad c_k \geq \frac{a_3}{\mu_q} \int_0^{2kT} |u_k'|^q + \frac{b_3}{\mu_q} \int_0^{2kT} |u_k|^q + \frac{1}{\omega_p}(a_4 \int_0^{2kT} |u_k'|^p + b_4 \int_0^{2kT} |u_k|^p)$$

$$- \int_0^{2kT} W(t,\, u_k).$$

On the other hand, using (3.2), (1.3) and (1.5), we have

$$(3.5) \int_0^{2kT} W_u'(t,\, u_k) \cdot u_k \leq a_3 \int_0^{2kT} |u_k'|^q + a_4 \int_0^{2kT} |u_k'|^p + b_3 \int_0^{2kT} |u_k|^q + b_4 \int_0^{2kT} |u_k|^p.$$

Thus, from (3.4) and (3.5) we obtain

$$(3.6) \qquad c_k \geq a_3(\frac{1}{\mu_q} - \frac{1}{\omega_p}) \int_0^{2kT} |u_k'|^q + b_3(\frac{1}{\mu_q} - \frac{1}{\omega_p}) \int_0^{2kT} |u_k|^q$$

$$+ \int_0^{2kT} (\frac{1}{\omega_p} W_u'(t,\, u_k) \cdot u_k - W(t,\, u_k)).$$

On the other hand using (1.7) and (1.8) we have that there exist constants $K_q,\ K_p > 0$ such that

$$(3.7) \qquad W(t,\, u) \leq K_q |u|^{\mu_q}, \quad \text{for each} \quad 0 < |u| \leq \alpha, \quad \text{and}$$

$$(3.8) \qquad W(t,\, u) \geq K_p |u|^{\mu_p}, \quad \text{for each} \quad |u| \geq \alpha, \quad \text{where} \ 0 < \alpha \leq 1.$$

Thus, from (1.7), (1.8), (3.7) and (3.8) we obtain

$$\int_0^{2kT} (\frac{1}{\omega_p} W_u'(t,\, u_k) \cdot u_k \quad - \quad W(t,\, u_k)) \geq (\frac{\mu_p}{\omega_p} - 1) K_p \int_0^{2kT} |u_k|^{\mu_p}$$

$$+ \quad ((1 - \frac{\mu_p}{\omega_p}) K_p + (\frac{\mu_q}{\omega_p} - 1) K_q) \int_0^{2kT} |u_k|^{\mu_q} \chi_{\{|u_k(t)| \leq \alpha\}}$$

hence, there are constants $c_1,\ c_2 > 0$ such that

$$(3.9) \int_0^{2kT} \frac{1}{\omega_p} W_u'(t,\, u_k) \cdot u_k - W(t,\, u_k) \geq c_1 ||u_k||_{L^{\mu_p}[0,\, 2kT]}^{\mu_p} - c_2 ||u_k||_{L^{\mu_p}[0,\, 2kT]}^{\mu_q} \geq -\alpha_0$$

where $\alpha_0 \in I\!R$ is independent of k. Since c_k is bounded independent of k, using (3.6) and (3.9) we obtain

$$(3.10) \qquad ||u_k||_{q,\, 2kT} \leq C \quad \text{for each} \quad k \in N$$

where C is a constant. For any function $u \in W^q_{2kT,\, P}$ we have

$$|u(t)| \leq |u(s)| + |\int_s^t u'(\sigma) d\sigma|, \quad t,\ s \in [0,\ 2kT]$$

and integrating over $[t-\frac{1}{2},\, t+\frac{1}{2}]$ we obtain

$$(3.11)\qquad \begin{aligned} |u(t)| &\leq \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} |u(s)|ds + \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} |\int_s^t u'(\sigma)d\sigma|ds \\ &\leq 2(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}} (|u'(s)|^{q} + |u(s)|^{q})ds)^{\frac{1}{q}} \end{aligned}$$

Hence there exists a constant K such that

$$(3.12)\qquad ||u||_{L^\infty(0,\,2kT)} \leq K||u||_{q,\,2kT},$$

thus using (3.10) and (3.12) we have

$$(3.13)\qquad ||u_k||_{L^\infty(0,\,2kT)} \leq KC$$

Since $p > \mu_q$ from (3.4), (3.7) and (3.13) we obtain a constant C such that

$$||u_k||_{p,\,2kT} \leq C.$$

Here it is convenient to define a new function v_k by translating u_k

$$v_k(t) = u_k(t+kT), \quad \text{where} \quad t \in [-kT,\, kT]$$

thus

$$(3.14)\qquad \int_{-kT}^{kT} (|v_k'|^p + |v_k|^p) \leq M$$

where M is positive constant.

Using a diagonal procedure it is not difficult to verify that there exists a subsequence that also we denote by $\{v_k\}$ such that,

$$v_k \rightharpoonup u \quad \text{in} \quad E_m, \quad \text{for each} \quad m \in \mathbb{N}$$

where $E_m =: W^{1,p}(-mT,\, mT)$.

On the other hand, we have that for each $\varphi \in E_m$

$$(3.15)\qquad \int_{-mT}^{mT} a(|v_k'|^p)|v_k'|^{p-2}v_k' \cdot \varphi' + \int_{-mT}^{mT} b(|v_k|^p)|v_k|^{p-2}v_k \cdot \varphi = \int_{-mT}^{mT} W_u'(t,\, v_k) \cdot \varphi +$$

$$a(|v_k'(mT)|^p)|v_k'(mT)|^{p-2}v_k'(mT)\cdot\varphi(mT) - a(|v_k'(-mT)|^p)|v_k'(-mT)|^{p-2}v_k'(-mT)\cdot\varphi(-mT)$$

Let $\varphi = v_k - u$ be in (3.15). Using the same argument of (3.12), we have that $\{v_k'\}$ is bounded in $[-mT,\ mT]$, hence

$$\lim_{k\to+\infty}\int_{-mT}^{mT} a(|v_k'|^p)|v_k'|^{p-2}v_k'\cdot(v_k'-u')=0,$$

thus using Lemma 2.1, we have that

$$v_k \to u \quad \text{(strong) in} \quad E_m, \quad \text{for each} \quad m,$$

hence we have using (3.15) that for each $\varphi\in E_{m,0}$

$$\int_{-mT}^{mT} a(|u'|^p)|u'|^{p-2}u'\cdot\varphi' + \int_{-mT}^{mT} b(|u|^p)|u|^{p-2}u\cdot\varphi = \int_{-mT}^{mT} W_u'(t,\ u)\cdot\varphi \tag{3.16}$$

where $E_{m,0}=\{\varphi\in E_m\ /\ \varphi(-mT)=\varphi(mT)=0\}$. Thus $a(|u'|^p)|u'|^{p-2}u'\in C^1(\mathbb{R},\ \mathbb{R}^n)$ and for each $t\in\mathbb{R}$

$$-(a(|u'(t)|^p)|u'(t)|^{p-2}u'(t))' + b(|u(t)|^p)|u(t)|^{p-2}u(t) = W_u'(t,\ u(t)), \tag{3.17}$$

furthermore, using (3.14) and the strong convergence of $\{v_k\}$ in compacts, it is easy to verify that $u\in W^{1,p}(\mathbb{R})$. Now we will verify that

$$u(-\infty)=u'(-\infty)=u(+\infty)=u'(+\infty)=0, \tag{3.18}$$

in fact, u satisfies (3.11), hence that $u(-\infty)=u(+\infty)=0$. So to verify that $u'(-\infty)=u'(+\infty)=0$ we observe that the following inequality is holds

$$|a(|u'(t)|^p)|u'(t)|^{p-2}u'(t)|\leq \tag{3.19}$$

$$2\left(\int_{t-\frac{1}{2}}^{t+\frac{1}{2}}|(a(|u'|^p)|u'|^{p-2}u')'|^{p'} + \int_{t-\frac{1}{2}}^{t+\frac{1}{2}}|a(|u'|^p)|u'|^{p-2}u'|^{p'}\right)^{\frac{1}{p'}},$$

furthermore

$$\lim_{t\to\pm\infty}\int_{t-\frac{1}{2}}^{t+\frac{1}{2}}|W_u'(s,\ u)|^{p'}ds=0 \tag{3.20}$$

because $u(\pm\infty)=0$ and $W_u'(t,\ 0)=0$ for each t. Thus using (3.17), (3.19), (3.20) and that $u\in W^{1,p}(\mathbb{R})$ we obtain that $u'(-\infty)=u'(+\infty)=0$, so that (3.18) holds. We still have that to prove that $u\neq 0$. Since $I_k'(v_k)=0$ we have

$$\int_{-kT}^{kT} a(|v_k'|^p)|v_k'|^p + \int_{-kT}^{kT} b(|v_k|^p)|v_k|^p = \int_{-kT}^{kT} W_u'(t,\ v_k)\cdot v_k. \tag{3.21}$$

Let $\beta_k>0$ be defined by

$$\beta_k =: \max\left\{\frac{W_u'(t,\ u)\cdot u}{|u|^q}\ /\ |u|\leq\|v_k\|_{L^\infty(-kT,\ kT)} \quad\text{and}\quad t\in\mathbb{R}\right\}$$

using (1.9), (1.10) and (3.13) we have that

$$a_1 \int_{-kT}^{kT} |v_k'|^q + b_1 \int_{-kT}^{kT} |v_k|^q \leq \beta_k \int_{-kT}^{kT} |v_k|^q \tag{3.22}$$

thus since $v_k \not\equiv 0$ we obtain

$$\beta_k \geq b_1 > 0, \quad \text{for each} \quad k \in \mathbb{N}, \tag{3.23}$$

consequently for a constant $C > 0$

$$||v_k||_{L^\infty(-kT,\, kT)} \geq C, \quad \text{for each} \quad k \in \mathbb{N}.$$

Finally we observe that if u is a $2kT$ periodic solution of System (P), then $u(t + jT)$, $j \in \mathbb{Z}$ is also solution. This allows us to traslate v_k if necessary so that its maximum is always achieved in the interval $[0,\ T]$. Then

$$||v_k||_{L^\infty(-kT,\, kT)} = ||v_k||_{L^\infty(0,\, T)}$$

and since v_k converges to u (strongly) in $L^\infty(0,\ T)$, the function u cannot be zero. That is, Problem (P) possesses a homoclinic orbit. The proof of Theorem 1.1 is complete.

§4. Examples.

Assume that W satisfies the hypothesis of Theorem 1.1, the following systems possess a Homoclinic Orbit $u : \mathbb{R} \to \mathbb{R}^n$:

$$(|u'|^{p-2}u')' - |u|^{p-2}u + W'_u(t,\ u) = 0 \tag{4.1}$$

where $p = q$.

$$((1 + \frac{1}{(1 + |u'|^p)^p})|u'|^{p-2}u')' - |u|^{p-2}u + W'_u(t,\ u) = 0 \tag{4.2}$$

where $p = q$. Observe that in this case we need $2p < \mu_p$.

$$(|u'|^{p-2}u')' + (|u'|^{q-2}u')' - |u|^{p-2}u - |u|^{q-2}u + W'_u(t,\ u) = 0 \tag{4.3}$$

where $p > q > 1$.

Remark 4.1. If in System (P) we consider $a(t) = t^{\frac{1-p}{p}}c(t)$, so that $c \in C(\mathbb{R}^+,\ \mathbb{R})$ is strictly increasing we can construct many other examples.

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