Multi-scale analysis of transport in porous media: model typologies

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In the ongoing quest for efficient models to describe transport in porous media, various approaches have emerged, each addressing the multi-scale nature of such systems in distinct ways (see the schematic representation in Fig. 1).

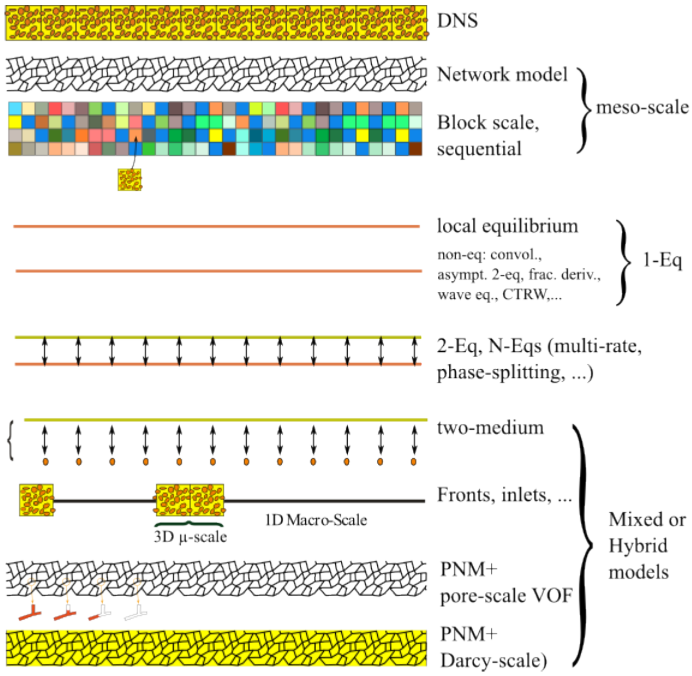

Figure 1: The ecosystem of porous medium models ().

The spectrum of models—ranging from pore-scale direct numerical simulations to fully averaged descriptions—encompasses several intermediate approaches:

- **Direct pore-scale simulations** are often impractical or infeasible, even with advances in high-performance computing, necessitating alternative methods.
- **Meso-scale models**, such as pore network or sequential models. Pore network models represent the medium as a graph of nodes and links, with transport equations approximated using simple, often analytical, solutions (e.g., Poiseuille flow for tubular links). Sequential models employ a multigrid-like approach, solving transport problems at each level with specific PDEs derived from upscaling tools (e.g., Stokes → Darcy → Darcy). These methods enable the analysis of large tomographic images, which are intractable for direct simulations.
- **Hybrid (or mixed) models**, which couple averaged equations with sub-scale modeling. These are widely used in reservoir engineering (e.g., fractured media) and electrochemistry (e.g., the Doyle–Fuller–Newman (DFN) model for Li-ion batteries; Doyle et al., 1993). A typical hybrid model would read

$$\varepsilon_\beta (\rho c_p)_\beta \frac{\partial T_\beta}{\partial t} + \varepsilon_\beta (\rho c_p)_\beta \mathbf{U}_\beta \cdot \nabla T_\beta = \nabla \cdot \left(\mathbf{K}^*_\beta \cdot \nabla T_\beta \right) - \varphi_{\beta\sigma} \tag{1}$$

for the homogenized β-phase, and a local micro-scale problem for the other phase (here σ)

$$(\rho c_p)_\sigma \frac{\partial T_\sigma}{\partial t} = \nabla \cdot (k_\sigma \nabla T_\sigma) \quad \text{in } V_\sigma. \tag{2}$$

Hybrid modeling, because of its need to simplify the pore-scale geometry, suffers from several limitations, which have been extensively discussed in recent works, especially in the case of the DFN model (Paten et al., 2026).

- **Phase-splitting or N-equation models**, where a single phase is macroscopically represented by multiple pseudo-phases. This approach captures parts of the full spectrum of characteristic lengths and times, offering an alternative to hybrid models—particularly when advection is significant. Originating from the mobile-immobile dispersion model (Coats and Smith, 1964), it is now generalized and formalized from a multi-scale perspective, enhancing model development based on porous structure characteristics. A typical general phase-splitting model (or N-equation model) would read:

$$\begin{bmatrix} \frac{\partial \phi_1 \mathbb{T}_1}{\partial t} \\ \vdots \\ \frac{\partial \phi_N \mathbb{T}_N}{\partial t} \end{bmatrix} + \begin{bmatrix} \phi_1 U_1 \frac{\partial \mathbb{T}_1}{\partial x} \\ \vdots \\ \phi_N U_N \frac{\partial \mathbb{T}_N}{\partial x} \end{bmatrix} + \mathrm{Pe}\,[C] \begin{bmatrix} \frac{\partial \mathbb{T}_1}{\partial x} \\ \vdots \\ \frac{\partial \mathbb{T}_N}{\partial x} \end{bmatrix} = [a] \begin{bmatrix} \frac{\partial^2 \mathbb{T}_1}{\partial x^2} \\ \vdots \\ \frac{\partial^2 \mathbb{T}_N}{\partial x^2} \end{bmatrix} - [h] \begin{bmatrix} \mathbb{T}_1 \\ \vdots \\ \mathbb{T}_N \end{bmatrix} \tag{3}$$

  where the matrices $[a]$ and $[h]$ play a fundamental role. The determination of these "effective properties", by solving upscaling closure problems or from inverse problems, is largely an open theoretical and numerical question. This is of crucial importance in many applications, for instance in modeling gas–liquid flows in distillation columns equipped with structured packing, for which several generalized Darcy's laws are coupled through non-linear exchange terms (Soulaine et al., 2014).

- **Fully averaged phase models**, which are efficient and widely adopted. However, accurately representing the full spectrum of characteristic lengths and times can introduce complex terms (e.g., spatial and temporal convolutions) and effective coefficients that are highly dependent on initial and boundary conditions.

This review examines the introduction of these models through elementary transport problems, such as Stokes flow, diffusion/dispersion, and two-phase flow. Concepts are illustrated with applications from engineering (e.g., fractured media, fuel cells, Li-ion batteries, heat exchangers), bioengineering (e.g., tissue characterization), and environmental science (e.g., flow in highly heterogeneous media). The discussion emphasizes the advantages and limitations of each approach, particularly in terms of accuracy and convergence toward appropriate transport solutions. Implementation challenges, often overlooked, are also addressed.

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