Nome: ________________________________ RA: ____________________

Métodos Matemáticos I (F520/MS550) - Teste 2

31 de março de 2014

1. (5 pontos) Considere o sistema de coordenadas curvilíneas paraboloidais, dado por

$$x = uv\cos\phi, \qquad y = uv \operatorname{sen}\phi, \qquad z = \frac{1}{2}\left(u^2 - v^2\right).$$

   (a) Mostre que tais coordenadas são ortogonais;

   (b) Calcule $\nabla \times \hat{\boldsymbol{u}}$;

   (c) Calcule $\nabla \cdot \hat{\boldsymbol{u}}$.

2. (5 pontos) Sobre monopolos magnéticos:

   (a) Mostre que $\boldsymbol{A} = -\dfrac{\cot\theta}{r}\hat{\boldsymbol{\varphi}}$ é uma solução de $\nabla \times \boldsymbol{A} = \dfrac{\hat{\boldsymbol{r}}}{r^2}$.

   (b) Mostre que $\boldsymbol{A} = -\dfrac{\varphi \operatorname{sen}\theta}{r}\hat{\boldsymbol{\theta}}$ é outra solução de $\nabla \times \boldsymbol{A} = \dfrac{\hat{\boldsymbol{r}}}{r^2}$.

   (c) Quais são os problemas destas soluções (além do fato já esperado de elas não estarem definidas para $r = 0$)?

Fórmulas possivelmente úteis:

coord. curvilíneas ortogonais

$$\boldsymbol{\nabla}\psi = \sum_i \frac{1}{h_i}\frac{\partial\psi}{\partial q_i}\hat{\boldsymbol{q}}_i$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{V} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}(V_1h_2h_3) + \frac{\partial}{\partial q_2}(V_2h_3h_1) + \frac{\partial}{\partial q_3}(V_3h_1h_2)\right]$$

$$\boldsymbol{\nabla}\times\boldsymbol{V} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{\boldsymbol{q}}_1 & h_2\hat{\boldsymbol{q}}_2 & h_3\hat{\boldsymbol{q}}_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1V_1 & h_2V_2 & h_3V_3 \end{vmatrix}$$

coord. cilíndricas

$$\boldsymbol{\nabla}\psi = \frac{\partial\psi}{\partial\rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial\psi}{\partial\varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial\psi}{\partial z}\hat{\boldsymbol{z}}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{V} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho V_\rho) + \frac{1}{\rho}\frac{\partial V_\varphi}{\partial\varphi} + \frac{\partial V_z}{\partial z}$$

$$\boldsymbol{\nabla}\times\boldsymbol{V} = \frac{1}{\rho}\begin{vmatrix} \hat{\boldsymbol{\rho}} & \rho\hat{\boldsymbol{\varphi}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ V_\rho & \rho V_\varphi & V_z \end{vmatrix}$$

coord. esféricas

$$\boldsymbol{\nabla}\psi = \frac{\partial\psi}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\varphi}\hat{\boldsymbol{\varphi}}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{V} = \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}(r^2V_r) + r\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + r\frac{\partial V_\varphi}{\partial\varphi}\right]$$

$$\boldsymbol{\nabla}\times\boldsymbol{V} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{\boldsymbol{r}} & r\hat{\boldsymbol{\theta}} & r\sin\theta\hat{\boldsymbol{\varphi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ V_r & rV_\theta & r\sin\theta V_\varphi \end{vmatrix}$$
