POSITIVE ORDERED SOLUTIONS OF A ANALOGUE OF NON-LINEAR ELLIPITIC EIGENVALUE PROBLEMS

Pedro C. Espinoza

RELATÓRIO TÉCNICO Nº 40/91

Abstract. Let $-\Delta u = \lambda f(u)$ be a nonlinear elliptic eigenvalue problem, where f change its sign. By using variational methods we show that its discrete analogue has ordered positive solutions.

Universidade Estadual de Campinas
Instituto de Matemática, Estatística e Ciência da Computação
IMECC – UNICAMP
Caixa Postal 6065
13.081 – Campinas – SP
BRASIL

Setembro – 1991

Positive Ordered Solutions of a Analogue of Non-Linear Elliptic Eigenvalue Problems

Pedro C. Espinoza
Facultad de Ciencias Matematicas. Universidad
Nacional Mayor de San Marcos Lima – Perú

Abstract.

Let $-\Delta u = \lambda f(u)$ be a nonlinear elliptic eigenvalue problem, where f change its sign. By using variational methods we show that its discrete analogue has ordered positive solutions.

0. Introduction

Let us consider the following elliptic eigenvalue problem

(0.1) $$-\Delta u = \lambda f(u) \text{ in } \Omega, u = 0 \text{ on } \partial\Omega$$

where λ is a positive real parameter, Ω a smooth bounded domain in $I\!R^N, f : [0,\infty) \longrightarrow I\!R$ a function that satisfies the following conditions

(0.2) $$\begin{cases} \text{a)} f \text{ is locally Lipschitz continuous} \\ \text{b) There exists real numbers } s_0 = 0 < s_1 < s_2 < \cdots < s_{2m-1} \text{ such that} \\ f(s_0) > 0 \text{ and } f(s_i) = 0 \; \forall\, i = 1,2,\ldots 2m-1 \text{ Furthermore } f \text{ changes} \\ \text{its sign i.e.} (-1)^i f(s) > 0 \text{ for s } \in (s_i, s_{i+1}), i = 0,1,\ldots,2(m-1) \\ \text{c)} F(s_{2i+1}) > F(s_{2i-1}) \; \forall\, i = 1,2,\ldots,m-1, \text{ where } F(t) = \int_0^t f(s)ds \end{cases}$$

The expression below

(0.3) $$Ax = \lambda h^2 \widetilde{f}(x),\; x \in I\!R^n$$

is the discrete analogue of (0.1) obtained by finite difference method (we restrict ourselves here, for purpose of simplicity, to $N = 1,2$). A is a M-matrix ([9], [10]) obtained in the discretization of the differential operator $-\Delta$, h is the mesh size and $\widetilde{f}(x) = (f(x_1),\ldots,f(x_n))$ is the Nemitskii operator associated with the scalar function f.

The solutions of (0.3) are the critical points of the function

(0.4) $$\varphi(\lambda, x) = \frac{1}{2} < Ax, x > -\lambda h^2 \sum_{i=1}^{n} F(x_i)$$

where $<,>$ is the usual scalar product of $I\!R^n$ and F is the primitive of the functions f described in (0.2)

φ is the discrete analogue of

(0.5) $$J(\lambda, u) = \frac{1}{2} \int_\Omega |\nabla u|^2 - \int_\Omega F(u)$$

the functional whose Euler equation is (0.1). φ and J are neither convex nor monotone operators. Concerning the study of (0.3), there exists an abundant literature applying techniques based on topological degree and global bifurcation theories. We mention the papers of Peitgen, Saupe and Schmitt ([6], [7]) where independent of condition (0.2) (c), they obtain continuum of montrivial solutions of (0.3). In spite of its power, this method does not avoid the occurrence of numerically irrelevant solutions "NIS", which do not approximate the solution of the corresponding continuous problem.

In this note, we will make use the variational method used in a paper of De Figueiredo ([2]) and also by Brown and Budin ([4]), in order to show the existence of $2m - 1$ ordered positive solutions of (0.3) in the cone $I\!R^n_+ = \{x \in I\!R^n : x_i \geq 0 \; \forall \; i\}$ for large λ.

Our approach is based strongly on the condition (0.2) (c) i.e. "the area condition posed in the nonlinearlity f" (apparently in this way it is possible to avoid the NIS).

Section 1 is devoted to the study of M-matrices and some properties of the solutions of (0.3).

Section 2 is devoted to see briefly the Palais-Smale condition and one Montain Pass Theorem, as well as the proof of the existence of $2m - 1$ ordered positive solutions of (0.3).

Acknowledgement

The author thank to profesor D'jairo G. De Figueiredo for his invitation to visit the Instituto de Matemática, Estatística e Ciencia da Computação (IMECC) - UNICAMP, moreover for his valuable suggestions during the Seminar developed with him.

This work was partially supported by "Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) - Brasil and Sociedade Brasileira de Matemática (SBM).

1. - Some remarks on M-matrices

Notations 1.1

(1.1) $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x_i \geq 0 \ \forall\, i = 1, \dots n\}$; $\overset{\circ}{\mathbb{R}}{}^n_+ =$ interior $(\mathbb{R}^n_+)$

(1.2) $x^+ = (x_1^+, \dots, x_n^+)$; $x^- = (x_1^-, \dots, x_n^-)$. Hence $x = x^+ - x^-$ and $x^+, x^- \in \mathbb{R}^n_+$.

(1.3) Let $x, y \in \mathbb{R}^n$; then we define $x \leq y$ if $x_i \leq y_i \ \forall\, i = 1, \dots, n$ and $x < y$ when $x \leq y$ and $x \neq y$.

(1.4) $< x, y >$ is the usual scalar product of $\mathbb{R}^n$; $||x|| = (< x, y >)^{1/2}$ and $||x||_\infty = \max\{|x_i|; i = 1, \dots n\}$

Definition 1.2

Let $A = [a_{ij}]$ be a real $n \times n$ matrix. We will say that A is a M-matrix if

$$
(1.5) \quad \begin{cases}
\text{a) } a_{ii} > 0, a_{ij} \leq 0 \text{ for } i \neq j \\
\text{b) } a_{ii} \geq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|, \text{ with strict inequality for at least one } i \\
\text{(hence } \sum_{j=1}^{m} a_{ij} \geq 0, \text{ with strict inequality for some } i \text{)} \\
\text{c) } A \text{ is irreducible.}
\end{cases}
$$

Lemma 1.3

Let A be a M-matrix. Then

a) A is non singular

b) $A^{-1}(\mathbb{R}^n_+) \subset \mathbb{R}^n_+$ and $A^{-1}(\mathbb{R}^n_+ - 0) \subset \overset{\circ}{\mathbb{R}}{}^n_+$

c) A has a unique positive eigenvalue λ_1 of multiplicity one with associated eigenvector $w \in \overset{\circ}{\mathbb{R}}{}^n_+$.

(see the proof in [10])

Lemma 1.4

The matrix A in (3.0), arising by discretization of the differential operator $-\Delta$ is a M-matrix which is symmetric, positive-definite. Moreover.

(1.6) $(< Ax, x >)^{1/2} = ||x||_A$ is a norm in $\mathbb{R}^n$

(1.7) $< Ax^+, x^- > = < Ax^-, x^+ > \leq 0$ for all x in $\mathbb{R}^n$.

Proof

For the proof of all statements except (1.6) and (1.7) see [10]. Now (1.6) is trivial, because A is a real symmetric positive definite matrix. Next (1.7) follows from

$$< Ax^+, x^- >= \sum_{i=1}^{n} a_{ii} x_i^+ x_i^- + \sum_{i \neq j} a_{ij} x_i^+ x_j^- .$$

using the fact that $a_{ij} \leq 0$ for $i \neq j$ and $x_i^+ x_j^- \geq 0$. □

We now give some properties concerning the solutions of (0.3).

Lemma 1.5

Let $f : I\!R \longrightarrow I\!R$ be a continuous function, with $f(0) > 0$ and define $f(s) = f(0)$ for $s \leq 0$.

Let

(1.8) $$Ax = \lambda \tilde{f}(x), \quad x \in I\!R^n, \lambda > 0$$

Then:

a) If x is solution of (1.8) for some $\lambda > 0$, then $x \in \overset{\circ}{I\!R}{}^n_+$

b) If, in addition, f is locally lipschitz and $f(\theta) \leq 0$ for some real $\theta \geq 0$. Then (1.8) has no solutions x such that $||x||_\infty = \theta$

c) Let us suppose that there exists a $\overline{\lambda} > 0$ such that (1.8) has a solution x_λ with $||x_\lambda||_\infty \leq$ constant for all $\lambda \geq \overline{\lambda}$. Then each i-incomponent $x_{\lambda i} \longrightarrow \alpha \in f^{-1}(0)$, as $\lambda \longrightarrow +\infty$

Proof

a) Let $x = (x_i, \ldots, x_n)$ be a solution of (1.8) and $x_i = \min\{x_j : j = 1, \ldots, n\}$. Suppose that $x_i \leq 0$. Since $a_{ij} \leq 0$ for $i \neq j$

$$(\sum_{j=1}^{n} a_{ij}) x_i \geq \sum_{j=1}^{n} a_{ij} x_j = \lambda f(x_i) = \lambda f(0) > 0.$$

Since A is a M-matrix the first term is non-positive. Thus we come to a contradiction.

b) Suppose that (1.8) has a solution x with $||x||_\infty = \theta$.

If $\theta = 0$, the contradictions is immediately.

If $\theta > 0$, since f is locally lipschitz continuous there exists $\sigma > \theta$ such that $f(s) + \sigma s$ is strictily increasing on $[0, \theta]$. Hence if $w = (\theta, \theta, \ldots, \theta)$, we have

$$(A + \lambda \sigma I)(w - x) \geq \lambda[\tilde{f}(w) + \sigma w - (\tilde{f}(x) + \sigma x)] \in \overset{\circ}{I\!R}{}^n_+$$

By Lemma 1.3 $\theta > x_i \ \forall i = 1, \ldots, n$ contradicting the assumption. (This idea is similar to the one in lema 6.2 [1]).

c) $\{x_\lambda\}$ is bounded $\forall \lambda \geq \overline{\lambda}$ and $A(\frac{x_\lambda}{\lambda}) = \widetilde{f}(x_\lambda)$.

Since every subset of $\{x_\lambda\}$ has a convergent subsequence converging to some $w \geq 0$, we have $\widetilde{f}(w) = 0$. Therefore $w_i \in f^{-1}(0)$ and $x_{\lambda i} \longrightarrow w_i$ when $\lambda \longrightarrow +\infty$. □

Remark 1.6

In case one replaces the condition $f(0) > 0$ by $f(0) = 0$ and $f'_+(0) > 0$ (see (f.1) [2]). The solutions of (0.3) will be in $I\!R^n_+$, provided we define $f(s) = -\mu s$ for $s \leq 0$, where $\mu > 0$ and satisfies $f(s) \geq -\mu s$ for $s \geq 0$

2 - The variational methods used to obtain ordered positive solutions of (0.3)

As observed before the critical points of the function

$$\varphi(x) = \frac{1}{2} < Ax, x > - \sum_{i=1}^{n} F(x_i), \text{ where } F(t) = \int_0^t t(s) \tag{2.1}$$

with f as in (0.2), are the solutions of (0.3). Hence in order to using variational methods, we shall need to establish some properties of φ.

The next proposition give a situation when φ is not bounded below.

Proposition 2.1

Suppose that

$$\lim_{s \to +\infty} \inf \frac{f(s)}{s} > \lambda_1, \tag{2.2}$$

where λ_1 is the (unique) positive eigenvalue of A. Then the functional φ described in (2.1) is unbounded below.

Proof

since $l = \liminf_{s \to} \frac{f(s)}{s} > \lambda_1, \ \forall \mu \in (\lambda_1, l)$, there exists $c_1 > 0$ such that $f(s) \geq \mu s - c_1$ for $s \geq 0$. Hence $\forall \sigma \in (\lambda_1, \mu)$, there exists $c_2 > 0$ such that
(2.3) $F(s) \geq \frac{1}{2}\sigma s^2 - c_2$ for $s > 0$. Therefore if w is the eigenvector associated to λ_1,

$$\varphi(tw) = \frac{1}{2} t^2 \lambda_1 ||w||^2 - \sum_{i=1}^{n} F(w_i) \leq \frac{1}{2} t^2 (\lambda_1 - \sigma) ||w||^2$$
$$+ nc_2 \longrightarrow -\infty \ \text{ when } |t| \longrightarrow +\infty. \ □$$

In order to study the critical points of functions of the type described in (2.1), we have to prove the Palais-Smale condition. It will be useful to have a direct criterion for it.

Definition 2.2 (Palais-Smale condition)

Let $\varphi : E \longrightarrow E$ be a C^1 functional, E a Banach space. We will say that φ satisfies the Palais-Smale condition, if every sequence $\{x_n\}$ in E which satisfies

$$(2.4) \qquad ||\varphi(x_n)||_E \leq \text{ constant and } \varphi'(x_n) \longrightarrow 0 \text{ in } E^*$$

prossesses a convergent (in norm) subsequence.

The next proposition is a adaptation of Lemma 6.3 [3], to the finite dimensional case and provides a direct criterion in order to know if (2.1) satisfies the Palais-Smale condition.

Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function. We assume that

$$(2.5) \qquad \lim_{s\to+\infty} \frac{f(s)}{s} = \alpha, \lim_{s\to-\infty} \frac{f(s)}{s} = \beta. \text{ Hence there exists}$$

$c_1 \geq 0$ and $c_2 \geq 0$ such that

$$(2.6) \qquad |f(s)| \leq c_1 s + c_2 \ \forall s \in \mathbb{R}$$

Proposition 2.3

Let φ be as in (2.1). We suppose that (2.5) hold and that

$$(2.7) \qquad Ax = \alpha x^+ - \beta x^-$$

has only the trivial solution. Then every sequence $\{x_k\}$ in $\mathbb{R}^n$, such that

$$(2.8) \qquad ||\varphi(x_k)|| \leq \text{ constant and } \varphi'(x_k) \longrightarrow 0 \text{ when } k \longrightarrow \infty$$

is bounded. [Hence φ satisfies the Palais-Smale condition].

Proof

Suppose by contradiction that there exists a subsequence of $\{x_k\}$ (to be denoted in the same way) such that

$$(2.9) \qquad \rho_k = ||x_k||_\infty \longrightarrow \infty, \text{ when } k \longrightarrow \infty.$$

The new sequence $y_k = \frac{x_k}{\rho_k}$ belongs to unit sphere centered at 0. Then it has a subsequence (also denoted in the same way) such that $y_k \longrightarrow y$. For this last sequence

$$(2.10) \qquad \frac{\varphi'(x_k)}{\rho_k} = A(y_k) - \frac{\tilde{f}(\rho_k y_k)}{\rho_k}$$

For each i-component
$\rho_k y_{ki} \longrightarrow (+\infty) sgn(y_i)$ if $y_i \neq 0$, hence

$$\frac{f(\rho_k y_{ki})}{\rho_k} \longrightarrow \begin{cases} \alpha y_i & \text{if } y_i > 0 \\ \beta y_i & \text{if } y_i < 0 \end{cases}$$

when $y_i = 0$, using (2.6), the convergence is to zero. Thus taking limits in (2.10) we have $Ay - (\alpha y^+ - \beta y^-) = 0$ with $y \neq 0$, a contradiction to the hypothesis. □

Remark 2.4

The functions of the form (2.1) with f bounded, satisfies the Palais-Smale condition, because in this case $\alpha = \beta = 0$ and A is a nonsingular matrix. We now give one version of Mountain Pass Theorem, that will be essential for the proof of our main result.

Theorem 2.5 (Mountain Pass Theorem)

Let x be a Banach space and $\varphi : x \longrightarrow \mathbb{R}$ a C^1 function that satisfies Palais-Smale condition. Suppose that φ has two local minima. Then φ has at least one more critical point. (see the proof in [3], [8])

Proposition 2.6

Let $f :\longrightarrow [0, \infty) \longrightarrow \mathbb{R}$ be a bounded continuous function such that $f(0) > 0$ and define $f(s) = f(0)$ for $s < 0$. We assume that φ, defined in (2.1), attains local minima at u and v in $\overset{\circ}{\mathbb{R}}{}^n_+$, where $u < v$. Then there exists a third critical point w of φ and $u < w < v$ (see notations 1.1)

Proof

By hypothesis $Au = \tilde{f}(u)$ and $Av = \tilde{f}(v)$, hence $Ay = \tilde{f}(y + u) - \tilde{f}(u)$ with $y = v - u > 0$.

The least equation suggests a modification of the function by cutting and translation i.e. to consider functions of the form

$$g(s) = f(s^+ + \mu_i) - f(u_i), \quad s \in \mathbb{R}, \quad i = 1, 2, \ldots n. \tag{2.11}$$

which are continuous.

Let

$$\psi(x) = \frac{1}{2} < Ax, x > - \sum_{i=1}^{n} G(x_i), x \in \mathbb{R}^n \tag{2.12}$$

where A is a M-matrix and $G(x_i) = F(x_i^+ + u_i) - F(u_i) - f(u_i) x_i^+$. Hence ψ is a C^1 function.

We will prove that the critical points of ψ belong to $\mathbb{R}^n_+$, that ψ attains its local minimum at $x = 0, s = v - u$ and for any critical point z_1 of $\psi, u + z_1$ is also critical point of ψ.

Let x be a critical point of ψ. Then

$$< Ax, y >= \sum_{i=1}^{n} [f(x_i^+ + u_i) - f(u_i)] y_i \quad \forall\, y \in \mathbb{R}^n$$

If $y = x^-$ the right side is zero, thus $< Ax^-, x^- >=< Ax^+, x^- > \leq 0$ consequently $x^- = 0$ and $x = x^+ \in \mathbb{R}^n_+$.

A straightforward computation gives

$$\text{(2.13)} \quad \psi(z+x) - \psi(z) = \varphi[v + x + (z+x)^-] - \varphi(v) - < A(z+x)^+, (z+x)^- > + \frac{1}{2} < A(z+x)^- 0, (x+x)^- >$$

where the two last terms are non-negative (Lemma 1.4). Taking into account that v is a local minimum of φ and as

$$\text{(2.14)} \qquad ||x + (z+x)^-|| \leq ||x||$$

it follows that $z = v - u$ is a local minimum of ψ. In order to prove that ψ attains its local minimum at $x = 0$ it is sufficient to make $z = 0$ i.e. $v = u$ in (2.13) and (2.14).

It is easy to see that $u + z_1$ is a critical point of ψ if z_1 is any other critical point of ψ.

Following the same reasoning, we can proof that function

$$\text{(2.15)} \qquad \theta(x) = \frac{1}{2} < Ax, x > - \sum_{i=1}^{n} H(x_i)$$

where $H(x_i) = G(z_i - x_i^+) - G(z_i) + g(z_i)x_i^+$, with $z = v - u$, has its critical points in $\mathbb{R}^n_+$, attains its local minimum at z and $x = 0$, moreover if y is any of its critical points then $z - y$ is a critical point of ψ. Hence $z - y + u$ will be a critical point of φ.

Finally, since $H'(s) = g(z_i - s^+) - g(z_i)$ is a continuous bounded function, θ satisfies the Palais-Smale condition (remark (2.4)). According to Mountain Pass theorem 2.5, θ has a third critical point $y \neq 0, z$. Consequentely $w = z - y + u = v - y$ is another critical point of φ and one can verify that $u < w < v$. □

Proposition 2.7
Let f be as in (0.2). Then (0.3) has $2m-1$ ordered positive solutions $x_\lambda^1 < x_\lambda^2 < \cdots < x_\lambda^{2m-1}$ in $\overset{\circ}{I\!R}{}^n_+$ such that $\|x_\lambda^i\|_\infty < s_{2i-1}$ for $i = 1, \dots, m$.

Proof
Let $f_k(s) = f(s)$ for $s \le s_{2k-1}$ and $f_k(s) = 0$ if $s \ge s_{2k-1}$
$F_k(t) = \int_0^t f_k(s)ds$, $k = 1, 2, \dots, m$ and

$$(2.16) \qquad \varphi_k(\lambda, x) = \frac{1}{2} < Ax, x > -\lambda h^2 \sum_{i=1}^n F_k(x_i) \ , \ x \in I\!R^n.$$

First we proof that φ_1 has a global minimum $u_\lambda \in \overset{\circ}{I\!R}{}^n_+$ with $\|u_\lambda\|_\infty < s_1$. Besides u_λ is a local minimum of φ_2.

As $< Ax, x >= \|x\|_A^2$ is a norm (Lemma 1.4)

$$\begin{aligned} \varphi_1(\lambda, x) &= \frac{1}{2}\|x\|_A^2 - \lambda h^2 \sum_{i=1}^n F_1(x_i) \ge \frac{1}{2}\|x\|_A^2 - \lambda c_1 \sum_{i=1}^n |x_i| \\ &\ge \frac{1}{2}\|x\|_A^2 - \lambda c_2 \|x\|_A. \end{aligned}$$

It follows then that there exists a constant $R = R(\lambda)$ such that $\varphi_1(\lambda, x) \ge 1$ for $\|x\|_A \ge R$. Thus $\varphi_1(\lambda, x)$ is bounded below and since it is C^1, it attains its global minimum at a point u_λ with $\|u_\lambda\|_A < R$. Since u_λ is a critical point of φ_1 by Lemma 1.5 $u_\lambda \in \overset{\circ}{I\!R}{}^n_+, \|u_\lambda\|_\infty < s_1$. Hence $0 < u_{\lambda i} < s_1 \ \forall i = 1, \dots n$ then taking $\delta = \frac{1}{2}(s_1 - \|u_\lambda\|_\infty)$

$$\varphi_2(\lambda, u_\lambda) = \varphi_1(\lambda, u_\lambda) \le \varphi_1(\lambda, x) = \varphi_2(\lambda, x) \text{ when } \|x - u_\lambda\|_\infty < \delta.$$

Second We show that the following function attains its global minimum at a point $z_\lambda > 0$ for λ large.

$$(2.17) \qquad \psi(\lambda, x) = \frac{1}{2} < Ax, x > -\lambda h^2 \sum_{l=1}^n G(x_i), \ x \in I\!R^n$$

where $G(x_i) = F_2(x_i^+ + u_i) - F_2(u_i) - f_2(u_i)x_i^+$ (here by brevity we denote $u_i = u_{\lambda i}$).
It is easy to see that $\psi(\lambda, x)$ is C^1 and bounded below. Hence it has a global minimum at a point z_λ and following the same argument done for the function in (2.12) we have that $z_\lambda \in \overset{\circ}{I\!R}{}^n_+$.

In order to prove that $z_\lambda \neq 0$, we observe that there exists $\delta \in (0, s_1)$ such that $f(s)s_3 < \frac{1}{2}[F(s_3) - F(s_1)]$ for $s \in [s_1 - \delta, s_1]$ (here we are using the condition (0.2) (c)).

By Lemma 1.4, $||u_\lambda||_\infty = \rho_2 \in [s_1 - \delta, s_1]$ $\forall \lambda \geq \alpha > 0$ hence taking $y_\lambda = (0, \ldots 0, s_3, 0 \ldots 0)$ (with s_3 at i-component of u_λ, corresponding $||u_\lambda||_\infty$)

$\psi(\lambda, y_\lambda) = \frac{1}{2}s_3^2(\sum_{j=1}^n a_{ij}) - \lambda h^2[F(s_3) - F(\rho_\lambda) - f(\rho_\lambda)s_3]$ then $\psi(\lambda, y_\lambda) \leq s_3^2 \max\{a_{ii} : i = 1, \ldots n\} - \frac{\lambda h^2}{2}[F(s_3) - F(s_1)] < 0$ for $\lambda \geq \lambda_2 > \alpha$ hence $z_\lambda > 0$.

Third

Now taking into account that $z_\lambda > 0$ is a critical point of $\psi(\lambda, x)$, we will show that $u_\lambda + z_\lambda$ is a local minimum of φ and by proposition 2.6 follows the rest.

Is easy to see that $u_\lambda + z_\lambda$ is a critical point of φ and by Lemma 1.5 $||u_\lambda + z_\lambda||_\infty < s_2$. Since $u_\lambda \in \mathring{I\!R}^n_+$ there exists $\varepsilon > 0$ such that the ball $\beta_\varepsilon(u_\lambda + z_\lambda)$ of center $u_\lambda + z_\lambda$ and readius ε, is contained in $\mathring{I\!R}^n_+$. Taking into acount (2.17)

$$\psi(\lambda, z_\lambda) = \varphi_2(\lambda, u_\lambda + z_\lambda) - \varphi_2(\lambda, u_\lambda) \text{ and}$$
$$\psi(\lambda, x + z_\lambda) = \varphi_2(\lambda, u_\lambda + z_\lambda + x) - \varphi_2(\lambda, u_\lambda) \ \forall\, x \in \beta_\varepsilon(u_\lambda + z_\lambda).$$

Hence φ_2 attains its local minimum at $u_\lambda + z_\lambda$. □

Remark

a) This work is an attempt to find alternative numerical procedures that will avoid the NIS in the resolution of (0.3).

b) Is still pending to study the convergence problems. When the solutions of (0.3) are been obtained by Variational Methods.

c) Another interesting problem is to study the analogue discrete of (0.1) by Finete Elements Methods, in the context of topological degree and global bifurcation theories.

References

[1] A. Ambrosetti and P. Hess. "Positive solutions of asymptotically linear elliptic eigenvalue problems". J. Math. Anal. Appl. 73 (1980) 411 - 422.

[2] D. G. De Figueiredo. "On the existence of multiple ordered solutions of nonlinear eigenvalue problems". Nonl. Anal. theory Meth. & Appl. 11, Nº 4 (1987). 481-492.

[3] D. G. De Figueiredo. "The Ekeland Variational principle with applications and detours". Data Inst. of Fund. Research. Springer-Verlag (1989).

[4] K. J. Brown and H. Budin. "On the existence of positive solutions for a class of semi linear elliptic boundary value problems" SIAM J. Math. Anal. 10, nº 5 (1979) 875 - 883.

[5] D. Greenspan and Seymour V. Parter. "Midly non linear elliptic partial differential equations and their numerical solutions II". Numer Math, 7, (1965) 129-146.

[6] H. O. Peitgen, D. Saupe and K. Schmitt. "Nonlinear elliptic boundary value problems versus their finite difference approximations: numerically irrelevant solutions". J. Reine Angew Mathematik 322 (1981) 74 - 117.

[7] H. O. Peitgen and K. Schmitt. "Positive and spurious solutions of nonlinear eigenvalue problems." Springer Lecture Notes in Math. 878 (1981) 275 - 324.

[8] P.H. Rabinowitz "Minimax methods in critical point theory with applications to differential equations" CBMS Lecture Notes Nº 65 AMS (1986).

[9] J. Schröder "M-matrices and generalizations using an operator theory approach." SIAM Review 20 (1978) 213 - 244.

[10] R. S. Varga "Matrix Iterative Analysis" Englewood Cliffs. New Jersey 1962.

RELATÓRIOS TÉCNICOS — 1991

01/91 **Um Método Numérico para Resolver Equações de Silvester e de Ricatti** — *Vera Lucia da Rocha Lopes* and *José Vitório Zago.*

02/91 **"Regge-Like" Relations for (Non-Evaporating) Black Holes and Cosmological Models** — *Vilson Tonin-Zanchin* and *Erasmo Recami.*

03/91 **The Exponential of the Generators of the Lorentz Group and the Solution of the Lorentz Force Equation** — *J. R. Zeni* and *Waldyr A. Rodrigues Jr.*

04/91 **Tensornorm Techniques for the (DF)-Space Problem** — *Andreas Defant* and *Klaus Floret.*

05/91 **Nonreversibility of Subsemigroups of Semi-Simple Lie Groups** — *Luiz San Martin.*

06/91 **Towards a General Theory of Convolutive Sets (With Applications to Fractals)** — *Jayme Vaz Jr.*

07/91 **Linearization of Holomorphic Mappings of Bounded Type** — *Jorge Mujica.*

08/91 **Topological Equivalence of Diffeomorphisms and Curves** — *M. A. Teixeira.*

09/91 **Applications of Finite Automata Representing Large Vocabularies** — *Cláudio L. Lucchesi* and *Tomasz Kowaltowski.*

10/91 **Torsion, Superconductivity and Massive Electrodinamics Cartan's Torsion Vector and Spin–0 Fields** — *L. C. Garcia de Andrade.*

11/91 **On The Continuity of Fuzzy Integrals** — *G. H. Greco* and *R. C. Bassanezi.*

12/91 **Optimal Chemical Control of Populations Developing Drug Resistance** — *M. I. S. Costa, J. L. Boldrini* and *R. C. Bassanezi.*

13/91 **Strict Monotonicity of Eigenvalues and Unique Continuation** — *Djairo G. de Figueiredo* and *Jean-Pierre Gossez.*

14/91 **Continuity of Tensor Product Operators Between Spaces of Bochner Integrable Functions** — *Andreas Defant* and *Klaus Floret.*

15/91 **Some Remarks on the Join of Spheres and their Particular Triangulations** — *Davide C. Demaria* and *J. Carlos S. Kiihl.*

16/91 **Sobre a Equação do Telégrafo e o Método de Riemann** — *L. Prado Jr.* and *E. Capelas de Oliveira.*

17/91 **Positive Solutions of Semilinear Elliptic Systems** — *Ph. Clément, D. G. de Figueiredo* and *E. Mitidiere.*

18/91 **The Strong Coupling Constant: Its Theoretical Derivation from a Geometric Approach to Hadron Structure** — *Erasmo Recami* and *Vilson Tonin-Zanchin.*

19/91 Time Analysis of Tunnelling Processes, and Possible Applications in Nuclear Physics — *Vladislavi S. Olkhovsiky* and *Erasmo Recami.*

20/91 Procedimento, Função, Objeto ou Lógica? — *M. Cecília Calani Baranauskas.*

21/91 The Relation Between 2-Spinors and Rotations — *W. A. Rodrighes Jr.* and *J. R. Zeni.*

22/91 Boundaries for Algebras of Analytic Functions on Infinite Dimensional Banach Spaces — *R. M. Aron, Y. S. Choi, M. L. Lourenço* and *O. W. Paques.*

23/91 Factorization of Uniformly Holomorphic Functions — *Luiza A. Moraes, Otilia W. Paques* and *M. Carmelina F. Zaine.*

24/91 Métrica de Prohorov e Robustez — *Mario Antonio Gneri.*

25/91 Cálculo de Funções de Green para a Equação de Schrödinger pelo Método de Expansão Tipo Sturm-Liouville — *L. Prado Jr.* and *E. Capelas de Oliveira.*

26/91 On the Weierstrass-Stone Theorem — *João B. Prolla.*

27/91 Sull'Equazione di Laplace nell'Universo di De Sitter — *E. Capelas de Oliveira* and *G. Arcidiacono.*

28/91 The Generalized Laplace Equation in Special Projective Relativity — *E. Capelas de Oliveira* and *G. Arcidiacono.*

29/91 The Projective D'Alembert Equation — *E. Capelas de Oliveira* and *G. Arcidiacono.*

30/91 The Generalized D'Alembert Equation in Special Projective Relativity — *E. Capelas de Oliveira* and *G. Arcidiacono.*

31/91 A General Algorithm for Finding the Minimal Angle between Subspaces — *Alvaro R. De Pierro* and *Alfredo N. Iusem.*

32/91 Scalar Curvature on Fibre Bundles — *Maria Alice B. Grou.*

33/91 Sur la Dimension des Algèbres Symétriques — *Rachid Chibloun, Artibano Micali* et *Jean Pierre Olivier.*

34/91 An Inverse Column-Updating Method for Solving Large-Scale Nonlinear Systems of Equations — *José M. Martínez* em *Mário C. Zambaldi.*

35/91 Parallel Implementations of Broyden's Method — *Francisco A. M. Gomes* and *José M. Martínez.*

36/91 Equivalência Elementar entre Feixes — *A. M. Sette* and *X. Caicedo.*

37/91 Unique Ergodicity for Degenerate Diffusions and the Accessibility Property of Control Systems — *Luiz San Martin.*

38/91 Unobservability of the Sign Change of Spinors Under a 2π Rotation in Neutron Interferometric Experiments — *J. E. Maiorino, J. R. R. Zeni* and *W. A. Rodrigues Jr.*

39/91 Disappearance of the Numerically irrelevant Solutions (NIS) in Non-Linear Elliptic Eigenvalue problems — *Pedro C. Espinoza.*