## Zakharov-Kuznetsov equation on $\mathbb{R}^n$:

$$(ZK) \quad \partial_t u + \partial_x^3 u + \partial_x \Delta_y u + \partial_x (u^2) = 0,$$

$$\Delta_y = \sum_{j=1}^{n-1} \partial_{y_j}^2, \quad u : \mathbb{R}_t \times \mathbb{R}_x \times \mathbb{R}_y^{n-1} \longrightarrow \mathbb{R}.$$

- Zakharov and Kuznetsov, 1974: Model for propagation of ionic sound waves in a magnetized plasma in 3D;
- Laedke and Spatschek, 1982: Derivation as a model in 2D;
- Lannes, Linares and Saut, 2012: Rigorous justification of (ZK) in 2D and 3D from the Euler-Poisson equations

Conserved quantities:

- Mass $M(t) = \int_{\mathbb{R}^n} u(t,x,y)^2 dx\,dy$ (= $L^2$-Norm$^2$)
- Energy $E(t) = \frac{1}{2} \int_{\mathbb{R}^n} |\nabla u(t,x,y)|^2 dx\,dy$

$$- \frac{2}{3} \int_{\mathbb{R}^n} u(t,x,y)^3 dx\,dy \quad (\sim H^1\text{-Norm}^2)$$

# Some results on the Cauchy-Problem:

(i) In two space dimensions:

- Faminskii, 1995: LWP and GWP in $H^1(\mathbb{R}^2)$ and above;
- Linares and Pastor, 2009: LWP in $H^s(\mathbb{R}^2)$ for $s > \frac{3}{4}$;

<u>Theorem</u> (G., Herr; 2013): The Cauchy-Problem for the ZK-equation is LWP in $H^s(\mathbb{R}^2)$ for $s > \frac{1}{2}$.

- Molinet and Pilod, 2013: LWP in $H^s(\mathbb{R}^2)$, $s > \frac{1}{2}$.

(ii) On the threedimensional problem:

- Linares and Saut, 2009: LWP in $H^s(\mathbb{R}^3)$ for $s > \frac{9}{8}$;
- Ribaud and Vento, 2012: LWP in $H^s(\mathbb{R}^3)$ for $s > 1$ (and in $B^1_{2,1}(\mathbb{R}^3)$);
- Molinet and Pilod, 2013: GWP in $H^s(\mathbb{R}^3)$ for $s > 1$.

(GWP in $H^1(\mathbb{R}^3)$ is still open!)

# Linear estimates

u resp. v solutions of the linear equations

| | |
|---|---|
| $(\partial_t + \partial_x^3 + \partial_x \partial_y^2) u = 0$ | $(\partial_t + \partial_x^3 + \partial_y^3) v = 0$ |
| $(\hat{=}\ (ZK))$ | $(\hat{=}\ (ZK_{sym.}))$ |

with initial datum $u_0$. The following space-time norms are $\lesssim \|u_0\|_{L^2_{xy}}$:

① Strichartz estimates with derivatives $\left(\frac{1}{p} + \frac{1}{q} = \frac{1}{2}\right)$:

| | | |
|---|---|---|
| $\|(3I_x^2 - I_y^2)^{\frac{1}{2p}} u\|_{L^p_t L^q_{xy}}$ | | $\|(I_x I_y)^{\frac{1}{2p}} v\|_{L^p_t L^q_{xy}}$ |
| $p=q=4$ in (Carbery, Kenig, Ziesler; 2012) | $\Leftarrow$ | (KPV, 91) |

② Maximal function estimate:

| | | |
|---|---|---|
| $\|(3I_x^2 - I_y^2)^{-\frac{1}{4}} u\|_{L^4_{xy} L^\infty_t}$ | $\Leftarrow$ | $\|(I_x I_y)^{-\frac{1}{4}} v\|_{L^4_{xy} L^\infty_t}$ |
| known before: | | (KPV, 93) + Fubini |
| $\|J^{-\frac{3}{4}-} u\|_{L^4_x L^\infty_{yT}}$ | | |
| (Linares, Pastor; 2009) | | |