

Proposta de Disciplina de Pós-graduação

Tópico de Geometria (MM813 ou MM814)

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Em 1995, Brualdi, Graves e Lawrence [12] introduziram o conceito de métricas em um espaço vetorial induzidas por uma ordem parcial. Estas são as métricas poset, que generalizam as métricas de Hamming e as métricas conhecidas por métricas de Niederreiter–Rosenbloom–Tsfasman ([55] e [70]).

A métrica de Hamming (determinada por um poset anti-cadeia) se assemelha (através de diversas de suas propriedades) à métrica Euclidiana, enquanto a métrica de Rosenbloom-Tsfasman metric (determinada por um poset cadeia) é uma ultramétrica. Um poset genérico mescla estas propriedades extremas, nas quais temos, em um caso, em que cada bola métrica possui um único centro e no outro extremo (ultramétrico) no qual todo ponto de uma bola é seu centro.

A mescla destas duas propriedades torna o estudo de espaços com métricas poset estimulante e atraente. Neste minicurso abordaremos diversas questões relacionadas a estes tópicos, diversas destas oriundas da teoria de Códigos Corretores de Erros.

A disciplina contará com notas de aula, que sumarizam diversos dos trabalhos que seguem abaixo.

A disciplina ter quatro horas de aula por semana, envolver leitura de textos, resolução de exercícios e uma prova.

Conteúdo da Disciplina

1. Métricas poset
 - (a) Ordens parciais sobre conjuntos finitos
 - (b) Algumas famílias especiais de posets
 - (c) Métricas definidas por posets
 - (d) Isometrias lineares e caracterização de órbitas
2. O caso simples: Posets hierárquicos
 - (a) Forma canônica sistemática de um código
 - (b) Distância mínima, raios de empacotamento, cobertura e Chebitchev
 - (c) Decodificação por síndrome

- (d) Códigos perfeitos e MDS
- 3. Um caso difícil importante: Cadeias disjuntas de mesmo comprimento
 - (a) Distância mínima e raio de empacotamento
 - (b) Identidade de MacWilliams
 - (c) Modelo de canal
- 4. O caso genérico: Invariantes de códigos
 - (a) Raio de cobertura e de empacotamento
 - (b) Códigos perfeitos e MDS
 - (c) Mergulhos e questões de complexidade
 - (d) Limitantes utilizando posets hierárquicos
- 5. Duality
 - (a) Dualidade e identidade de Mac-Williams
 - (b) I -bolas e identidade de tipo Mac-Williams generalizada
 - (c) Esquemas associativos e dualidade de posets
- 6. Extensões
 - (a) Métricas poset em módulos
 - (b) Métricas poset-blocos
 - (c) Métricas definidas por grafos orientados
 - (d) Métricas de Lee–Niederreiter–Rosenbloom–Tsfasman

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