ON FORCES WHICH DEPEND ON THE ACCELERATION

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Abstract. We present some criticisms which appeared in the literature against force laws which depend on the acceleration of the test body and answer them. In particular we show that the superposition principle for the acceleration no longer holds for these laws and emphasize that only some careful experiments can tell us if this principle is always valid or if it is only an approximation.

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Abstract. We present some criticisms which appeared in the literature against force laws which depend on the acceleration of the test body and answer them. In particular we show that the superposition principle for the acceleration no longer holds for these laws and emphasize that only some careful experiments can tell us if this principle is always valid or if it is only an approximation.

Key Words. Weber's law, superposition principle, forces which depend on the acceleration, Newton's second law, Mach's ideas, principle of the parallelogram of forces.

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This work is written in order to answer some criticisms which have appeared in the literature against force laws that depend on the acceleration of the test body. In order to make our arguments completely clear we will always utilize a concrete example of a force law of this type: Weber's electrodynamics, [1]. Besides being an extremely powerful model of interaction for point charges, it is also the oldest law to appear in the literature (1846) that depends on the velocity and acceleration of the charges. In modern language Weber's law reads

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}_{12}}{r_{12}^2}\left[1 + \frac{1}{c^2}\left(\vec{v}_{12}\cdot\vec{v}_{12} - \frac{3}{2}(\hat{r}_{12}\cdot\vec{v}_{12})^2 + (\vec{r}_{12}\cdot\vec{a}_{12})\right)\right] , \qquad (1)$$

where $\vec{F}_{21}$ is the force exerted on q_1 by q_2, $\vec{r}_{12} \equiv \vec{r}_1 - \vec{r}_2$, $r_{12} \equiv |\vec{r}_{12}|$, $\hat{r}_{12} \equiv \vec{r}_{12}/r_{12}$, $\vec{v}_{12} \equiv d\vec{r}_{12}/dt$, $\vec{a}_{12} \equiv d\vec{v}_{12}/dt$ and c is the ratio of electromagnetic to electrostatic units of charge (and which was found experimentally by Weber and Kohlrausch to have the same value as light velocity in vacuum). With this law Weber *derived* Ampère's law for the force between current elements and also Faraday's law of induction. Moreover with Newton's law it can be easily shown that Eq. (1) satisfies the principles of conservation of energy, linear and angular momentum, [2-4].

The first criticism against laws of this kind was given by Przeborski, [5,6]. In order to answer his reasoning we consider Eq. (1) with motions only along the X axis (the generalization to three dimensions is straightforward), so that it reads $F_{21} = A_{12} + B_{12}a_1$, where A_{12} and B_{12} are functions of the relative position and velocity of q_1 and q_2, and also of a_2, but they do not depend on a_1. Equating this with Newton's second law yields for the acceleration of q_1 due to q_2 the value

$$a_{1,2} = \frac{A_{12}}{m_1 - B_{12}} . \qquad (2)$$

If q_1 were interacting only with q_3 this would indicate analogously

$$a_{1,3} = \frac{A_{13}}{m_1 - B_{13}} . \qquad (3)$$

On the other hand if q_1 were interacting simultaneously with q_2 and q_3 we would obtain, with Newton's second law:

$$a_{1,23} = \frac{A_{12} + A_{13}}{m_1 - (B_{12} + B_{13})} . \qquad (4)$$

It is easily seem from these expressions that $a_{1,23} \neq a_{1,2} + a_{1,3}$ unless $B_{12} = B_{13} = 0$. And this was the argument of Przeborski against laws like Eq. (1): With laws of this type the superposition principle for force and for acceleration cannot be maintained simultaneously.

To answer this we first agree with Przeborski in this conclusion. But to us this is not a problem or flaw of equations like Eq. (1). On the contrary, this is a necessary characteristic of these equations, and if they are correct (or at least approximate) expressions for the law of interaction this means that one of the principles of superposition will need to be abandoned. As a matter of principle there is no problem with this possibility, and we can only know if this happens observing nature. As was correctly pointed out by Mach, [7], the law for the paralellogram of forces or of accelerations (or equivalently the principle of superposition) is an *experimental* proposition (perhaps only approximately valid). As such there is no logical necessity in its validity. In everyday experiences we observe that the superposition principle is approximately valid but perhaps this doesn't work for extremely large accelerations. In Eq. (1) the term in the acceleration is $(\vec{r}_{12} \cdot \vec{a}_1)/c^2$ times the Coulomb force. Due to the large value of c^2 this term will only be relevant or for large distances or for large accelerations. As an example of the possible relevance of this term we showed recently that when we apply Eq. (1) to gravitation this term becomes the responsible for inertia due to the extremely distant galaxies, [3].

We next discuss Waldron's paper, [8]. He also claims that an equation like Eq. (1) leads to a contradiction. The main part of his argument is, literally: "If the force is multiplied by a factor n (e.g. by multiplying by n the voltage on an electrode), the acceleration will be multiplied by the same factor, and [Newton's second law] will become

$$n\vec{F} = m(n\vec{a}) \text{ ."} \qquad (5)$$

Then he shows that in this case instead of Eq. (2) one would have

$$a_{1,2} = \frac{A_{12}}{m_1 - nB_{12}} \ . \qquad (6)$$

As this equation contradicts (2), he concludes that "the acceleration cannot figure in a force law in a universe in which Newton's second law of motion holds good."

Although he did not perceive that, in the cern of his argument there is the same idea of Przeborski, namely, that if we multiply a force by n then the acceleration will also be multiplied by n (the same superposition principle for forces and accelerations). We take his own example to show that the flaw in his reasoning is his assumption that when we multiply the coefficient in front of the force by n then this is equivalent to n times the force when we have an equation like (1). In another work, [4], we showed using Eq. (1) that the force inside a capacitor according to Weber's law is given by

$$\vec{F} = -q_1 \frac{\sigma}{\varepsilon_0} \left\{ \hat{x} + \frac{1}{c^2} \left[\frac{v_1^2}{2} \hat{x} - x_1 \vec{a}_1 + 2 x_1 a_{1x} \hat{x} - v_{1x} (v_{1y} \hat{y} + v_{1z} \hat{z}) \right] \right\} . \quad (7)$$

In this equation q_1 is the test charge and $\pm\sigma$ are the surface charge densities of the infinite plates (located at $\pm x_0$, respectively). Supposing motion only along the X axis (the generalization to three dimensions is straightforward) and equating this to $m_1 a_1$ yields (with $\alpha \equiv -q_1\sigma/\varepsilon_0$, $\beta \equiv 1 + v_1^2/(2c^2)$)

$$a_{1,\sigma} = \frac{\alpha\beta}{m_1 - \alpha x_1/c^2} . \quad (8)$$

If instead of this voltage we had n times this voltage, which is equivalent to multiply σ by n, we would obtain

$$a_{1,n\sigma} = \frac{n\alpha\beta}{m_1 - n\alpha x_1/c^2} . \quad (9)$$

Of course $a_{1,n\sigma} \neq n a_{1,\sigma}$, unless $x_1 = 0$. This shows that according to Weber's law if we multiply the coefficient of the force by n (as in this case when we multiplied the voltage or σ by n) not necessarily the acceleration will be multiplied by n. Also the force will not be multiplied by n because it is a function also of the acceleration of the test body. If we want to multiply the acceleration given by (8) n times then the force will need to be n times greater. But this is not equivalent to multiply σ by

n. In particular, the new value of σ in order to obtain $na_{1,\sigma}$ is given by $-\varepsilon_0 mna_{1,\sigma}c^2/(q_1\beta c^2 + q_1 na_{1,\sigma}x_1)$, which is different from $n\sigma$. Anyway with this value of σ the acceleration will be $na_{1,\sigma}$ and the force will also be nF, where F is given by (7) with motion only along the X axis.

In retrospect we see that there is no problem to have a force law which depends on the acceleration and to satisfy at the same time Newton's second law. But when this happens the relation between force and acceleration is no longer linear so that twice the coefficient of the force doesn't mean ncessarily a double acceleration. The superposition principle for the acceleration no longer holds for these interactions although we can still maintain the superposition principle for the forces and Newton's second law. Only careful observations can show us if nature behaves like this.

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