AO

$$xy' + \left(1 + \frac{1}{x}\right)y = e^{1/x} \qquad x>0 \qquad \underline{\text{linear}}$$

$$y' + \left(\frac{1}{x} + \frac{1}{x^2}\right)y = \frac{1}{x}e^{1/x}$$

$\boxed{2}$

fator integrante

$$e^{\int \frac{1}{x} + \frac{1}{x^2}} = e^{\ln x - x^{-1}} = xe^{-\frac{1}{x}}$$

$\boxed{2}$

$$\underbrace{xe^{-1/x}y' + xe^{-1/x}\left(\frac{1}{x} + \frac{1}{x^2}\right)y}= \frac{1}{x}e^{1/x} \cdot xe^{-1/x} = 1$$

$\boxed{2}$

$$\frac{d}{dx}\left[xe^{-1/x}y\right] = 1$$

$\boxed{2}$

integra $\int \cdot\, dx$

$$xe^{-1/x}y = x + C$$

$$\boxed{y = e^{1/x} + \frac{Ce^{1/x}}{x}}$$

$\boxed{2}$

A1 $\underbrace{(2xy)}_{M}dx + \underbrace{(y^2-x^2)}_{N}dy = 0$

$M_y = 2x \neq N_x = -2x$ $\Rightarrow$ não é exata $\boxed{2}$

$\frac{M_y - N_x}{N} = \frac{2x-(-2x)}{y^2-x^2} \neq f(x)$

$\frac{N_x - M_y}{M} = \frac{-2x-2x}{2xy} = \frac{-4x}{2xy} = -\frac{2}{y} = f(y)$

fator integrante

$e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = y^{-2}$ $\boxed{2}$

$\underbrace{(2xy^{-1})}_{M}dx + (\underbrace{1-x^2y^{-2}}_{N})dy = 0$ $\boxed{2}$

$M_y = -2xy^{-2} = N_x = -2xy^{-2} \Rightarrow$ edo é exata.

$\Rightarrow$ existe $F(x,y) = C$ tal que

$F_x = M$ e $F_y = N$

$F = \int M\,dx = \int 2xy^{-1}dx = x^2y^{-1} + g(y)$

$F_y = -x^2y^{-2} + g'(y) = N = 1 - x^2y^{-2}$

$\Rightarrow g'(y) = 1 \Rightarrow g(y) = y$ $\boxed{2}$

Solução $F(x,y) = C$

$$\boxed{x^2y^{-1} + y = C}$$ $\boxed{2}$

A2. $y' = \frac{y}{x} + e^{y/x} \qquad x > 0$

↑

é homogênea [2]

$v = \frac{y}{x} \qquad vx = y$

Substituindo:

$v'x + v = y'$ [2]

$v'x + v = v + e^{v}$

$v'x = v + e^{v} - v = e^{v}$

$\int e^{-v}\,dv = \int \frac{1}{x}\,dx$ [2]

$-e^{-v} = \ln x + C$ [2]

$\boxed{-e^{-y/x} = \ln x + C}$ [2]

15 (

$$y^{(3)} - y^{(2)} - y^{(1)} + y = 2e^{-x} + 3e^{x}$$

Homogênea associada $y^{(3)} - y^{(2)} - y^{(1)} + y = 0$

eq. característica $r^3 - r^2 - r + 1 = 0$ candidata à solução $y = e^{rx}$

$$(r^2-1)(r-1) = (r+1)(r-1)^2 = 0 \quad \Rightarrow r^2(r-1) - (r-1) = 0$$

$r = -1$ e $r = 1$ com mult. 2 $\Rightarrow \boxed{y_c = c_1 e^{-x} + (c_2 + c_3 x)e^{x}}$

multiplica-se $e^{rx}$ por um polinômio de grau um a menos que a multiplic.

2

$$y_P = x^s A e^{-x} + x^l B e^{x} = Axe^{-x} + Bx^2 e^{x}$$

$s = 1$, $l = 2$

para eliminar repetições c/ termos em $y_c$

2

$$y_P' = Ae^{-x} - Axe^{-x} + 2Bxe^{x} + Bx^2e^{x}$$

$$y_P'' = -Ae^{-x} - Ae^{-x} + Axe^{-x} + 2Be^{x} + 2Bxe^{x} + 2Bxe^{x} + Bx^2e^{x}$$

$$= -2Ae^{-x} + Axe^{-x} + 2Be^{x} + 4Bxe^{x} + Bx^2e^{x}$$

$$y_P''' = 2Ae^{-x} + Ae^{-x} - Axe^{-x} + 2Be^{x} + 4Be^{x} + 4Bxe^{x} + 2Bxe^{x} + Bx^2e^{x}$$

$$= 3Ae^{-x} - Axe^{-x} + 6Be^{x} + 6Bxe^{x} + Bx^2e^{x}$$

2

Substituir na e.d.o.

$$y_P''' - y_P'' - y_P' + y_P = (3\cancel{A}e^{-x} - Axe^{-x} + 6Be^{x} + 6Bxe^{x} + Bx^2e^{x}) -$$

$$-(-2\cancel{A}e^{-x} + Axe^{-x} + 2Be^{x} + 4Bxe^{x} + Bx^2e^{x}) - (\cancel{A}e^{-x} - Axe^{-x} + 2Bxe^{x} + Bx^2e^{x})$$

$$+ (Axe^{-x} + Bx^2e^{x}) = 2e^{-x} + 3e^{x}$$

$$3Ae^{-x} + 2Ae^{-x} - Ae^{-x} = 2e^{-x} \Rightarrow 4Ae^{-x} = 2e^{-x}$$

$$\Rightarrow \boxed{A = \tfrac{1}{2}}$$

2

$$6Be^{x} - 2Be^{x} = 4Be^{x} = 3e^{x}$$

$$4B = 3 \quad \boxed{B = \tfrac{3}{4}}$$

$$\boxed{y_P = \frac{x}{2}e^{-x} + \frac{3}{4}x^2e^{x}}$$

2

B1

$$y''' + y' = tg\,x$$

homogênea associada $y''' + y' = 0$ candidata à solução $e^{rx}$

eq. característica $r^3 + r = 0$ $r(r^2+1) = 0$ $r = 0$ $r = \pm i$

$$y_c(x) = c_1 e^{0x} + c_2 \cos x + c_3 \operatorname{sen} x$$

1

2

---

Variação de parâmetros: $y_p = u_1 + u_2 \cos x + u_3 \operatorname{sen} x$

sistema:
$$\begin{cases} u_1' + u_2' \cos x + u_3' \operatorname{sen} x = 0 \\ 0 + u_2'(-\operatorname{sen} x) + u_3' \cos x = 0 \\ 0 - u_2' \cos x - u_3' \operatorname{sen} x = tg\,x \end{cases}$$

$$W = \begin{vmatrix} 1 & \cos x & \operatorname{sen} x \\ 0 & -\operatorname{sen} x & \cos x \\ 0 & -\cos x & -\operatorname{sen} x \end{vmatrix} = 1 \cdot ((-\operatorname{sen} x)(-\operatorname{sen} x) - (-\cos x)\cos x) = 1$$ 2

$$u_1' = \begin{vmatrix} 0 & \cos x & \operatorname{sen} x \\ 0 & -\operatorname{sen} x & \cos x \\ tg\,x & -\cos x & -\operatorname{sen} x \end{vmatrix} = tg\,x \begin{vmatrix} 0 & \cos x & \operatorname{sen} x \\ 0 & -\operatorname{sen} x & \cos x \\ 1 & -\cos x & -\operatorname{sen} x \end{vmatrix} = tg\,x \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix}$$

$$= tg\,x$$ 2

$$u_2' = \begin{vmatrix} 1 & 0 & \operatorname{sen} x \\ 0 & 0 & \cos x \\ 0 & tg\,x & -\operatorname{sen} x \end{vmatrix} = tg\,x \begin{vmatrix} 1 & 0 & \operatorname{sen} x \\ 0 & 0 & \cos x \\ 0 & 1 & -\operatorname{sen} x \end{vmatrix} = -tg\,x \cos x = -\operatorname{sen} x$$ 2

$$u_3' = \begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\operatorname{sen} x & 0 \\ 0 & -\cos x & tg\,x \end{vmatrix} = tg\,x \begin{vmatrix} 1 & \cos x & 0 \\ 0 & -\operatorname{sen} x & 0 \\ 0 & -\cos x & 1 \end{vmatrix} = tg\,x(-\operatorname{sen} x) = \frac{-\operatorname{sen}^2 x}{\cos x}$$

$$= \frac{\cos^2 x - 1}{\cos x} = \cos x - \sec x$$

---

$u_1 = \int tg\,x\,dx = -\ln|\cos x|$ $u_2 = \int \operatorname{sen} x = +\cos x$

2

$u_3 = \int \cos x - \sec x = \operatorname{sen} x - \ln|\sec x + tg\,x|$

$$\int \sec x \frac{(\sec x + tg\,x)}{\sec x + tg\,x} dx \qquad u = \sec x + tg\,x \qquad du = \sec^2 x + \sec x\, tg\,x$$

$$y_p = -\ln|\cos x| + \underbrace{\cos^2 x + \operatorname{sen}^2 x}_{1} - (\ln|\sec x + tg\,x|)\operatorname{sen} x.$$

B2

$$x^2y'' + xy' + y = \ln x$$

homogênea associada: $x^2y'' + xy' + y = 0$

Verificamos que $y_1 = \cos(\ln x)$ e $y_2 = \text{sen}(\ln x)$ são soluções da homogênea associada.

$y_1' = \dfrac{-\text{sen}(\ln x)}{x}$ $\quad y_1'' = -\dfrac{1}{x^2}(-\text{sen}(\ln x)) + \left(\dfrac{-\cos(\ln x)}{x}\right)\left(\dfrac{1}{x}\right)$

$= \dfrac{1}{x^2}\text{sen}(\ln x) - \dfrac{1}{x^2}\cos(\ln x)$

$y_2' = \dfrac{\cos(\ln x)}{x}$ $\quad y_2'' = \left(-\dfrac{1}{x^2}\right)\cos(\ln x) + \dfrac{1}{x}\left(-\dfrac{\text{sen}(\ln x)}{x}\right)$

$= -\dfrac{1}{x^2}\left(\cos\ln x + \text{sen}\ln x\right)$ $\boxed{2}$

$x^2y_1'' + xy_1' + y_1 = \dfrac{x^2}{x^2}(\text{sen}\ln x - \cos\ln x) + \dfrac{x}{x}(-\text{sen}\ln x) + \cos\ln x = 0$

$x^2y_2'' + xy_2' + y_2 = \dfrac{x^2}{x^2}(-\cos\ln x - \text{sen}\ln x) + \dfrac{x}{x}\cos\ln x + \text{sen}\ln x = 0$

//

Variação de Parâmetros $\quad y_p = u_1\cos\ln x + u_2\,\text{sen}\ln x$

sistema: $\begin{cases} u_1'\cos\ln x + u_2'\,\text{sen}\ln x = 0 \\ -u_1'\dfrac{\text{sen}\ln x}{x} + u_2'\dfrac{\cos\ln x}{x} = \dfrac{\ln x}{x^2} \end{cases}$

$W = \begin{vmatrix} \cos\ln x & \text{sen}\ln x \\ -\dfrac{\text{sen}\ln x}{x} & \dfrac{\cos\ln x}{x} \end{vmatrix} = \dfrac{\cos^2\ln x}{x} + \dfrac{\text{sen}^2\ln x}{x} = \dfrac{1}{x}$ $\boxed{2}$

$u_1' = \dfrac{\begin{vmatrix} 0 & \text{sen}\ln x \\ \dfrac{\ln x}{x^2} & \dfrac{\cos\ln x}{x} \end{vmatrix}}{(1/x)} = \dfrac{x\ln x}{x^2}\begin{vmatrix} 0 & \text{sen}\ln x \\ 1 & \dfrac{\cos\ln x}{x} \end{vmatrix} =$

$= -\dfrac{\ln x}{x}\text{sen}\ln x$ $\boxed{2}$

$\boxed{2}$

$u_2' = \dfrac{\begin{vmatrix} \cos\ln x & 0 \\ -\dfrac{\text{sen}\ln x}{x} & \dfrac{\ln x}{x^2} \end{vmatrix}}{1/x} = \dfrac{x\ln x}{x^2}\begin{vmatrix} \cos\ln x & 0 \\ -\dfrac{\text{sen}\ln x}{x} & 1 \end{vmatrix} = \dfrac{\ln x}{x}\cos\ln x$

$w = \ln x \quad dw = \dfrac{1}{x}dx$

$u_1 = \int -w\,\text{sen}\,w\,dw$ $\quad u_2 = \int w\cos w\,dw = w$ $\boxed{2}$

$w = u \quad dw = du$ $\quad w = u \quad dw = du$

$-\text{sen}\,w\,dw = dv \quad v = \cos w$ $\quad \cos w\,dw = dv \quad v = \text{sen}\,w$

$u_2 = w\,\text{sen}\,w + \cos w = \ln x\,\text{sen}\ln x + \cos\ln x$

$u_1 = w\cos w - \text{sen}\,w$

$u_1 = \ln x\cos\ln x - \text{sen}\ln x$

$y_p = (\ln x\,\text{sen}(\ln x) + \cos\ln x)\,\text{sen}\ln x + (\ln x\cos(\ln x) - \text{sen}\ln x)\cos\ln x = \ln x$

CO $\mathcal{L}$ $y'' + 4y' + 4y = 1 + \delta(t-2)$ $\quad y(0) = 2 \quad y'(0) = 3$

$$s^2 Y(s) - s\,y(0) - y'(0) + 4s\,Y(s) - 4y(0) + 4Y(s) = \frac{1}{s} + e^{-2s}$$

$$\underbrace{(s^2 + 4s + 4)}_{(s+2)^2} Y(s) = 2s + 3 + 8 + \frac{1}{s} + e^{-2s}$$

$$Y(s) = \overbrace{\frac{2s+11}{(s+2)^2}}^{(a)} + \overbrace{\frac{1}{s(s+2)^2}}^{(b)} + \overbrace{\frac{e^{-2s}}{(s+2)^2}}^{(c)} \qquad \boxed{2}$$

(a) $$\mathcal{L}^{-1}\left\{\frac{2s+11}{(s+2)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{2\cancel{(s+2)}}{(s+2)^{\cancel{2}}}\right\} + \mathcal{L}^{-1}\left\{\frac{7}{(s+2)^2}\right\} =$$

Usando: $\mathcal{L}^{-1}\{F(s-a)\} = e^{at} f(t)$ e $\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n$

$$= 2e^{-2t} + 7te^{-2t} \qquad \boxed{2}$$

(b) $$\mathcal{L}^{-1}\left\{\frac{1}{s(s+2)^2}\right\} = \int_0^t e^{-2\tau}\,\underbrace{\tau}_{u}\, d\tau \underset{\text{partes}}{=} -\frac{e^{-2\tau}}{2}\tau\Big|_0^t + \left(-\frac{e^{-2\tau}}{4}\right)\Big|_0^t =$$

Usando: $\mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(\tau)\,d\tau$

$\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\} = e^{-2t}\,t$

$$= -\frac{e^{-2t}}{2}t - \frac{e^{-2t}}{4} + \frac{1}{4} \qquad \boxed{2}$$

(c) $$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{(s+2)^2}\right\} = u(t-2)\,f(t-2) = u(t-2)\,e^{-2(t-2)}(t-2)$$

Usando $\mathcal{L}^{-1}\{e^{-cs}F(s)\} = u(t-c)\,f(t-c)$

$$F(s) = \frac{1}{(s+2)^2} \longrightarrow f(t) = e^{-2t}\,t \qquad \boxed{2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \boxed{y(t) = \underbrace{2e^{-2t} + 7te^{-2t}}_{(a)} \underbrace{- \frac{e^{-2t}}{2}t - \frac{e^{-2t}}{4} + \frac{1}{4}}_{(b)} + \underbrace{u_2(t)\,e^{-2(t-2)}(t-2)}_{(c)}} \qquad \boxed{2}$$

(C1) $y'' - 3y' + 2y = \delta(t-1) + e^{2t} \quad y(0) = 0 \quad y'(0) = 1$

$\mathcal{L}$

$$s^2 Y(s) - s y(0) - y'(0) - 3(sY(s) - y(0)) + 2Y(s) = e^{-s} + \frac{1}{s-2}$$

$$\underbrace{(s^2 - 3s + 2)}_{(s-2)(s-1)} Y(s) = 1 + e^{-s} + \frac{1}{s-2}$$

$$Y(s) = \overbrace{\frac{1}{(s-2)(s-1)}}^{(a)} + \overbrace{\frac{e^{-s}}{(s-2)(s-1)}}^{(b)} + \overbrace{\frac{1}{(s-2)^2(s-1)}}^{c} \quad \boxed{2}$$

(a) $$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-2} - \frac{1}{s-1}\right\} = e^{2t} - e^{t}$$

$$\frac{A}{s-2} + \frac{B}{s-1} = \frac{A(s-1) + B(s-2)}{(s-2)(s-1)} \Rightarrow (A+B)s + (-A-2B) = 1$$

$A+B=0$ e $-A-2B=1$ $\quad A=-B \Rightarrow B=-1 \quad A=1$ $\boxed{2}$

(b) $$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{(s-2)(s-1)}\right\} = u(t-1)\left[e^{2(t-1)} - e^{(t-1)}\right]$$

Usando: $\mathcal{L}^{-1}\{e^{-cs}F(s)\} = u(t-c)f(t-c)$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-1)}\right\} = e^{2t} - e^{t} \quad \boxed{2}$$

(c) $$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2} \cdot \frac{1}{s-1}\right\} = e^{2t}t * e^{t} = \int_0^t e^{2\tau}\tau \, e^{(t-\tau)} d\tau$$

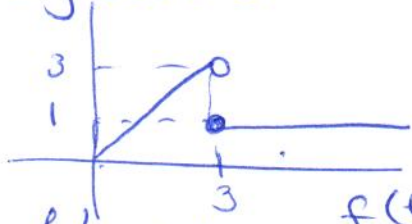
$$= e^{t}\int_0^t \underbrace{e^{\tau}}_{dv}\underbrace{\tau}_{u} \, d\tau = e^{t}\left[\tau e^{\tau}\Big|_0^t - e^{\tau}\Big|_0^t\right] =$$

$$e^{t}\left[te^{t} - e^{t} + 1\right] \quad \boxed{2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \boxed{y(t) = \underbrace{e^{2t} - e^{t}}_{a} + \underbrace{u_1(t)\left[e^{2(t-1)} - e^{(t-1)}\right]}_{b} + \underbrace{te^{2t} - e^{2t} + e^{t}}_{c}} \quad \boxed{2}$$

C2 $\quad y'' + 2y' + y = f(t) \quad y(0) = y'(0) = 0$

$f(t) = (1 - u_3(t))t + u_3(t)$




$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\} - \mathcal{L}\{u_3(t).\underset{f(t-3)}{t}\} + \mathcal{L}\{u_3(t)\}$

$f(t-3) = t \qquad v = t-3 \Rightarrow f(v) = v+3.$

$\mathcal{L}\{u_3(t).t\} = e^{-3s}\mathcal{L}\{f(t)\} = e^{-3s}\mathcal{L}\{t+3\} =$
$= e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s}\right)$ $\boxed{2}$

$\mathcal{L}\{f(t)\} = \frac{1}{s^2} - e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s}\right) + \frac{e^{-3s}}{s}$

$s^2Y(s) - s\underset{0}{y(0)} - \underset{0}{y'(0)} + 2[sY(s) - \underset{0}{y(0)}] + Y(s) = \mathcal{L}\{f(t)\}$

$(s^2 + 2s + 1)Y(s) = \frac{1}{s^2} - \frac{e^{-3s}}{s^2} - \frac{2e^{-3s}}{s}$ $\boxed{2}$

$Y(s) = \underbrace{\frac{1}{s^2(s+1)^2}}_{a} - \underbrace{\frac{e^{-3s}}{s^2(s+1)^2}}_{b} - \underbrace{\frac{2e^{-3s}}{s(s+1)^2}}_{c}$

(c) $\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = e^{-t}.t$ (translação em s) $\boxed{2}$

$\mathcal{L}^{-1}\left\{\frac{\frac{1}{(s+1)^2}}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(\tau)d\tau = \int_0^t e^{-\tau}\underset{u}{\tau}\,d\tau$ $\quad dv$

$= -\tau e^{-\tau}\Big|_0^t - \int_0^t -e^{-\tau}d\tau = -te^{-t} - e^{-t} + 1$

$2\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s(s+1)^2}\right\} = 2u_3(t)\left[-(t-3)e^{-(t-3)} - e^{-(t-3)} + 1\right]$

(a) $\mathcal{L}^{-1}\left\{\frac{\frac{1}{s(s+1)^2}}{s}\right\} = \int_0^t -\tau e^{-\tau} - e^{-\tau} + 1\,d\tau = -(-te^{-t} - e^{-t} + 1)$
$+ (e^{-t} - 1) + t = te^{-t} + 2e^{-t} - 2 + t$ $\boxed{2}$

(b) $\mathcal{L}^{-1}\left\{e^{-3s}\frac{1}{s^2(s+1)^2}\right\} = u_3(t)\left[(t-3)e^{-(t-3)} + 2e^{-(t-3)} + t - 5\right]$

translação em t.

$\mathcal{L}^{-1}\{Y(s)\} = y(t) = 2u_3(t)\left[(t-3)e^{-(t-3)} + e^{-(t-3)} - 1\right] +$ $\boxed{2}$
$+ te^{-t} + 2e^{-t} - 2 + t - u_3(t)\left[(t-3)e^{-(t-3)} + 2e^{-(t-3)} + t - 5\right]$
$= u_3(t)(t-3)e^{-(t-3)} + 3u_3(t) - tu_3(t) + (t+2)e^{-t} + t - 2$

DO

$$f(t) = \begin{cases} \operatorname{sen} t & 0 \le t < \pi \\ t^2 & \pi \le t < 4 \\ 2 & t \geq 4 \end{cases}$$

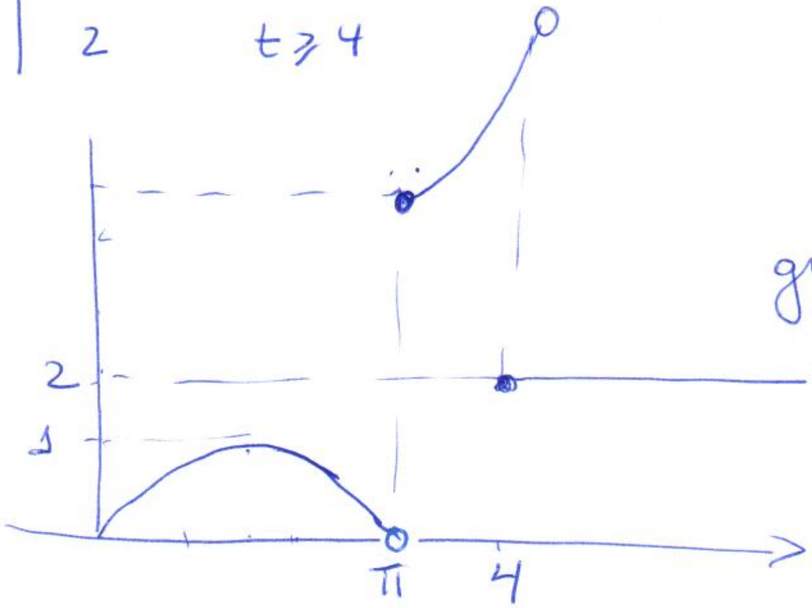



gráfico fora de escala.

2

$$f(t) = (1 - u_\pi(t)) \operatorname{sen} t + u_\pi(t) t^2 - u_4(t) t^2 + 2u_4(t)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\operatorname{sen} t - u_\pi(t)\operatorname{sen} t\} + \mathcal{L}\{u_\pi(t)t^2\} - \mathcal{L}\{u_4(t)t^2\} + 2\mathcal{L}\{u_4(t)$$

2

$$\mathcal{L}\{\operatorname{sen} t - u_\pi(t)\operatorname{sen} t\} = \mathcal{L}\{\operatorname{sen} t\} - \mathcal{L}\{u_\pi(t) \underbrace{\operatorname{sen} t}_{f(t-\pi)}\}$$

2

$$= \frac{1}{s^2+1} - e^{-\pi s}\mathcal{L}\{f(t)\}$$

Se $f(t-\pi) = \operatorname{sen} t \quad t - \pi = v$

$$= \frac{1}{s^2+1} - e^{-\pi s} \cdot \left(-\frac{1}{s^2+1}\right)$$

$\to f(v) = \operatorname{sen}(v+\pi) = -\operatorname{sen} v$

$$\mathcal{L}\{u_\pi(t)t^2\} = e^{-\pi s}\mathcal{L}\{t^2 + 2\pi t + \pi^2\} = e^{-\pi s}\left(\frac{2}{s^3} + \frac{2\pi}{s^2} + \frac{\pi^2}{s}\right)$$

$f(t-\pi) = t^2 \quad v = t - \pi \Rightarrow f(v) = (v+\pi)^2 = v^2 + 2\pi v + \pi^2$

$$\mathcal{L}\{u_4(t)t^2\} = e^{-4s}\mathcal{L}\{t^2 + 8t + 16\} = e^{-4s}\left(2/s^3 + 8/s^2 + 16/s\right)$$

$f(t-4) = t^2 \quad v = t - 4 \Rightarrow f(v) = (v+4)^2 = v^2 + 8v + 16.$ 2

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1} + e^{-\pi s}\left(\frac{2}{s^3} + \frac{2\pi}{s^2} + \frac{\pi^2}{s}\right)$$

$\leftarrow$ 2

$$- e^{-4s}\left(2/s^3 + 8/s^2 + \frac{16}{s}\right) + 2\frac{e^{-4s}}{s}.$$

pois $2\mathcal{L}\{u_4(t)\} = 2\dfrac{e^{-4s}}{s}.$

$\hookrightarrow$ tabela.

D1 $\quad tx'' - 2x' + 9tx = 0 \quad x(0)=0$

$\downarrow \mathcal{L}$

Usar: $\mathcal{L}\{t f(t)\} = -F'(s)$

$\mathcal{L}\{tx''\} - 2\mathcal{L}\{x'\} + 9\mathcal{L}\{tx\} = 0$

$-[s^2X(s) - s\underbrace{x(0)}_{0} - x'(0)]' - 2[sX(s) - \underbrace{x(0)}_{0}] - 9X'(s) = 0$ $\boxed{2}$

$-[2sX(s) + s^2X'(s)] - 2sX(s) - 9X'(s) = 0$

regra do produto

$(s^2+9)X'(s) + 4sX(s) = 0$

$X'(s) + \dfrac{4s}{s^2+9}X(s) = 0$ $\boxed{2}$

Fator integrante:

$\mu = e^{\int \frac{4s}{s^2+9}ds} \qquad \begin{matrix} u = s^2+9 \\ du = 2s\,ds \end{matrix} \quad \Rightarrow \mu(s) = e^{2\ln u} = u^2 = (s^2+9)^2$

Multiplica pelo fator integrante:

$(s^2+9)^2X'(s) + 4s(s^2+9)X(s) = 0$

$\dfrac{d}{ds}\left[(s^2+9)^2X(s)\right] = 0 \quad \Rightarrow$ $\boxed{2}$

$(s^2+9)^2X(s) = C \Rightarrow X(s) = \dfrac{C}{(s^2+9)^2}$

$x(t) = \mathcal{L}^{-1}\left\{\dfrac{C}{(s^2+9)^2}\right\} = \dfrac{C}{9}\mathcal{L}^{-1}\left\{\dfrac{3}{s^2+9}\cdot\dfrac{3}{s^2+9}\right\} =$ $\boxed{2}$

$= \dfrac{C}{9}\,\text{sen}\,3t * \text{sen}\,3t$ $\quad \overset{\text{CONTAS}}{=} \dfrac{C}{9}\int_0^t \text{sen}\,3\tau\,(\text{sen}(3t-3\tau))\,d\tau$

$\int_0^t \text{sen}\,3\tau[\text{sen}3t\cos3\tau - \text{sen}3\tau\cos3t]d\tau = \text{sen}3t\int_0^t \dfrac{\text{sen}\,6\tau}{2} - \cos3t\int_0^t\dfrac{1-\cos6\tau}{2}$

$\text{sen}3t\left(-\dfrac{\cos6\tau}{12}\Big|_0^t\right) - \cos3t\left(\dfrac{1}{2}\tau - \dfrac{\text{sen}\,6\tau}{12}\right)\Big|_0^t = -\dfrac{\text{sen}3t\cos6t}{12} + \dfrac{\text{sen}3t}{12}$

$-\dfrac{(\cos3t)t}{2} + \dfrac{\text{sen}\,6t}{12} \Rightarrow \boxed{x(t) = -\dfrac{C}{12\cdot9}\left(-\text{sen}3t\cos6t + \text{sen}3t \;\; 6t\cos3t + \cos3t\,\text{sen}6t\right)}$ $\boxed{2}$

D2 - $\mathcal{L}^{-1}\left\{\arctan \frac{5}{s+2}\right\}$

Usar: $\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$ $\boxed{2}$

$F(s) = \arctan \frac{5}{s+2}$ regra da cadeia.

$$F'(s) = \frac{1}{1+\left(\frac{5}{s+2}\right)^2} \cdot \left(\frac{5}{s+2}\right)' = \frac{(s+2)^2}{(s+2)^2+5^2}\left(-5(s+2)^{-2}\right)$$

$$= \frac{-5}{(s+2)^2+5^2}$$ $\boxed{4}$

$$\mathcal{L}^{-1}\{F'(s)\} = \mathcal{L}^{-1}\left\{\frac{-5}{(s+2)^2+5^2}\right\} = -t\,f(t)$$

$$= -e^{-2t}\operatorname{sen} 5t \Rightarrow e^{-2t}\operatorname{sen} 5t = t\,f(t)$$ $\boxed{2}$

$$\boxed{f(t) = \frac{e^{-2t}\operatorname{sen} 5t}{t}}$$ $\boxed{2}$