Course

MM842 Topics in PDEs I

New trends in elliptic regularity theory and free boundary problems

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Abstract.

In this series of Lectures we will address some recent progresses in regularity theory of elliptic PDEs and Free Boundary Problems, which were developed in the recent decades. Such topics include:

(1) Harmonic Functions: A brief review.

(2) Schauder theory via Maximum Principle for Laplacian operator.

(3) Schauder theory via Compactness Method for Laplacian operator.

(4) Schauder theory for operators with variable coefficients.

(5) De Giorgi's Theory for operators in divergent form.

(6) Krylov-Safonov's Theory for operators in non-divergent form.

(7) Calderón–Zygmund's Theory: $W^{2,p}$ estimates for Laplacian operator $(1 < p < \infty)$.

(8) Obstacle type problems for Laplacian operator.

(9) Jets problems (for Laplacian operator) via singular perturbation methods.

(10) Dead-core type problems ($p-$Laplacian operator $(2 \leq p < \infty)$).

The insights behind the expositions consist of explaining their mathematical relevance, intrinsic difficulties in being overcome and applications in other classes of problems (*e.g.* existence/regularity for problems driven $p-$Laplacian type operators as $p \to \infty$, where arises the infinity Laplace operator). In the end of the Lectures we will present new directions and some mathematical expectations for the next years in such subjects of research.

These Lectures are accessible for all students in the Master and Doctoral levels, Post-doctoral Researches in Analysis/PDEs and also Geometry, as well as other interested people with minimal knowledge about Functional Analysys, Measure Theory and Poisson's equation.

References

[1] Qing Han & Fanghua Lin, *Elliptic partial differential equations*. Second edition. Courant Lecture Notes in Mathematics, 1. Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2011. x+147 pp. ISBN: 978-0-8218-5313-9.

[2] Xavier Fernández-Real & Xavier Ros-Oton, *Regularity Theory for Elliptic PDE*. Book, 2020.

[3] Peter Lindqvist, *Notes on the infinity Laplace equation*. SpringerBriefs in Mathematics. BCAM Basque Center for Applied Mathematics, Bilbao; Springer, [Cham], 2016. ix+68 pp. ISBN: 978-3-319-31531-7; 978-3-319-31532-4.

[4] Eduardo V. Teixeira, *Elliptic regularity and free boundary problems: an introduction*. Publicaçẽs Matemáticas do IMPA. [IMPA Mathematical Publications] 26 Colóquio Brasileiro de Matemática. [26th Brazilian Mathematics Colloquium] Instituto Nacional de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, 2007. ii+205 pp. ISBN: 978-85-244-0252-4

[5] Eduardo V. Teixeira, *Introdução à teoria de regularidade elíptica: uma abordagem geométri*. III ENAMA, Maringá, 2009.

[6] Eduardo V. Teixeira, *Um convite à análise geométrica de EDPs elípticas de 2a ordem*. IV EBED, João Pessoa, 2011.

[7] Lihe Wang, *Regularity Theory*. Lecture Notes - Korea Winter School, 2013.

[8] Noemí Wolanski, *Introdución a los problemas de frontera libre*. Cursos y Seminarios de Matemática - Serie B, Fascículo 2, Publicaciones del Departamento de Matemática, FCEN-UBA, 2007. ISSN 1851-1481.