A Finite Element Model for Three Dimensional Hydraulic Fracturing

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Motivation

- Hydraulic fracturing is a method extensively used in oil industry
- To enhance petroleum recovery, fractures are artificially created inside the porous media by applying hydraulic pressure produced by an injected fluid
- Usually, numerical simulations use simple models (2d or pseudo3d) which are not suited for high permeability and/or low efficiency fluids (e.g.water).
- The purpose is to obtain a more sofisticated numerical model, including the fluid flow inside the fracture, the elastic response of the porous media and leak-off.
- The simulation is part of a research project between Unicamp and PETROBRAS

Summary

Introduction

Model Construction

- Mathematical Model
- Finite Element Model
- Algorithm for Temporal Evolution

Numerical Simulation

Conclusions

Mathematical Model

- **Fluid flow inside the fracture: conservation of mass**

  Law relating the fluid flow with the variation the fracture opening

- **Elastic response of the porous media**

  Formula relating the fracture opening and the pressure on the fracture walls

- **Fluid filtration to the porous media**

Numerical Model

- **For the fluid flow inside the fracture**

  finite element Galerkin method + finite difference time integration

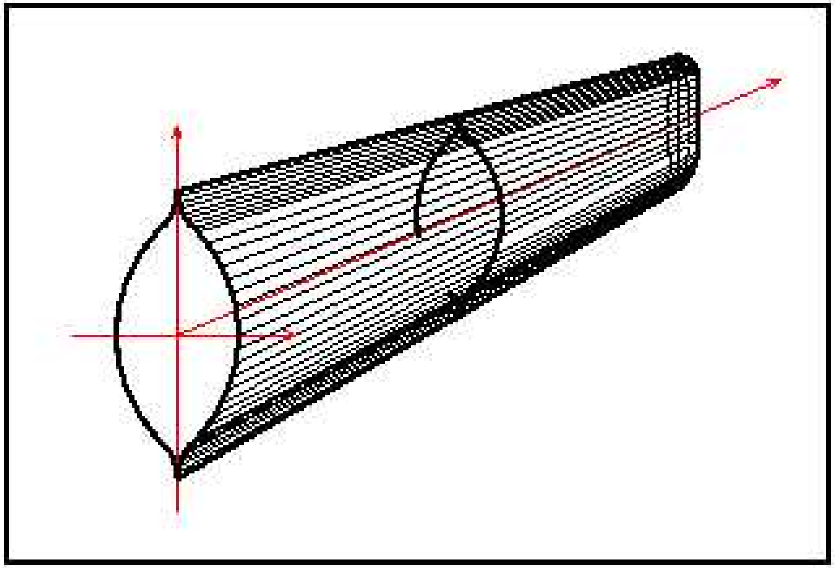
- **For the relation between the pressure and fracture opening**

  finite element approximation + analytical formulas

Computational implementation: PZ enviroment

- Based on finite element techniques for the numerical solution of PDE
- Object oriented philosophy

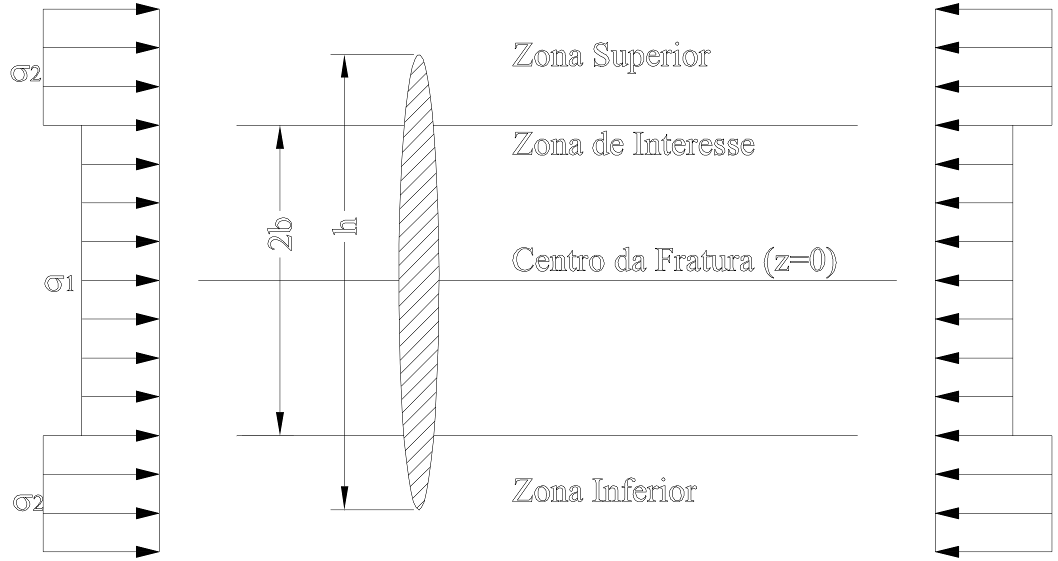
Geometric Aspects



- **Axis** x: main propagation direction
- **Axis** z: fracture hight
- **Axis** y: fracture opening width

$$|y| \leq \frac{w(x,z,t)}{2},$$

Section orthogonal to the propagation direction



- σ: in-situ stress distribution
- $h/2$: frature hight
- $H_r = b$: reservoir hight

Matemathical Model

State variable: fracture opening $w = w(x, z, t)$

- **Opening displacement equation (Bui, 1977)**

$$p(x,z,t) - \sigma(x,z) = \frac{G}{4\pi(1-\nu)} \int_\Omega \left[\frac{\partial}{\partial x}\left(\frac{1}{r}\right)\frac{\partial w}{\partial x^l} + \frac{\partial}{\partial z}\left(\frac{1}{r}\right)\frac{\partial w}{\partial z^l} \right] dx^l dz^l$$

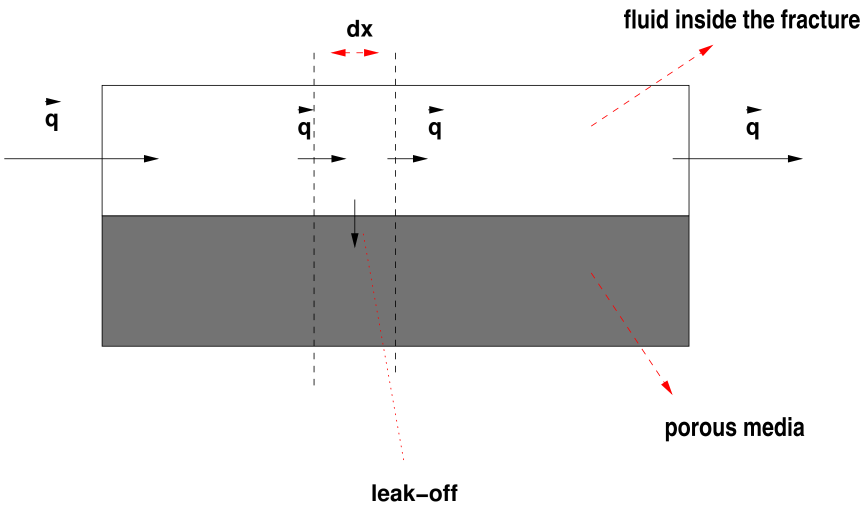
  - $r = \sqrt{(x-x^l)^2 + (z-z^l)^2}$ $\quad$ G, ν – shear modulus and Poisson's ration
  - Ω: computational region on the plane $x \times z$

- **Leak-off: Carter's model (1957)**

$$q_l(t) = A\left(v_{sp}\delta(t-\tau) + \frac{\alpha}{2\sqrt{t-\tau}} \right)$$

Fluid loss, per time unit, on area A exposed during time period τ

- **Fluid Flow: Conservation of mass**



$$div(\overrightarrow{q}) + \frac{\partial w}{\partial t} + Q_l = 0.$$

- **Fluid flow models**
  - Newtonian fluid

$$\overrightarrow{q} = -\frac{w^3}{12\mu}\nabla p$$

  - Non Newtonian fluids (power law): $0 < \alpha < 1$

$$\overrightarrow{q} = \left(\frac{2\alpha}{1+2\alpha}\right)\left(\frac{1}{k}\right)^{1/\alpha}\left(\frac{w}{2}\right)^{\frac{1+2\alpha}{\alpha}} ||\nabla p||^{\frac{1-\alpha}{\alpha}} \nabla p,$$

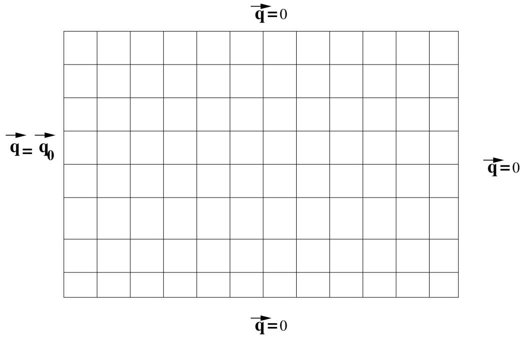
Numerical Approximation Schemes

For the conservation law

- **Galerkin Method**
  - Spatial discretization by bilinear finite elements

$$w(x,z,t^n) = \sum_i w_i^n \Psi_i(x,z)$$

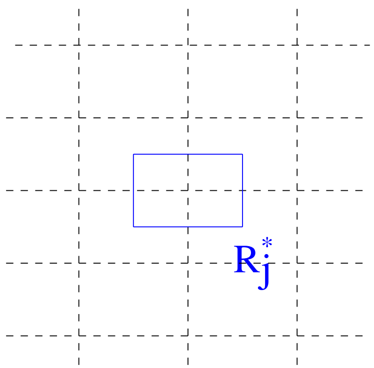
  - Computational region and boundary conditions



- **Temporal discretization**
  - Implicit Euler scheme + mass lumping

$$-\int_{\Omega} \nabla\Psi_i . \overrightarrow{q}^{n+1} dxdy + \overline{\Psi}_i \frac{w_i^{n+1} - w_i^n}{\Delta t} + \overline{\Psi}_i \int_{t_n}^{t_n+1} Q_{li} dt + \overrightarrow{q}\Psi_i|_{\partial\Omega} = 0$$

For the opening-preassure integral equation



p = constant on staggered cells R_j^*

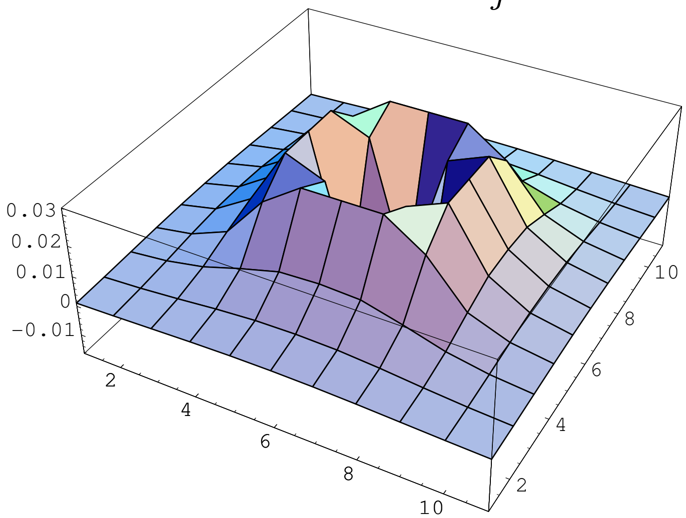
$$\begin{aligned} p_j^n &= \sigma_j + \frac{G}{4\pi(1-\nu)} \frac{1}{m_j^*} \int_{R_j^*} \int_{\Omega} \left[\frac{\partial}{\partial x^l} \left(\frac{1}{r}\right) \frac{\partial w}{\partial x} + \frac{\partial}{\partial y^l} \left(\frac{1}{r}\right) \frac{\partial w}{\partial z} \right] dxdzdx^l dz^l \\ &= \sigma_j + \frac{G}{4\pi(1-\nu)} \frac{1}{m_j^*} \sum_i T_{ij} w_i^n, \end{aligned}$$

Opening-preassure connection matrix

p = T w + Σ

conection matrix

$$T_{ij} = \int_{R_j^*} \frac{\partial}{\partial x}\left(\int_\Omega \frac{1}{r}\frac{\partial \Psi^i}{\partial x^l} dx^l\ dz^l\right) dx\ dz + \int_{R_j^*} \frac{\partial}{\partial z}\left(\int_\Omega \frac{1}{r}\frac{\partial \Psi^i}{\partial y^l} dx^l\ dz^l\right) dx\ dz.$$



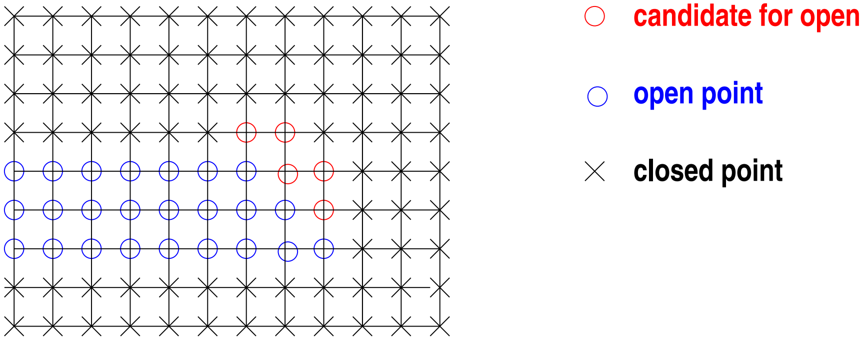
Algorithm for temporal integration

$$(\mathbf{w}^n, \mathbf{p}^n) \longrightarrow (\mathbf{w}^{n+1}, \mathbf{p}^{n+1})$$

First step: Algorithm for fracture evolution

- **Point classification**

$$\begin{cases} \text{open} \quad w(x_i, y_i, t^n) > 0 \\ \text{candidate for open } w(x_i, y_i, t^n) = 0 \text{ but } Res_i > \epsilon \\ \text{closed: otherwise} \end{cases}$$



- **zero flux**: If some of the four nodes of the cell boundary is closed

Second step: Newton's method + GMRES

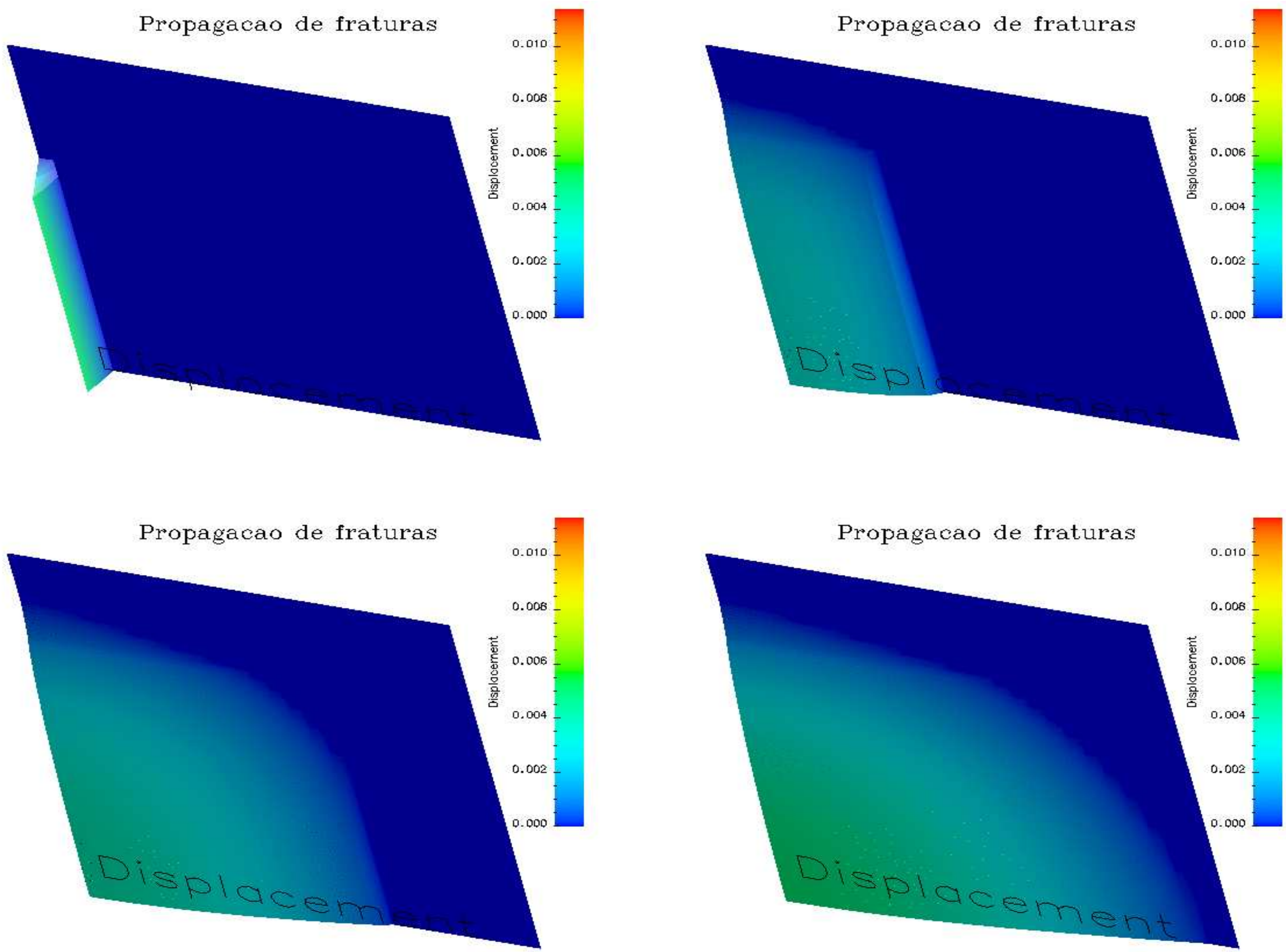
$$Res^{(m)} = \mathcal{G}(\mathbf{w}^{n+1,(m)}, \mathbf{p}^{n+1,(m)}) + \tilde{\mathbf{F}}_n^{n+1}$$

$$\begin{aligned}
\mathcal{G}_i(\mathbf{w}, \mathbf{p}) &= -\Delta t \int_{\Omega} \nabla \Psi_i \overrightarrow{q}\, dxdy + \overline{\Psi}_i w_i \\
\left(\tilde{\mathbf{F}}_n^{n+1}\right)_i &= -\overline{\Psi}_i w_i^n + \Delta t \Psi_i \overrightarrow{q}^{n+1}\big|_{\partial\Omega}
\end{aligned}$$

$$\begin{pmatrix} \mathcal{K}^w & \mathcal{K}^p \\ -T & I \end{pmatrix} \begin{pmatrix} \mathbf{w}^{n+1,(m+1)} - \mathbf{w}^{n+1,(m)} \\ \mathbf{p}^{n+1,(m+1)} - \mathbf{p}^{n+1,(m)} \end{pmatrix} = \begin{pmatrix} Res^m \\ 0 \end{pmatrix}$$

Third step: Leak-off and pos-processing

Numerical results



Conclusions and Next Steps

- Demontration of the numerical viability of hydraulic fracture simulation by combining fluid flow inside fractures, elastic response of the porous media and leak-off;
- Pseudo3D simulations indicate some advantages in including the leakoff term during Newton's iterations. For the 3D model, this would require a more elaborated control of fracture opening;
- There is a great interest in PETROBRAS to consider horizontal wells simulations;
- The next step of the project is to extend the formulation to consider 3D non-planar fracture propagation.