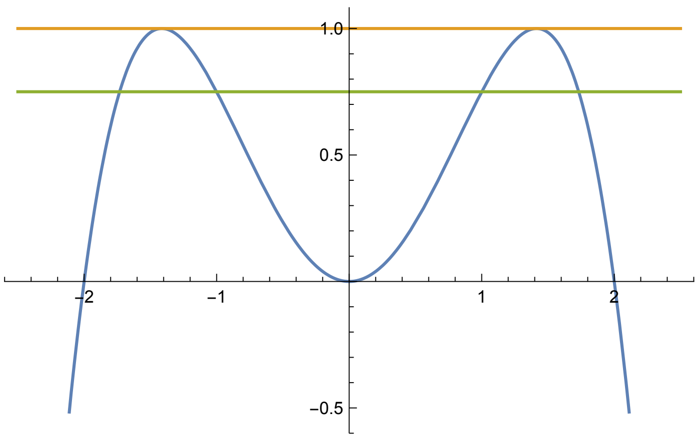
Questão 1

```
Quit
```

```
V = V0 (x^2/a^2 - x^4/(4 a^4))
```

$V0\left(\frac{x^2}{a^2}-\frac{x^4}{4 a^4}\right)$

```
Plot[{V /. {V0 → 1, a → 1}, 1, 3/4}, {x, -2.5, 2.5}]
```




```
D[V, x]
Solve[% == 0, x]
V /. %
```

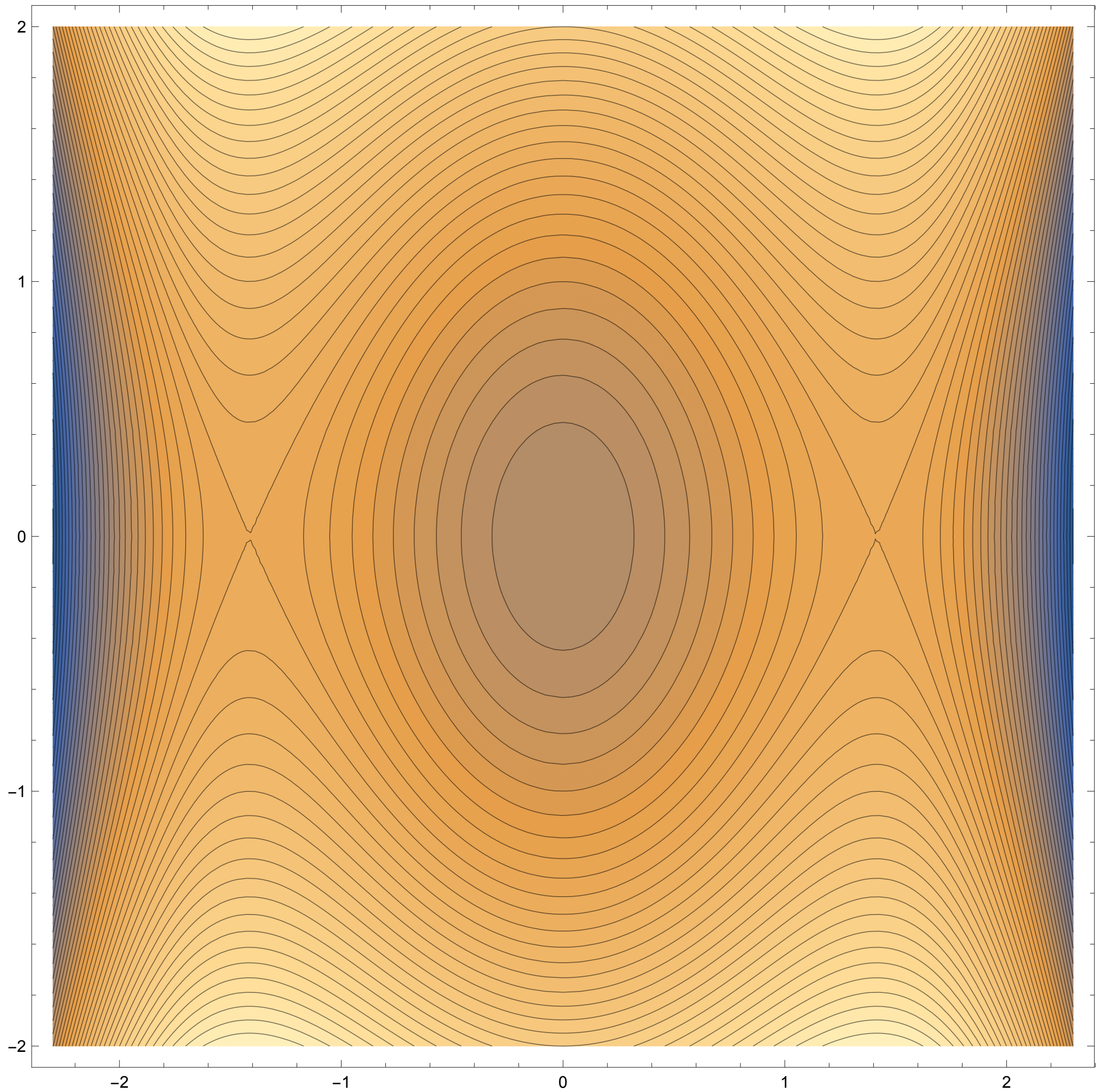
$V0\left(\frac{2 x}{a^2}-\frac{x^3}{a^4}\right)$

$\{\{x\to 0\},\{x\to -\sqrt{2}\, a\},\{x\to \sqrt{2}\, a\}\}$

$\{0, V0, V0\}$

```
(V + m v^2/2) /. {V0 → 1, a → 1, m → 1}
ContourPlot[%, {x, -2.3, 2.3}, {v, -2, 2},
 Contours → Table[i, {i, -3, 3, 0.1}], ImageSize → 600, PlotPoints → 30]
```

$$\frac{v^2}{2} + x^2 - \frac{x^4}{4}$$



```
Solve[V + m v^2/2 == e, v]
% // FullSimplify[#, {m > 0, V0 > 0, a > 0, e > 0}] &
```

$$\left\{\left\{v\to -\frac{\sqrt{4\,a^4\,e-4\,a^2\,V0\,x^2+V0\,x^4}}{\sqrt{2}\,a^2\,\sqrt{m}}\right\},\left\{v\to \frac{\sqrt{4\,a^4\,e-4\,a^2\,V0\,x^2+V0\,x^4}}{\sqrt{2}\,a^2\,\sqrt{m}}\right\}\right\}$$

$$\left\{\left\{v\to -\frac{\sqrt{\frac{4\,a^4\,e-4\,a^2\,V0\,x^2+V0\,x^4}{m}}}{\sqrt{2}\,a^2}\right\},\left\{v\to \frac{\sqrt{\frac{4\,a^4\,e-4\,a^2\,V0\,x^2+V0\,x^4}{m}}}{\sqrt{2}\,a^2}\right\}\right\}$$

```
Solve[V + m v^2/2 == V0, v]
% // FullSimplify[#, {m > 0, V0 > 0, a > 0, e > 0}] &
```

$$\left\{\left\{v\to -\frac{\sqrt{V0}\,\left(2\,a^2-x^2\right)}{\sqrt{2}\,a^2\,\sqrt{m}}\right\},\left\{v\to \frac{\sqrt{V0}\,\left(2\,a^2-x^2\right)}{\sqrt{2}\,a^2\,\sqrt{m}}\right\}\right\}$$

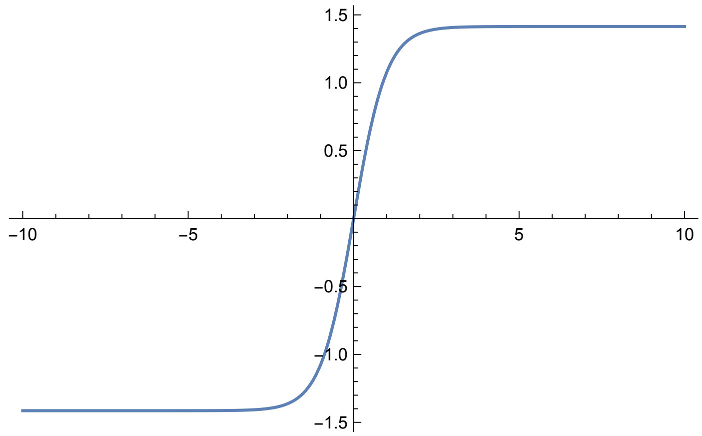
$$\left\{\left\{v\to \frac{\sqrt{\frac{V0}{m}}\,\left(-2\,a^2+x^2\right)}{\sqrt{2}\,a^2}\right\},\left\{v\to \frac{\sqrt{\frac{V0}{m}}\,\left(2\,a^2-x^2\right)}{\sqrt{2}\,a^2}\right\}\right\}$$

```
Integrate[1/(Sqrt[V0/m] (2 a^2 - x^2)/(Sqrt[2] a^2)), x] // FullSimplify[#, {m > 0, V0 > 0, a > 0, e > 0}] &
Solve[% == t, x] // FullSimplify[#, {m > 0, V0 > 0, a > 0, e > 0, t ∈ Reals}] &
```

$$a\,\sqrt{\frac{m}{V0}}\,\mathrm{ArcTanh}\left[\frac{x}{\sqrt{2}\,a}\right]$$

$$\left\{\left\{x\to \sqrt{2}\,a\,\mathrm{Tanh}\left[\frac{t\,\sqrt{\frac{V0}{m}}}{a}\right]\right\}\right\}$$

$$\text{Plot}\left[\sqrt{2}\, a\, \text{Tanh}\left[\frac{t\sqrt{\frac{V0}{m}}}{a}\right] /. \{V0 \to 1, a \to 1, m \to 1\}, \{t, -10, 10\}\right]$$



$$\text{Solve}\left[V + \frac{m\, v^2}{2} == \frac{3}{4} V0, v\right]$$

```
% // FullSimplify[#, {m > 0, V0 > 0, a > 0, e > 0}] &
```

$$\left\{\left\{v \to -\frac{\sqrt{V0}\, \sqrt{3\, a^4 - 4\, a^2\, x^2 + x^4}}{\sqrt{2}\, a^2\, \sqrt{m}}\right\}, \left\{v \to \frac{\sqrt{V0}\, \sqrt{3\, a^4 - 4\, a^2\, x^2 + x^4}}{\sqrt{2}\, a^2\, \sqrt{m}}\right\}\right\}$$

$$\left\{\left\{v \to -\frac{\sqrt{\frac{V0\,(3\, a^4 - 4\, a^2\, x^2 + x^4)}{m}}}{\sqrt{2}\, a^2}\right\}, \left\{v \to \frac{\sqrt{\frac{V0\,(3\, a^4 - 4\, a^2\, x^2 + x^4)}{m}}}{\sqrt{2}\, a^2}\right\}\right\}$$

$$\text{Integrate}\left[\frac{1}{\frac{\sqrt{\frac{V0\,(3\, a^4 - 4\, a^2\, x^2 + x^4)}{m}}}{\sqrt{2}\, a^2}}, x\right] \text{ // FullSimplify}[\#, \{m > 0, V0 > 0, a > 0, e > 0\}]\ \&$$

$$\frac{a^3 \sqrt{2 - \frac{2\, x^2}{a^2}}\, \sqrt{3 - \frac{x^2}{a^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{3}\, a}\right], 3\right]}{\sqrt{\frac{V0\,(3\, a^4 - 4\, a^2\, x^2 + x^4)}{m}}}$$

$$\text{Solve}\left[V + \frac{m\, 0^2}{2} == \frac{3}{4} V0, x\right]$$

$$\{\{x \to -a\}, \{x \to a\}, \{x \to -\sqrt{3}\, a\}, \{x \to \sqrt{3}\, a\}\}$$

$$\frac{1}{\sqrt{\frac{m\,a^2}{V0}}}\;\frac{4\,a}{\frac{\sqrt{\frac{V0\,(3\,a^4-4\,a^2\,x^2+x^4)}{m}}}{\sqrt{2}\,a^2}}$$ /. {x → a ξ} // FullSimplify[#, {m > 0, V0 > 0, a > 0, e > 0}] &

```
Integrate[%, {ξ, 0, 1}]
NIntegrate[%, {ξ, 0, 1}]
```

$$\frac{4\sqrt{2}}{\sqrt{3-4\,\xi^2+\xi^4}}$$

$$4\sqrt{\frac{2}{3}}\;\text{EllipticK}\left[\frac{1}{3}\right]$$

5.66295

Questão 2

```
Quit
```

$$\frac{r}{z} == \text{Tan}[\alpha]$$

$$z == r\,\text{Cot}[\alpha]$$

$$\frac{r}{z} == \text{Tan}[\alpha]$$

$$z == r\,\text{Cot}[\alpha]$$

$$T = \frac{m}{2}\left(r'[t]^2 + r[t]^2\,\theta'[t]^2 + \text{Cot}[\alpha]^2\,r'[t]^2\right)$$

```
V = m g r[t] Cot[α]
L = T - V // FullSimplify
```

$$\frac{1}{2}\,m\left(r'[t]^2 + \text{Cot}[\alpha]^2\,r'[t]^2 + r[t]^2\,\theta'[t]^2\right)$$

$$g\,m\,\text{Cot}[\alpha]\,r[t]$$

$$\frac{1}{2}\,m\left(-2\,g\,\text{Cot}[\alpha]\,r[t] + \text{Csc}[\alpha]^2\,r'[t]^2 + r[t]^2\,\theta'[t]^2\right)$$

```
D[L, θ[t]]
pθ == D[L, θ'[t]]
Solve[%, θ'[t]]
```

0

$$p\theta == m\,r[t]^2\,\theta'[t]$$

$$\left\{\left\{\theta'[t] \to \frac{p\theta}{m\,r[t]^2}\right\}\right\}$$

```
D[L, r'[t]]
D[%, t]
D[L, r[t]]
eq = % - %% /. {θ'[t] → pθ/(m r[t]^2)} // FullSimplify
```

$$m\,\mathrm{Csc}[\alpha]^2\, r'[t]$$

$$m\,\mathrm{Csc}[\alpha]^2\, r''[t]$$

$$\frac{1}{2} m \left(-2\, g\, \mathrm{Cot}[\alpha] + 2\, r[t]\, \theta'[t]^2\right)$$

$$\frac{p\theta^2}{m\, r[t]^3} - m\left(g\,\mathrm{Cot}[\alpha] + \mathrm{Csc}[\alpha]^2\, r''[t]\right)$$

```
eq /. r → Function[t, r0]
Solve[% == 0, pθ] // FullSimplify
Solve[%% == 0, r0] // FullSimplify
```

$$\frac{p\theta^2}{m\, r0^3} - g\, m\, \mathrm{Cot}[\alpha]$$

$$\left\{\left\{p\theta \to -\sqrt{g}\, m\, r0^{3/2} \sqrt{\mathrm{Cot}[\alpha]}\right\}, \left\{p\theta \to \sqrt{g}\, m\, r0^{3/2} \sqrt{\mathrm{Cot}[\alpha]}\right\}\right\}$$

$$\left\{\left\{r0 \to \frac{p\theta^{2/3}\,\mathrm{Tan}[\alpha]^{1/3}}{g^{1/3}\, m^{2/3}}\right\}, \left\{r0 \to -\frac{(-1)^{1/3}\, p\theta^{2/3}\,\mathrm{Tan}[\alpha]^{1/3}}{g^{1/3}\, m^{2/3}}\right\}, \left\{r0 \to \frac{(-1)^{2/3}\, p\theta^{2/3}\,\mathrm{Tan}[\alpha]^{1/3}}{g^{1/3}\, m^{2/3}}\right\}\right\}$$

```
ω = pθ/(m r0^2) /. {pθ → Sqrt[g] m r0^(3/2) Sqrt[Cot[α]]} // FullSimplify
pθ/(m r0^2) /. {r0 → pθ^(2/3) Tan[α]^(1/3)/(g^(1/3) m^(2/3))} // FullSimplify
```

$$\frac{\sqrt{g}\,\sqrt{\mathrm{Cot}[\alpha]}}{\sqrt{r0}}$$

$$\frac{g^{2/3}\, m^{1/3}}{p\theta^{1/3}\,\mathrm{Tan}[\alpha]^{2/3}}$$

```
eq /. r → Function[t, r0 + ϵ δr[t]]
Series[%, {ϵ, 0, 1}] // FullSimplify
% /. {pθ → √g m r0^(3/2) √Cot[α]} // FullSimplify
%% /. {r0 → pθ^(2/3) Tan[α]^(1/3) / (g^(1/3) m^(2/3))} // FullSimplify
```

$$\frac{p\theta^2}{m\,(r0 + \epsilon\,\delta r[t])^3} - m\,(g\,\mathrm{Cot}[\alpha] + \epsilon\,\mathrm{Csc}[\alpha]^2\,\delta r''[t])$$

$$\left(\frac{p\theta^2}{m\,r0^3} - g\,m\,\mathrm{Cot}[\alpha]\right) + \left(-\frac{3\,p\theta^2\,\delta r[t]}{m\,r0^4} - m\,\mathrm{Csc}[\alpha]^2\,\delta r''[t]\right)\epsilon + O[\epsilon]^2$$

$$-\frac{m\,(3\,g\,\mathrm{Cot}[\alpha]\,\delta r[t] + r0\,\mathrm{Csc}[\alpha]^2\,\delta r''[t])\,\epsilon}{r0} + O[\epsilon]^2$$

$$\left(-\frac{3\,g^{4/3}\,m^{5/3}\,\delta r[t]}{p\theta^{2/3}\,\mathrm{Tan}[\alpha]^{4/3}} - m\,\mathrm{Csc}[\alpha]^2\,\delta r''[t]\right)\epsilon + O[\epsilon]^2$$

```
Ω = Sqrt[(3 g Cot[α])/(r0 Csc[α]^2)] // FullSimplify[#, {0 < α < π/2, r0 > 0, g > 0, m > 0}] &
```

$$\sqrt{3}\,\sqrt{\frac{g\,\mathrm{Cos}[\alpha]\,\mathrm{Sin}[\alpha]}{r0}}$$

```
Sqrt[((3 g^(4/3) m^(5/3))/(pθ^(2/3) Tan[α]^(4/3)))/(m Csc[α]^2)] // FullSimplify[#, {0 < α < π/2, r0 > 0, g > 0, m > 0}] &
```

$$\frac{\sqrt{3}\,\mathrm{Sin}[\alpha]}{\left(\frac{p\theta\,\mathrm{Tan}[\alpha]^2}{g^2\,m}\right)^{1/3}}$$

```
Ω/ω // FullSimplify[#, {0 < α < π/2, r0 > 0, g > 0, m > 0}] &
```

$$\sqrt{3}\,\mathrm{Sin}[\alpha]$$

```
Ω/ω /. {α → π/3} // FullSimplify[#, {0 < α < π/2, r0 > 0, g > 0, m > 0}] &
```

$$\frac{3}{2}$$

```
eq1 = eq /. {α → π/3, m → 1, g → 1, pθ → 1} //
  FullSimplify[#, {0 < α < π/2, r0 > 0, g > 0, m > 0}] &
r01 = pθ^(2/3) Tan[α]^(1/3) / (g^(1/3) m^(2/3)) /. {α → π/3, m → 1, g → 1, pθ → 1}
N[r01]
```

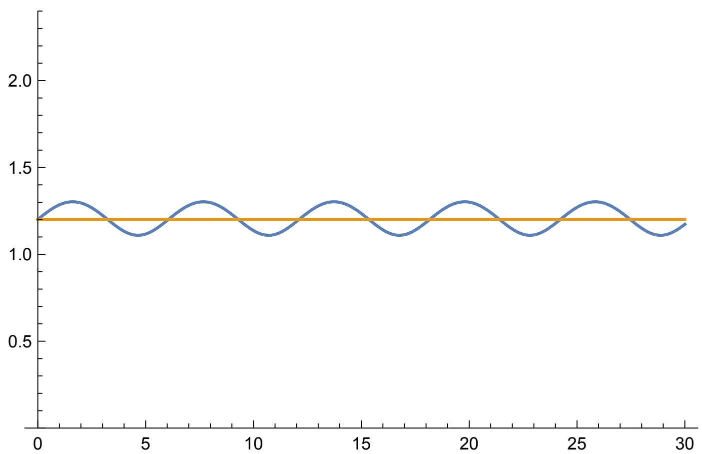
$$-\frac{1}{\sqrt{3}}+\frac{1}{r[t]^3}-\frac{4\,r''[t]}{3}$$

$$3^{1/6}$$

1.20094

```
NDSolve[{eq1 == 0, r[0] == r01, r'[0] == 0.1}, r, {t, 0, 30}]
rsol = %[[1, 1, 2]];
Plot[{rsol[t], r01}, {t, 0, 30}, PlotRange → {0, 2 r01}]
```

{{r → InterpolatingFunction[Domain: {{0., 30.}} Output: scalar]}}

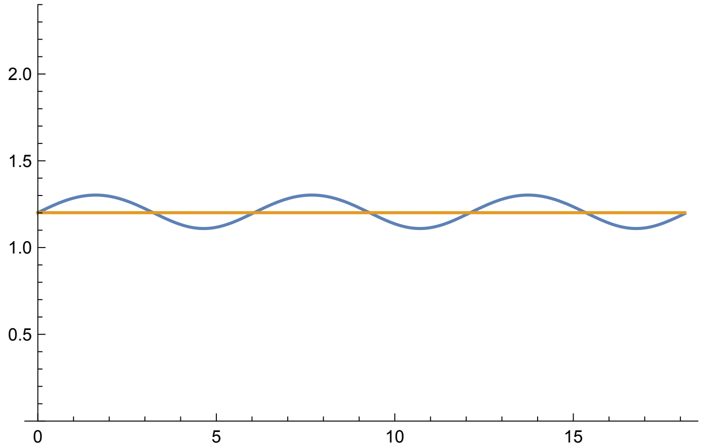



```
2 π/ω /. {α → π/3, m → 1, g → 1, pθ → 1} /. r0 → r01 // N
2 π/Ω /. {α → π/3, m → 1, g → 1, pθ → 1} /. r0 → r01 // N
```

9.06192

6.04128

```
Plot[{rsol[t], r01}, {t, 0, 2 × 9.06192130944158`}, PlotRange → {0, 2 r01}]
```



Questão 4

```
Quit
```

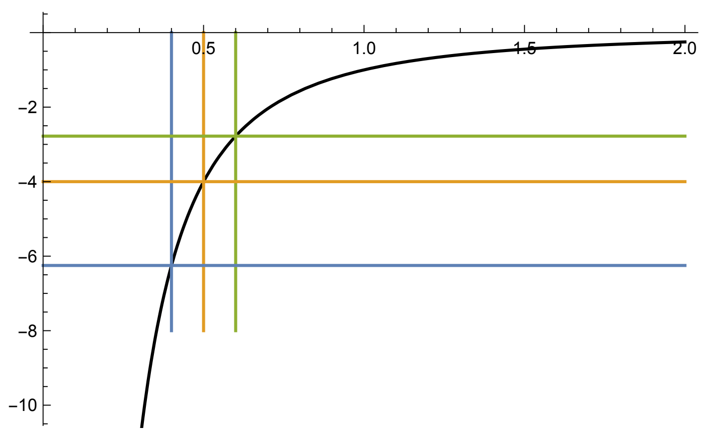
$V[r\_] = -\frac{1}{r}$ (* r em unidades de k=GM *)

```
F[r_] = -V'[r]
```

$-\frac{1}{r}$

$-\frac{1}{r^2}$

```
linhas1 = ParametricPlot[{{0.4, y}, {0.5, y}, {0.6, y}}, {y, -8, 0}];
linhas2 = Plot[{F[0.4], F[0.5], F[0.6]}, {r, 0, 2}];
Plot[F[r], {r, 0, 2}, PlotStyle → Black];
Show[%, linhas1, linhas2, PlotRange → {-10, 0}]
```



```
F[r0 + δr] + F[r0 - δr]
Series[%, {δr, 0, 2}]
```

$$-\frac{1}{(r0-\delta r)^2}-\frac{1}{(r0+\delta r)^2}$$

$$-\frac{2}{r0^2}-\frac{6\,\delta r^2}{r0^4}+O[\delta r]^3$$

```
Plot[{F[1 + δr] + F[1 - δr], 2 F[1]}, {δr, 0, 0.1}]
```