Scattering of solutions and stability of solitary waves for the generalized BBM-ZK equation

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Contents

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BBM equation

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Assumptions: $f \in C^2(\mathbb{R})$ such that $f(0) = 0$ and $f(s) = O(|s|^{p+1})$ as $|s| \to +\infty$.

Theorem

$\forall\ u_0 \in H^s(\mathbb{R}^2)$, $s > 1$, $\exists\ T = T(\|u_0\|_s) > 0$ & $\exists!\ u \in C\left([0,T); H^s(\mathbb{R}^2)\right)$ of the BBM-ZK Eq. with $u(0) = u_0$.

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$$I(u(t)) = \int_{\mathbb{R}^2} u \,\mathrm{d}x\mathrm{d}y = I(u_0),$$

$$E(u(t)) = -\frac{1}{2}\int_{\mathbb{R}^2} \varepsilon u^2 + \beta u_y^2 + 2F(u)\,\mathrm{d}x\mathrm{d}y = E(u_0),$$

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where $F' = f$. *Furthermore,* $u \in C\left([0,T); H^{s_1,s_2}(\mathbb{R}^2)\right)$, *where* $s_1 \geq 0$ *and* $s_2 = s - s_1$.

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Lemma

Let $N \gg 1$, $t \neq 0$ and $J = [-N, N]$. Then there exists a constant $C > 0$, independent of t and N, such that

$$\left| \int_J \int_{\mathbb{R}} \mathrm{e}^{-\mathrm{i}\left(t\xi\left(\frac{\varepsilon+\beta\eta^2}{1+\xi^2}\right) - x\xi - y\eta\right)} \mathrm{d}\eta \, \mathrm{d}\xi \right| \leq C t^{-1/2} N^{3/2}.$$

Let $S(t)$ be the C_0-group of unitary operators for

$$u_t + \varepsilon u_x - (u_{xt} + \beta u_{yy})_x = 0.$$

Lemma

1. *Let $s_1, s_2 > 1/2$, $\theta = (2(s_1+1))^{-1} \in [0, 1/3)$ and $u_0 \in H^{s_1,s_2}(\mathbb{R}^2) \cap L^1(\mathbb{R}^2)$. Then there exists a constant $C > 0$, depending only on s_1, such that for all $t \in \mathbb{R}$, we have*

$$|S(t)u_0|_\infty \le C(1+|t|)^{\frac{3\theta-1}{2}} \left(|u_0|_1 + \|u_0\|_{(s_1,s_2)}\right).$$

2. *Let $s > 1/2$, $\theta = (2(s+1))^{-1} \in [0, 1/3)$ and $u_0 \in H^{2s}(\mathbb{R}^2) \cap L^1(\mathbb{R}^2)$. Then there exists a constant $C > 0$, depending only on s, such that for all $t \in \mathbb{R}$, we have*

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Let $1 < q \leq \infty$ *and* $1/q + 1/q' = 1$.

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Suppose that $s > 1$, $\mu = (1 - 3\theta)/2$, $\theta = (2(s+1))^{-1} \in [0, 1/3)$ *and*

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Then there exists $\delta > 0$ *and* $C > 0$ *such that for any* $u_0 \in H^{2s}(\mathbb{R}^2) \cap L^1(\mathbb{R}^2)$ *satisfying* $|u_0|_1 + \|u_0\|_{2s} < \delta$, *there is a unique solution* $u \in C(\mathbb{R}; H^{2s}(\mathbb{R}^2))$ *of the BBM-ZK Eq. with* $u(0) = u_0$ *such that*

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Theorem

Let $u(t)$ be the solution of the BBM-ZK Eq., then there exists a unique solution $u^{\pm}(t) \in H^{2s}(\mathbb{R}^2)$, $s \geq 1/2$, of the linearized equation of the BBM-ZK Eq. such that

$$\|u(t) - u^{\pm}(t)\|_{2s} \to 0,$$

as $t \to \pm\infty$.

Contents

Solitary wave solution of the BBM-ZK Eq. of the form $u(x, y, t) = \varphi_c(x - ct, y)$, vanishing at infinity, where $c > 0$ is the wave velocity:

$$-c\partial_x^2\varphi_c + (c - \varepsilon)\varphi_c + \beta\partial_y^2\varphi_c - F(\varphi_c) = 0. \qquad (*)$$

Theorem

Let $c > 0$ and $\beta < 0$, then equation $(*)$ possesses a cylindrically symmetric positive solution $\varphi_c \in H^1(\mathbb{R}^2)$, which is called a ground state of $(*)$.

Moreover, $\varphi_c \in H^\infty(\mathbb{R}^2)$, $\partial_r \varphi_c(r) < 0$, for all $r \neq 0$ where $r = |(\sqrt{c}x, \sqrt{-\beta}y)|$, and there is a $\sigma > 0$ such that for all $\alpha \in \mathbb{N}^2$ with $|\alpha| \leq 2$, $|\partial^\alpha \varphi_c(x, y)| \leq C_\alpha \mathrm{e}^{-\sigma r}$.

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Define the the functional S_c and the linearized operator L_c around φ_c by

$$S_c(\varphi_c) = E(\varphi_c) + cQ(\varphi_c)$$

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Hypothesis I

The ground state solution of equation $(*)$ is unique and the curve $c \mapsto \varphi_c$ defined for $c > \varepsilon$ is C^1 with values in $H^2(\mathbb{R}^2)$. Moreover there are positive constants C and ϱ, such that

$$\left|\frac{\mathrm{d}\varphi_c}{\mathrm{d}c}(x,y)\right| \leq C\mathrm{e}^{-\varrho|(x,y)|},$$

for all $(x,y) \in \mathbb{R}^2$ and $c > \varepsilon$.

Hypothesis II

The operator L_c, for $c > \varepsilon$, has a unique simple negative eigenvalue λ_c with a corresponding cylindrically symmetric positive eigenfunction χ_c such that for all $(x, y) \in \mathbb{R}^2$,

$$|\chi_c(x, y)| \leq C_c \mathrm{e}^{-\rho|(x,y)|},$$

for some positive constants C_c and ρ. In addition, the mapping $c \mapsto \chi_c$ is continuous with values in $H^2(\mathbb{R}^2)$. Furthermore the essential spectrum of L_c is positive and bounded away from zero and the null space of L_c is spanned by $\partial_x \varphi_c$ and $\partial_y \varphi_c$.

Hypothesis III

$\mathcal{L}(\frac{\mathrm{d}\varphi_c}{\mathrm{d}c}) \in L^1(\mathbb{R}^2)$, where $\mathcal{L} = I - \partial_x^2$.

Theorem

Suppose that the Hypotheses I-III hold. Then φ_c *is stable if and only if* $d''(c) > 0$*, where*

$$d(c) = E(\varphi_c) + cQ(\varphi_c).$$

$$A(t) = \int_{\mathbb{R}^2} \partial_x^{-1} u(x,y,t) \mathcal{L}\vartheta(x-\gamma(t),y)\, \mathrm{d}x\, \mathrm{d}y,$$

where $\vartheta = \frac{\mathrm{d}\varphi_c}{\mathrm{d}c} + s'(c)\chi_c$.

Theorem

Let $u_0 \in H^{s_1,s_2}(\mathbb{R}^2) \cap L^\infty(\mathbb{R}_y; L^1(\mathbb{R}_x)) \cap L^1(\mathbb{R}^2)$, $s_1, s_2 > 1/2$,

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$$\left|\partial_x^{-1} u(z,y,t)\right|_\infty \le C(1+|t|)^{\frac{3\theta+1}{2}},$$

where $\theta = (2(s_1+1))^{-1} \in (0,1/3)$ and C is a constant depending only on

$$\sup_{t\ge 0} \|u(t)\|_1 + \|u_0\|_{L_y^\infty L_x^1} + |u_0|_1.$$

Contents

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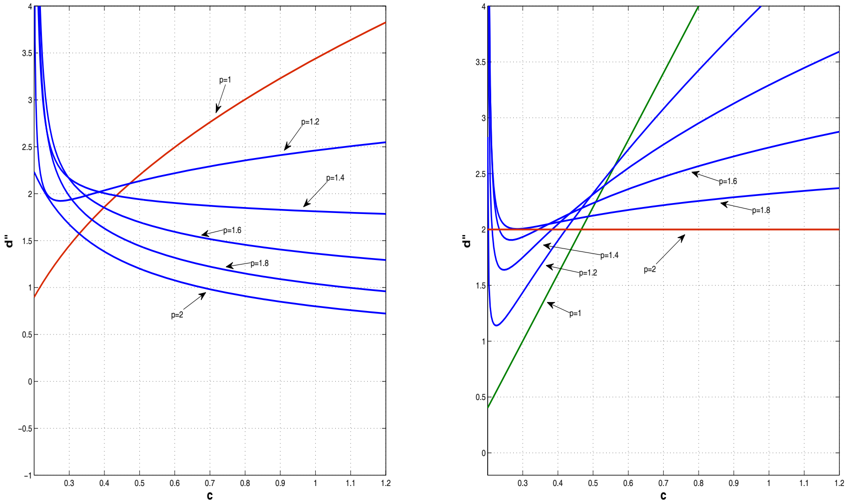
5. $p > 4$, $p\varepsilon \neq 4$ and $\{c > \varepsilon\} \cap \{a_1(p) < c < a_2(p)\}$, where
$$a_1(p) = \frac{p\varepsilon - 4 - \sqrt{p^2\varepsilon(\varepsilon-1) - 8p\varepsilon + 16(\varepsilon+1)}}{p^2-16}p$$
and
$$a_2(p) = \frac{p\varepsilon - 4 + \sqrt{p^2\varepsilon(\varepsilon-1) - 8p\varepsilon + 16(\varepsilon+1)}}{p^2-16}p.$$

When $\varepsilon = 1$, we observe that φ_c, with $c > 1$, is stable for $p \leq 2$ or $p \in [3, 4)$ and

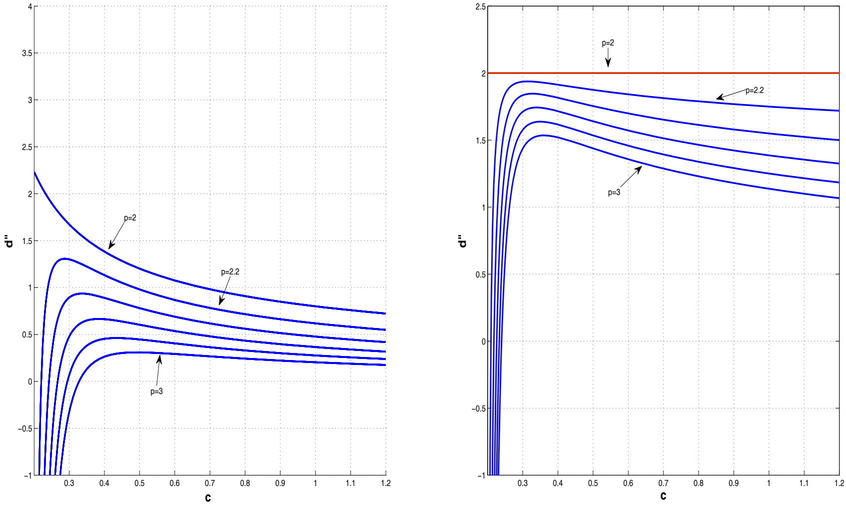
$$c > \left(\frac{4 - p + \sqrt{8 - 2p}}{16 - p^2} \right) p,$$

and unstable otherwise.

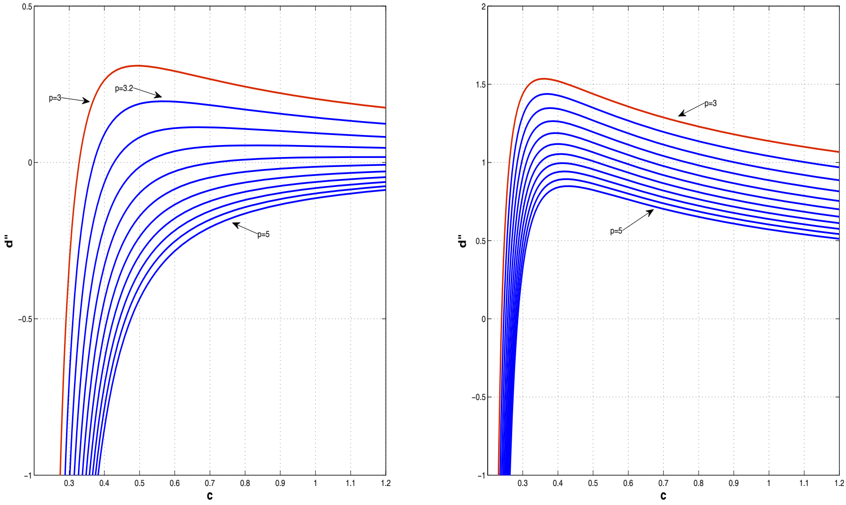
The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 1 - 2$ and $\varepsilon = 0.2$



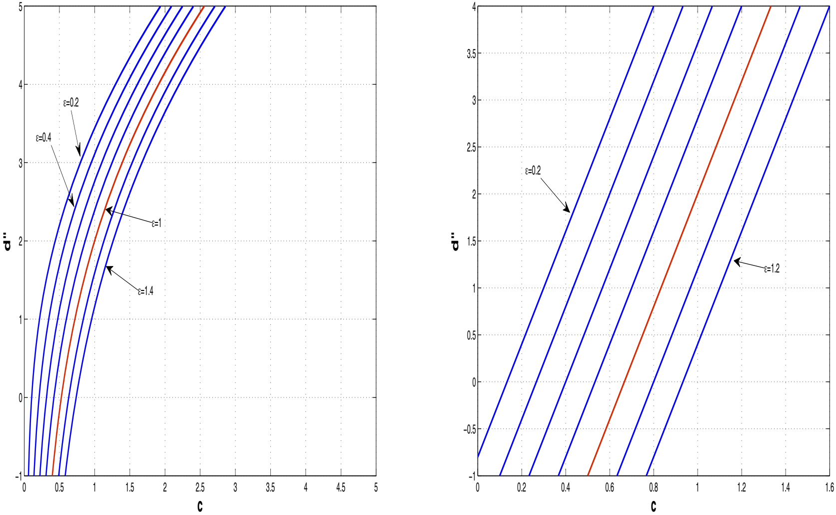
The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 2 - 3$ and $\varepsilon = 0.2$



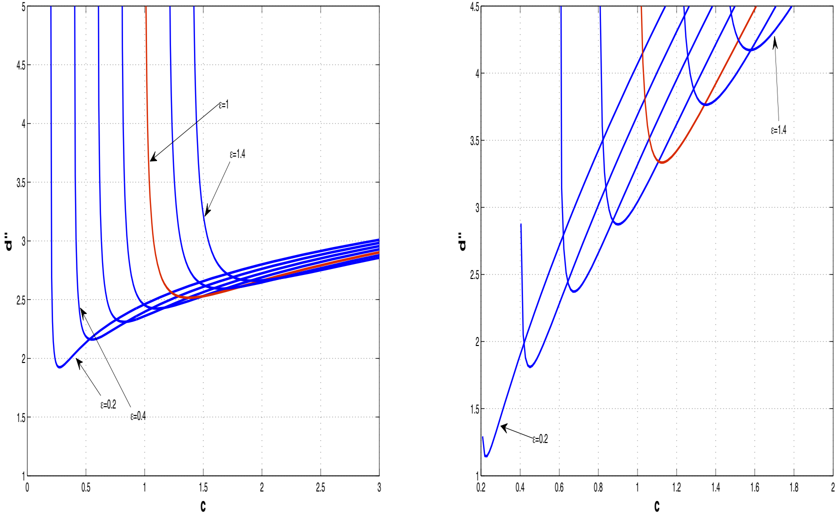
The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 3 - 5$ and $\varepsilon = 0.2$



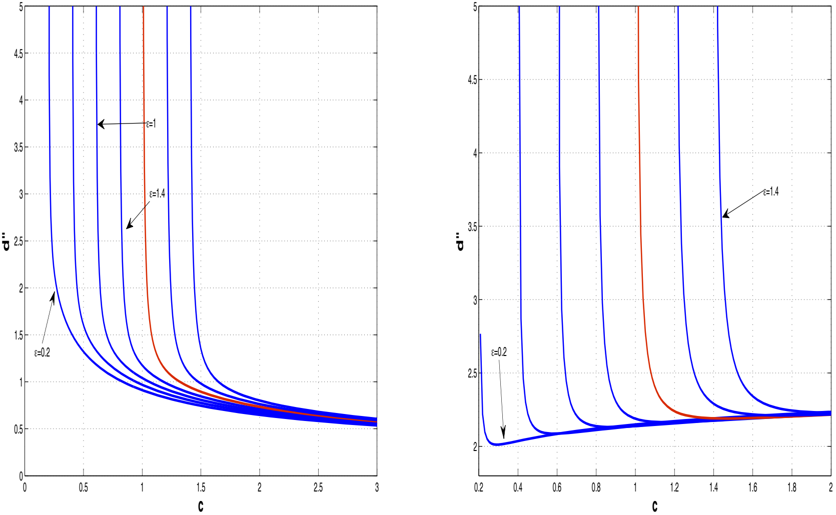
The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 1$



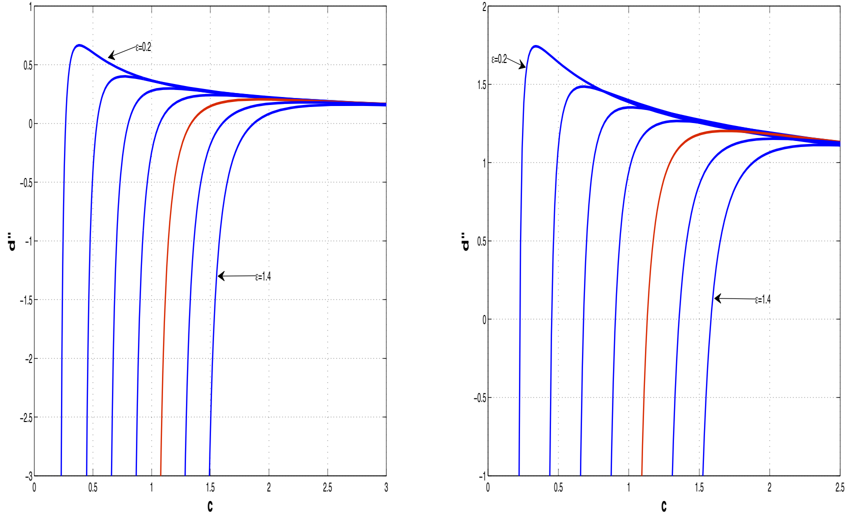
The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 1.2$



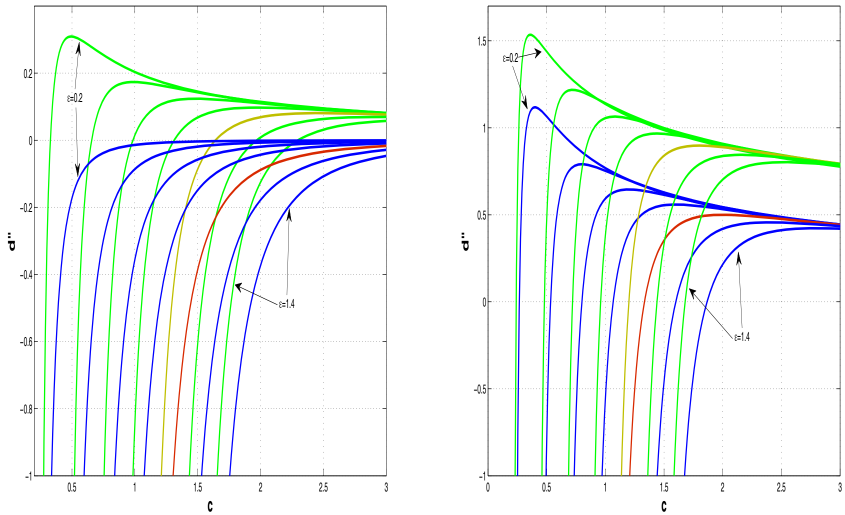
The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 1.9$



The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 2.6$



The gBBM-ZK (left) and 2D-gBBM (right) equations with $p = 3, 4$



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Thank you