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THE FEYNMAN-DYSON PROOF OF MAXWELL EQUATIONS AND MAGNETIC MONOPOLES

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ABSTRACT – Using a violation of the Jacobi Identity[3,4] we are able to generalize the Feynman's Proof of the Maxwell Equations including magnetic monopoles.

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The Feynman - Dyson proof of Maxwell equations and magnetic monopoles

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Abstract. Using a violation of the Jacobi Identity[3,4] we are able to generalize the Feynman's Proof of the Maxwell Equations including magnetic monopoles.

In 1990 Dyson[1] published a proof due to Feynman that the Maxwell equations follow from Newton's equation

$$m\,\ddot{x}_j = F_j(x, \dot{x}, t) \tag{1}$$

and the quantum mechanical canonical rules

$$[x_j, x_k] = 0 \tag{2}$$

$$m[x_j, \dot{x}_k] = i\,\hbar\,\delta_{jk}\,. \tag{3}$$

Soon after, Lee[2] extended the Feynman's proof to non - abelian gauge fields, obtaining the Yang-Mills equations. In his paper, Lee suggested that magnetic monopoles can be introduced, through Feynman's approach using the dual Lorentz force equation

$$F_j = B_j - \varepsilon_{jk\ell}\dot{x}_k E_\ell\,. \tag{4}$$

It is possible to obtain the magnetic monopoles without postulating the dual Lorentz force. This is shown below.

In his proof Feynman have used twice the well known Jacobi Identity

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0\,. \tag{5}$$

Magnetic monopoles appear when we have a violation of Jacobi Identity for the kinetic momenta $p_k = m\,\dot{x}_k$.

We follow Dyson-Feynman[1] closely and point out the necessary changes to include magnetic monopoles.

From (1) and (3) we have

$$[x_j, F_k] = -m[\dot{x}_j, \dot{x}_k]\,. \tag{6}$$

Now, we use the Jacobi Identity (5) for operators x_j and $\dot{x}_k$ in the form

$$[x_\ell, [\dot{x}_j, \dot{x}_k]] + [\dot{x}_j, [\dot{x}_k, x_\ell]] + [\dot{x}_k, [x_\ell, \dot{x}_j]] = 0\,. \tag{7}$$

From (3) it's easy to see that the two last terms in the left-handed side of above equation, vanish.

So (7) can be writen

$$[x_\ell, [\dot{x}_j, \dot{x}_k]] = 0\,. \tag{8}$$

This equation means that the comutator $[\dot{x}_j, \dot{x}_k]$ is a function of x and t only. So, from (6) and (8) we can define the magnetic field H as

$$[x_j, F_k] = (\frac{-i\,\hbar}{m})\varepsilon_{jk\ell}\,H_\ell \tag{9}$$

and the eletric field as

$$E_j = F_j - \varepsilon_{jk\ell}\,\dot{x}_k\,H_\ell \tag{10}$$

and, of course, H_ℓ and E_j are also functions of x and t only.

Substituting (6) and (9) in the Jacobi Identity in the form

$$\varepsilon_{jk\ell}[\dot{x}_\ell, [\dot{x}_j, \dot{x}_k]] = 0\,. \tag{11}$$

We conclude that

$$[\dot{x}_\ell, H_\ell] = 0 \tag{12}$$

which is equivalent to

$$\mathrm{div}\ \vec{H} = 0\,. \tag{13}$$

Now, as shown by Jackiw[3] and Wu and Zee[4], the existence of magnetic monopoles implies the violation of Jacobi Identity (11) and this is the very definition of magnetic charge, namely

$$\operatorname{div} \vec{H} = \frac{1}{\hbar^2}\, \varepsilon_{jk\ell}[p_\ell, [p_j, p_k]] = \rho_{\text{mag}} \tag{14}$$

where we have rewritten (11) in terms of kinetic momenta $p_j = m\,\dot{x}_j$.

Using (6) we can rewrite (9) as

$$H_\ell = \frac{-im^2}{\hbar^2} \varepsilon_{jk\ell}[\dot{x}_j, \dot{x}_k]\,. \tag{15}$$

The total time derivative of (15) is

$$\frac{\partial H_\ell}{\partial t} + \dot{x}_m \frac{\partial H_\ell}{\partial x_m} = \frac{-im^2}{\hbar^2} \varepsilon_{jk\ell}[\ddot{x}_j, \dot{x}_k]\,. \tag{16}$$

After some calculations on the right-hand side the above equation we get

$$\frac{\partial H_\ell}{\partial t} - \varepsilon_{jk\ell} \frac{\partial E_j}{\partial x_k} = -\dot{x}_\ell \frac{\partial H_k}{\partial x_k}\,. \tag{17}$$

The right-handed side of this equation defines the magnetic current, using (14)

$$-\dot{x}_\ell\, \rho_{\text{mag}} = j_\ell \tag{18}$$

and so we obtain the second generalized Maxwell equation

$$\frac{\partial H_\ell}{\partial t} - \varepsilon_{jk\ell} \frac{\partial E_j}{\partial x_k} = j_\ell\,. \tag{19}$$

The other two non-homogeneous Maxwell equations

$$\operatorname{div} \vec{E} = \rho_{\text{eletric}} \tag{20}$$

$$\operatorname{cur}\ell\, \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}_{\text{eletric}} \tag{21}$$

are interpreted in Feynman-Dyson approach as defining the very eletric charge and current.

This have caused a certain uneasiness[5–10] because apparently there is no physical or mathematical principle to fix the non-homogeneous equations such that the complete set of Maxwell equations results Lorentz invariant.

Nevertheless, we agree with Farina and Vaydia[5], and Hojman and Shepley[10] that it is necessary to introduce a parameter with units of velocity. This arbitrary parameter is shown to be independent of the observer[11] using weaker assumptions on isotropy and homogeneity of space than the original conditions used by Einstein, obtaining in this way the Lorentz transformations. But, unfortunately we can not yet fix the non-homogeneous equations from the postulates (1), (2), (3).

Another shortcoming is related to a Lagrangian formulation of magnetic monopoles theories. Hojman and Shepley[10] have shown that if we don't have a Lagrangian for a physical system we can't quantize it.

However the monopole theory, where the monopole didn't arise from a change of the topology of the world manilfold, is an example of a quantum system for which there doesn't exist a Lagrangian[12] giving simultaneonsly the field equations and the equations of motion of changes and monopoles. So, it would be interesting to investigate how and why this kind of monopole overrides the Hojman and Shepley's theorem. To end we call the reader's attention that we have shown elsewhere[13] that the equations of motion for both charges and monopoles follows directly from the generalized Maxwell equations without any ad-hoc postulate, a result complementar to the above one.

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