MONOTONICALLY DOMINATED OPERATORS ON CONVEX CONES

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ABSTRACT - Let X be a compact Hausdorff space and let $(\mathcal{C}, d)$ be a metric convex cone. The convex cone $C(X; \mathcal{C})$ of all continuous functions from X into $(\mathcal{C}, d)$ is endowed with the topology of uniform convergence. Our purpose is to establish convergence results and quantitative estimates for sequences $\{T_n\}_{n\geq 1}$ of convex conic operators on $C(X; \mathcal{C})$ which are monotonically dominated.

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§ 1. INTRODUCTION

Let X be a compact Hausdorff space and let $(\mathcal{C}, d)$ be a metric convex cone. The convex cone $C(X; \mathcal{C})$ of all continuous functions from X into $(\mathcal{C}, d)$ is endowed with the topology of uniform convergence, determined by the metric

$$d(F, G) = \sup\{d(F(x), G(x))\ ;\ x \in X\}$$

for all $F, G \in C(X; \mathcal{C})$.

Our purpose is to establish convergence results and quantitative estimates for sequences $\{T_n\}_{n \geq 1}$ of convex conic operators on $C(X; \mathcal{C})$ which are monotonically dominated, i.e., for some positive linear operator S_n on $C(X; \mathbb{R})$ we have

$$D(T_n F, T_n G) \leq S_n(D(F, G)), \quad n = 1, 2, 3, \ldots$$

where $D(F, G)$ is the function $t \longmapsto d(F(t), G(t)), t \in X$.

For the definition of metric convex cones and of convex conic operators on them, see the survey paper of Prolla [4] in these Proceedings.

§ 2. CONVERGENCE RESULTS

For each $K \in \mathcal{C}$, we denote by K^* the element of $C(X;\ \mathcal{C})$ defined by $K^*(t) = K$, for all $t \in X$.

We recall that a linear operator S on $C(X;\ I\!R)$ is called **positive** (or monotone) if $f \geq 0$ implies $S\, f \geq 0$.

Lemma 1. *Let $\{S_n\}_{n\geq 1}$ be a sequence of positive linear operators on the space $C(X;\ I\!R)$ such that $S_n g \to g$, for all $g \in C(X; I\!R)$. If $F \in C(X; \mathcal{C})$, then $(S_n(D(F,\ [F(x)]^*)),\ x) \to 0$, uniformly in $x \in X$.*

Proof. Let $F \in C(X; \mathcal{C})$ and $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta(3 + \delta) < \varepsilon$. By the compactness of X and the uniform continuity of F, there exists a finite set $\{x_1,\ x_2, \ldots, x_m\} \subset X$ such that, given $x \in X$ there is $i \in \{1,\ 2, \ldots, m\}$ such that $d(F(x),\ F(x_i)) < \delta$. Choose n_o so that $n \geq n_o$ implies

$$(S_n e_0,\ x) < 1 + \delta,$$
$$(S_n\ D(F,\ [F(x_i)]^*),\ x) < d(F(x),\ F(x_i)) + \delta,$$

for all $x \in X$ and all $i = 1,\ 2, \ldots, m$, where e_0 denotes the real-valued function $e_0(t) = 1$, for all $t \in X$.

Let now $x \in X$. Choose $i \in \{1, 2, \ldots m\}$ so that $d(F(x),\ F(x_i)) < \delta$. It follows that

$$D(F, [F(x)]^*) \leq D(F,\ [F(x_i)]^*) + \delta e_0.$$

Then, for all $n \geq n_0$ we have

$$\begin{aligned}(S_n D(F,\ [F(x)]^*),\ x) &\leq (S_n D(F,\ [F(x_i)]^*),\ x) + \delta(S_n e_0,\ x)\\ &\leq d(F(x),\ F(x_i)) + \delta + (1+\delta) < \varepsilon.\end{aligned}$$

□

Definition 1. Let T be a convex conic operator on $C(X; \mathcal{C})$, and let S be a linear operator on $C(X;\ I\!R)$. We say that T is **monotonically dominated by** S if

$$D(TF,\ TG) \leq S(D(F,\ G))$$

for all functions $F,\ G \in C(X;\ \mathcal{C})$.

Notice that, if T is monotonically dominated by S, then S is positive.

Remark. If T is monotonically dominated by S, then $D(TK^*, TL^*) \leq d(K,\ L).\ S(e_0)$, for all $K,\ L \in \mathcal{C}$.

Example 1. Let J be a finite set, and for each $k \in J$, let $t_k \in X$ and $\psi_k \in C^+(X)$ be given. The convex conic operator T, defined on $C(X;\mathcal{C})$, by

$$(TF,\ x) = \sum_{k\in J} \psi_k(x),\ F(t_k)$$

for all $F \in C(X;\mathcal{C})$ and $x \in X$ is called an operator of interpolation type. Since

$$\begin{aligned} d((TF,\ x),\ (TG,\ x)) &= d(\sum_{k\in J}\psi_k(x)F(t_k),\ \sum_{k\in J}\psi_k(x)G(t_k)) \\ &\leq \sum_{k\in J}\psi_k(x)d(F(t_k),\ G(t_k)) \\ &= (S(D(F,\ G)),\ x) \end{aligned}$$

for all $x \in X$ and $F,\ G \in C(X;\mathcal{C})$, where, for each $f \in C(X;\ \mathbb{R})$,

$$(Sf,\ x) = \sum_{k\in J}\psi_k(x)f(t_k),$$

it follows that T is monotonically dominated by S.

Lemma 2. *Let $\{T_n\}_{n\geq 1}$ be a sequence of convex conic operators on $C(X;\mathcal{C})$ such that each T_n is monotonically dominated by a linear operator on $C(X;\ \mathbb{R})$. Assume that*

(1) $S_n e_0 \to e_0$,

(2) $T_n K^* \to K^*$, for every $K \in \mathcal{C}$.

Then $(T_n[F(x)]^*,\ x) \to F(x)$, uniformly in $x \in X$, for every $F \in C(X;\mathcal{C})$.

Proof. Let $F \in C(X;\mathcal{C})$ and $\varepsilon > 0$ be given. Choose $\delta > 0$ such that $\delta(3+\delta) < \varepsilon$. As in the proof of Lemma 1 there exist $x_1,\ x_2,\ldots,x_m$ in X such that, given $x \in X$ there is x_i such that $d(F(x),\ F(x_i)) < \delta$. By (1) and (2) we can choose n_0 so that $n \geq n_0$ implies

(3) $(S_n e_0,\ x) < 1 + \delta$, and

(4) $d((T_n[F(x_i)]^*,\ x),\ F(x_i)) < \delta$,

for all $x \in X$ and all $i = 1,\ 2,\ldots,m$.

Let now $x \in X$. Choose $x_i \in X$ such that $d(F(x),\ F(x_i)) < \delta$. It follows that $D([F(x)]^*,\ [F(x_i)]^*) \leq \delta e_0$. Since each S_n is positive and linear, (3) implies

$$(S_n D([F(x)]^*,\ [F(x_i)]^*),\ x) \leq \delta(S_n e_0,\ x) \leq \delta(1+\delta)$$

for all $n \geq n_0$.

From (4) and the hypothesis that T_n is monotonically dominated by S_n, we have, for $n \geq n_0$

$$\begin{aligned} d((T_n[F(x)]^*,\ x),\ F(x)) &\leq d((T_n[F(x)]^*,\ x),\ (T_n[F(x_i)]^*,\ x)) + \\ &+\ d((T_n[F(x_i)]^*,\ x),\ F(x_i)) + d(F(x_i),\ F(x)) \\ &\leq (S_n D([F(x)]^*,\ [F(x_i)]^*),\ x) + 2\delta < \delta(1+\delta) + 2\delta < \varepsilon. \end{aligned}$$

Hence $(T_n[F(x)]^*, x) \to F(x)$, uniformly in $x \in X$.

□

Theorem 1. *Let $\{T_n\}_{n\geq 1}$ be a sequence of convex conic operators on $C(X; \mathcal{C})$ suc[h] that each T_n is monotonically dominated by a linear operator S_n on $C(X; \mathbb{R})$. As[-]sume that*

(1) $S_n g \to g$, for every $g \in C(X; \mathbb{R})$,

(2) $T_n K^* \to K^*$, for every $K \in \mathcal{C}$.

Then $T_n F \to F$, for every $F \in C(X; \mathcal{C})$.

Proof. Let $F \in C(X; \mathcal{C})$ and $\varepsilon > 0$ be given. By Lemma 1 and Lemma 2, choose n[...] so that $n \geq n_0$ implies $(S_n D(F, [F(x)]^*), x) < \varepsilon/2$ and $d((T_n[F(x)]^*, x), F(x)) <$ $\varepsilon/2$, for every $x \in X$. Since T_n is monotonically dominated by S_n it follows that for $n \geq n_0$

$$\begin{aligned} d((T_nF, x), F(x)) &\leq d((T_nF, x), (T_n[F(x)]^*, x)) \\ &+ d((T_n[F(x)]^*, x), F(x)) \\ &\leq (S_nD(F, [F(x)]^*), x) + \frac{\delta}{2} < \varepsilon \end{aligned}$$

for every $x \in X$. Therefore, $T_nF \to F$.

□

Let φ be a non negative bounded function defined on $X \times X$, which satisfies th[e] following conditions:

(A) φ_x is continuous, for each $x \in X$, where $\varphi_x(t) = \varphi(x, t)$, for all $t \in X$;

(B) $\inf \{\varphi(x, y); (x, y) \in M\} > 0$ for every compact and non empty set M of th[e] complement of the diagonal set $\Delta = \{(t, t); t \in X\}$ in $X \times X$.

Let $\{S_n\}_{n\geq 1}$ be a sequence of positive linear operators on $C(X; \mathbb{R})$.

We denote by α_n the function defined by

$$\alpha_n(x) = (S_n(\varphi_x), x)$$

for all $x \in X$. Notice that α_n depends on S_n and φ.

Example 2. When X is a compact metric space with metric $\tilde{d}$, then $\varphi(x, t) =$ $\tilde{d}(x, t)$, for $t, x \in X$, satisfies (A) and (B).

More generally, if X is a compact subset of a metric space $(Y, \tilde{d})$ then, for eac[h] $\rho > 0$, $\varphi(x, t) = [\tilde{d}(x, t)]^\rho$, for $x, t \in X$, satisfies the conditions (A) and (B).

Corollary 1. *Let $\{S_n\}_{n\geq 1}$ and $\{T_n\}_{n\geq 1}$ be as in Theorem 1. Assume that φ satisfies (A) and (B), and*

(1) $S_n e_0 \to e_0$,

(2) $\alpha_n(x) \to 0$, uniformly in $x \in X$,

(3) $T_n K^* \to K^*$, for every $K \in \mathcal{C}$.

Then $T_n F \to F$, for every $F \in C(X; \mathcal{C})$.

Proof. By Nishishiraho ([2], Theorem 1), the hypothesis (1) and (2) imply that $S_n g \to g$, for every $g \in C(X; \mathbb{R})$. It remains to apply Theorem 1.

□

Example 3. Let X be a compact Hausdorff space. Let $g_1, g_2, \ldots, g_m$ be elements of $C^+(X)$ such that there exist bounded functions $\beta_1, \beta_2, \ldots, \beta_m$ on X such that, if we define

$$\varphi(x, t) = \sum_{i=1}^{m} \beta_i(x) g_i(t)$$

for all $x,\ t \in X$, then $\varphi(x,\ t) \geq 0$, $\varphi(x,\ x) = 0$ and (B) is satisfied.

Then $\mathcal{H} = \{e_0,\ g_1,\ g_2, \ldots, g_m\}$ is a Korovkin system in $C(X;\ \mathbb{R})$. Indeed, let $\{S_n\}_{n\geq 1}$ be a sequence of positive linear operators on $C(X,\ \mathbb{R})$. Assume that $S_n e_0 \to e_0$ and $S_n g_i \to g_i$, for every $i = 1,\ 2, \ldots, m$. Then

$$\alpha_n(x) = (S_n(\varphi_x),\ x) \to 0,$$

uniformly in $x \in X$, since $\varphi(x,\ x) = 0$. By Nishishiraho ([2], Theorem 1) it follows that $S_n g \to g$, for all $g \in C(X;\ \mathbb{R})$. Therefore, $\mathcal{H}$ is a Korovkin system in $C(X;\ \mathbb{R})$.

Corollary 2. *Let $\{S_n\}_{n\geq 1}$ and $\{T_n\}_{n\geq 1}$ be as in Theorem 1. Let $g_1,\ g_2, \ldots, g_m$ be as in Example 2. Assume that $S_n h \to h$, for every $h \in \{e_0,\ g_1,\ g_2, \ldots, g_m\}$, and that $T_n K^* \to K^*$, for all $K \in \mathcal{C}$. Then $T_n F \to F$, for all $F \in C(X; \mathcal{C})$.*

Proof. By Example 2, $\mathcal{H} = \{e_0,\ g_1,\ g_2, \ldots, g_m\}$ is a Korovkin system in $C(X;\ \mathbb{R})$. Thus, $S_n h \to h$, for all $h \in \mathcal{H}$, implies $S_n g \to g$, for all $g \in C(X;\ \mathbb{R})$. It remains to apply Theorem 1.

□

§ 3. QUANTITATIVE ESTIMATES FOR MONOTONICALLY DOMINATED OPERATORS

Notice that $\mathcal{C}$ being a metric space and X being compact, every element $F \in C(X; \mathcal{C})$ is in fact uniformly continuous. In the particular case that X is a compact metric space, say with metric $\tilde{d}$, this means that for every $\varepsilon > 0$, there exists $\delta > 0$ such that $x,\ t \in X$, $\tilde{d}(x,\ t) < \delta$ implies $d(F(x),\ F(t)) < \varepsilon$.

The modulus of continuity of $F \in C(X; \mathcal{C})$ is then defined as

$$\omega(F, \delta) = \sup\{d(F(x), F(t)); x, t \in X, \tilde{d}(x, t) \leq \delta\}$$

for every $\delta > 0$. By uniform continuity of F, we have $\omega(F, \delta) \to 0$ as $\delta \to 0$. Notice also that $\omega(F, \delta)$ is monotonically increasing, i.e., $\delta_1 \leq \delta_2$ implies $\omega(F, \delta_1) \leq \omega(F, \delta_2)$.

Let us consider the following condition:

$(*)$ There exists a constant p with $0 < p \leq 1$ such that $\omega(F, \lambda\delta) \leq [1 + \lambda^{\frac{1}{p}}]\omega(F, \delta)$, for all $F \in C(X; \mathcal{C})$ and all $\delta, \lambda > 0$.

Example 4.

(4.1) Let X be a compact convex subset of a metric linear space $(Y, \tilde{d})$. Suppose that $\tilde{d}$ is invariant, i.e., $\tilde{d}(x + z, t + z) = \tilde{d}(x, t)$, for all $x, t, z \in Y$, and that $\tilde{d}(\beta x, 0) \leq \beta\tilde{d}(x, 0)$, for all $x \in Y$ and all β with $0 \leq \beta \leq 1$. Then $(*)$ holds for $p = 1$.

(4.2) Suppose that $\tilde{d}$ has the property that if $\tilde{d}(x, t) = a + b$, where $a > 0$ and $b > 0$. Then there exists a point $z \in X$ such that $\tilde{d}(x, z) = a$ and $\tilde{d}(z, t) = b$. Then $(*)$ holds for $p = 1$.

(4.3) Let X be a compact convex subset of a q-normed linear space $(Y, |||\cdot|||)$ with $0 < q < 1$. Then $(*)$ holds for $p = q$.

We recall that a q-normed linear space, with $0 < q \leq 1$, is a linear space Y with a real-valued function $|||\cdot||| : Y \to [0, \infty)$ such that (see Köthe [1]):

(a) $|||y||| = 0 \Leftrightarrow y = 0$,

(b) $|||\lambda y||| = |\lambda|^q \cdot |||y|||$,

(c) $|||y + z||| \leq |||y||| + |||z|||$,

for all $\lambda \in \mathbb{R}$ and $y, z \in Y$. If $q = 1$, we obtain the concept of a norm.

The class of q-normed spaces includes the spaces ℓ^q, for $0 < q < 1$, with the q-norm defined by $|||(x_n)_{n=1}^{\infty}||| = \sum_{n=1}^{\infty} |x_n|^q$.

Lemma 3. *Assume that $(*)$ holds. Let $F \in C(X; \mathcal{C})$ and $\delta > 0$ be given. Then*

$$d(F(x), F(t)) \leq \left[1 + \left(\frac{\tilde{d}(x, t)}{\delta}\right)^{\frac{1}{p}}\right]\omega(F, \delta)$$

for every pair, x and t, of elements of X.

Proof. If $\tilde{d}(x, t) \geq \delta$, then

$$d(F(x), F(t)) \leq \omega(F, \tilde{d}(x, t)) \leq \left[1 + \left(\frac{\tilde{d}(x, t)}{\delta}\right)^{\frac{1}{p}}\right]\omega(F, \delta).$$

If $\tilde{d}(x, t) \leq \delta$ then

$$d(F(x), F(t)) \leq \omega(F, \delta) \leq \left[1 + \left(\frac{\tilde{d}(x, t)}{\delta}\right)^{\frac{1}{p}}\right]\omega(F, \delta).$$

□

If $\{S_n\}_{n\geq 1}$ is a sequence of positive linear operators on $C(X, I\!R)$, let

$$\alpha_n(x) = (S_n[\tilde{d}(x, \cdot)]^{\frac{1}{p}}, x)$$

for all $x \in X$, where p is given by condition $(*)$.

Theorem 3. *Assume that $(*)$ holds. Let $\{T_n\}_{n\geq 1}$ be a sequence of convex conic operators on $C(X; \mathcal{C})$ such that each T_n is monotonically dominated by a linear operator S_n on $C(X; I\!R)$. Then*

$$d((T_n F, x), F(x)) \leq \left[(S_n e_0, x) + \frac{1}{\delta^{\frac{1}{p}}}\alpha_n(x)\right]\omega(F, \delta) + d((T_n[F(x)]^*, x), F(x))$$

for every $F \in C(X, \mathcal{C})$, $x \in X$ and $\delta > 0$.

Proof. Let $F \in C(X; \mathcal{C})$ and $\delta > 0$ be given. By Lemma 3, we have for x, $t \in X$

$$d(F(x), F(t)) \leq \left[1 + \left(\frac{\tilde{d}(x, t)}{\delta}\right)^{\frac{1}{p}}\right]\omega(F, \delta).$$

It follows that, for $x \in X$

$$D(F, [F(x)]^*) \leq \left[e_0 + \frac{1}{\delta^{\frac{1}{p}}}(\tilde{d}_x)^{\frac{1}{p}}\right]\omega(F, \delta),$$

where the function $\tilde{d}_x$ is defined by $\tilde{d}_x(t) = \tilde{d}(x, t)$, for all $t \in X$.

Since S_n is positive and linear

$$(S_n D(F, [F(x)]^*), x) \leq [(S_n e_0, x) + \frac{1}{\delta^{\frac{1}{p}}}\alpha_n(x)]\omega(F, \delta).$$

Now T_n is monotonically dominated by S_n, and therefore

$$\begin{aligned} d((T_n F, x), F(x)) &\leq d((T_n F, x), (T_n[F(x)]^*, x)) + d((T_n[F(x)]^*, x), F(x)) \\ &\leq (S_n D(F, [F(x)]^*), x) + d((T_n[F(x)]^*, x), F(x)) \\ &\leq [(S_n e_0, x) + \frac{1}{\delta^{\frac{1}{p}}}\alpha_n(x)]\omega(F, \delta) + d((T_n[F(x)]^*, x), F(x)) \end{aligned}$$

for all $x \in X$.

$\square$

Corollary 3. *Let $\{S_n\}_{n\geq 1}$ and $\{T_n\}_{n\geq 1}$ be as in Theorem 3. If $S_n e_0 = e_0$ and $T_n K^* = K^*$, for all $K \in \mathcal{C}$ and $n \in \mathbb{N}$ then*

$$d((T_n F, x), F(x)) \leq [1 + \frac{1}{\delta^{\frac{1}{p}}} \alpha_n(x)] \omega(F, \delta)$$

for every $F \in C(X, \mathcal{C})$, $x \in X$ and $\delta > 0$.

Corollary 4. *Let $\{S_n\}_{n\geq 1}$ and $\{T_n\}_{n\geq 1}$ be as in Corollary 3. At every point $x \in X$ where $\alpha_n(x) > 0$, we have*

$$d((T_n F, x), F(x)) \leq 2\omega(F, \alpha_n(x))$$

for every $F \in C(X; \mathcal{C})$ and $n = 1, 2, \ldots$

Proof. Make $\delta = \alpha_n(x)$ in Corollary 3 and notice that $\alpha_n(x) \leq [\alpha_n(x)]^{\frac{1}{p}}$, since $p \leq 1$.

$\square$

Remark. We write $\alpha_n(x) = 0(n^{-1})$, uniformly in $x \in X$, if there is some constant $k > 0$ such that $n\alpha_n(x) \leq k$ for all $n = 1, 2, 3, \ldots$, and all $x \in X$.

Theorem 4. *Assume that $(*)$ holds. Let $\{T_n\}$ be a sequence of convex conic operators on $C(X, \mathcal{C})$ such that each T_n is monotonically dominated by a linear operator S_n on $C(X, \mathbb{R})$. Assume that*

(i) $S_n e_0 \to e_0$,

(ii) $T_n K^* \to K^*$, for every $K \in \mathcal{C}$,

(iii) $\alpha_n(x) = 0(n^{-1})$, uniformly in $x \in X$.

Then $T_n F \to F$, for every $F \in C(X; \mathcal{C})$.

Proof. Let $F \in C(X, \mathcal{C})$ and $\varepsilon > 0$ be given. By (i), (ii) and Lemma 2 choose n_1 so that $n \geq n_1$ implies

(1) $(S_n e_0, x) < 1 + \varepsilon/2$,

(2) $d((T_n[F(x)]^*, x), F(x)) < \varepsilon/2$,

for all $x \in X$. By (iii) there is some constant $k > 0$ such that

(3) $n\alpha_n(x) \leq k$,

for $n = 1, 2, \ldots$ and all $x \in X$. Since $\omega(F, \delta) \to 0$ as $\delta \to 0$, we can choose n_2 such that $n \geq n_2$ implies

(4) $\omega(F, n^{-p}) \leq \varepsilon/2(1 + k + \varepsilon/2)^{-1}$.

By Theorem 3 and (1) - (4), it follows that for $n \geq n_0 = \max\{n_1, n_2\}$

$$\begin{aligned} d((T_nF, x), F(x)) &\leq [(S_n e_0, x) + \frac{1}{\delta^{\frac{1}{p}}}\alpha_n(x)]\omega(F, \delta) + d((T_n[F(x)]^*, x), F(x)) \\ &= [(S_n e_0, x) + n\alpha_n(x)]\omega(F, n^{-p}) + d((T_n[F(x)]^*, x), F(x)) \\ &< (1 + k + \varepsilon/2]\omega(F, n^{-p}) + \varepsilon/2 < \varepsilon, \end{aligned}$$

for all $x \in X$. □

Corollary 5. *Let* $\{S_n\}_{n\geq 1}$ *and* $\{T_n\}_{n\geq 1}$ *be as in Theorem 4. Assume that* $S_n e_0 = e_0$, $T_n K^* = K^*$, *for all* $K \in \mathcal{C}$ *and* $n \in \mathbb{N}$, *and* $\alpha_n(x) = 0(n^{-1})$, *uniformly in* $x \in X$. *Then* $T_n F \to F$, *for every* $F \in C(X; \mathcal{C})$.

REFERENCES

[1] G. Köthe, *Topologische lineare Räume I*, Springer-Verlag, Berlin-Göttingen-Heidelberg 1960.

[2] T. Nishishiraho, Convergence of positive linear approximation processes, *Tôhoku Math. J.* **35**(1983), 441-458.

[3] J. B. Prolla, Approximation of Continuous Convex-Cone-Valued Functions by Monotone Operators, to appear in *Studia Math.* **102**.

[4] J. B. Prolla, Uniform Approximation of Continuous Convex-Cone-Valued Functions, these *Proceedings*.

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