PROVA 1 – MATUTINO.

Questão 1:

$xy' - y = (xy)^{1/2}$ $\qquad x > 0 \quad y > 0$.

$y' - \frac{1}{x} y = \left(\frac{y}{x}\right)^{1/2}$

a) $\boxed{y' + p(x)\, y = f(x)\, y^n \text{ – Bernoulli} \quad v = y^{1-n}}$ 0.2

b) $y^{-1/2} y' - \frac{1}{x} y^{1/2} = \frac{1}{x^{1/2}}$ $\qquad v = y^{1/2}$

$v' = \frac{1}{2} y^{-1/2} y'$

$2v' - \frac{1}{x} v = \frac{1}{x^{1/2}}$

$v' - \frac{1}{2x} v = \frac{1}{2x^{1/2}}$ 05 $\Rightarrow \mu(x) = e^{\int -\frac{1}{2x}} = e^{-\frac{1}{2}\ln x}$

$= x^{-1/2}$

$x^{-1/2} v' - \frac{1}{2x} \cdot x^{-1/2} v = \frac{1}{2x^{1/2}} \cdot x^{-1/2} = \frac{1}{2x}$

0.5

$\frac{d}{dx}\left[x^{-1/2} v\right] = \frac{1}{2x} \Rightarrow x^{-1/2} v = \frac{1}{2} \ln x + C$

$v = \boxed{y^{1/2} = \frac{1}{2} x^{1/2} \ln x + C x^{1/2}}$

c) $y(1) = 3 \qquad 3^{1/2} = \frac{1}{2} \cancel{\ln 1} + C$

$\boxed{y^{1/2} = \frac{1}{2} x^{1/2} \ln x + (3x)^{1/2}}$ 0.3

Questão 2:

a) $x^2y'' - x(x+2)y' + (x+2)y = 0$. $y_1 = x$ é sol

$y_2 = v \cdot x$ $\quad y_2' = v'x + v$ $\quad y_2'' = v''x + 2v'$

$x^2(v''x + 2v') - x(x+2)(v'x+v) + (x+2)vx = 0$. 05

$v''x^3 + 2v'x^2 - (x^2+2x)v'x = v''x^3 - v'x^3 = 0$ 05

Red. de ordem $\quad w = v' \quad w' = v''$

$w'x^3 - wx^3 = 0 \qquad w' - w = 0 \qquad w = Ce^x$

$w = v' = Ce^x \Rightarrow v = Ce^x + D$ 05

$y_2 = (Ce^x + D)x$

$$\boxed{y(x) = c_1x + c_2e^x \cdot x}$$

b) $y_p(x) = u_1x + u_2xe^x$

04 $\begin{cases} u_1'x + u_2'xe^x = 0 \\ u_1' + u_2'(e^x + xe^x) = f(x) = xe^x \end{cases}$ ← -04 se errado f(x)

$x^2y'' - x(x+2)y' + (x+2)y = x^3e^x \Rightarrow f(x) = \dfrac{x^3e^x}{x^2}$

02 $W = \begin{vmatrix} x & xe^x \\ 1 & (e^x + xe^x) \end{vmatrix} = xe^x + x^2e^x - xe^x = x^2e^x$

03 $u_1' = \dfrac{\begin{vmatrix} 0 & xe^x \\ xe^x & e^x + xe^x \end{vmatrix}}{x^2e^x} = \dfrac{-x^2e^{2x}}{x^2e^x} = -e^x$

0.3 $u_2' = \dfrac{\begin{vmatrix} x & 0 \\ 1 & xe^x \end{vmatrix}}{x^2e^x} = \dfrac{x^2e^x}{x^2e^x} = 1$

$u_1 = -e^x$ e $u_2 = x$ $\qquad$ podemos assumir ambas const. de integração igual a 0.

03 $y_p = -\underline{xe^x} + x^2e^x$

dentro de y_c

$$\boxed{y(x) = c_1x + c_2xe^x + x^2e^x}$$

3. $y^{(4)} - 2y^{(3)} + 9y^{(2)} - 3y^{(1)} + 9y = \cos x + x^4 e^{3x}$

a) $r^4 - 2r^3 + 9r^2 - 3r + 9 = (r-3)^2(r^2+1)$

$r = 3$ c/ mult 2

$r = \pm i$

03 02

$y_c = (c_0 + c_1 x)e^{3x} + c_2 \cos x + c_3 \operatorname{sen} x$

b) $y_p = x^s(A\cos x + B \operatorname{sen} x) + x^j(C + Dx + Ex^2 + Fx^3 + Gx^4)e^{3x}$

05 05

s, j menores inteiros que eliminam repetição c/ termos de y_c

$\Rightarrow s = 1$

$j = 5$

4. $y'' + 6y' + 9y = u_2(t)\, e^{-3(t-2)}(t-2)$.

$y(0) = y'(0) = 0$

$s^2 X(s) + 6sX(s) + 9X(s) =$

$= \mathcal{L}\{u_2(t) f(t-2)\} = e^{-2s}\mathcal{L}\{f(t)\}$.

$f(t-2) = e^{-3(t-2)}(t-2)$

$f(t) = e^{-3t}\cdot t$ e $\mathcal{L}\{te^{-3t}\} = \frac{1}{(s+3)^2}$

teorema translação ems

~~$s^2X(s)$~~ $X(s)(s^2+6s+9) = e^{-2s}\frac{1}{(s+3)^2}$.

1.0 $\boxed{X(s) = e^{-2s}\frac{1}{(s+3)^4}}$ $G(s)$

$\mathcal{L}^{-1}\{X(s)\} = y(t) = u_2(t)\, g(t-2)$

1.0 $g(t) = \mathcal{L}^{-1}\{G(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+3)^4}\right\} = \frac{e^{-3t}t^3}{3!}$

$\boxed{y(t) = u_2(t)\frac{e^{-3(t-2)}\cdot(t-2)^3}{3!}}$

0.5

$\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} = \frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{s^4}\right\}$

$= \frac{1}{3!}t^3$

5) $\mathcal{L}^{-1}\left\{ \ln \frac{s-2}{s+2} \right\} = \mathcal{L}^{-1}\{F(s)\} = f(t)$

$$F'(s) = \frac{1}{\left(\frac{s-2}{s+2}\right)} \cdot \frac{s+2-(s-2)}{(s+2)^2} = \frac{s+2}{s-2} \cdot \frac{4}{(s+2)^2} =$$

$$= \frac{4}{(s-2)(s+2)} = \frac{A}{(s-2)} + \frac{B}{s+2} = \frac{A(s+2)+B(s-2)}{(s-2)(s+2)}$$

$A(s+2) + B(s-2) = 4$

$s = 2 \Rightarrow A = 1$

$s = -2 \Rightarrow B = -1$

$$\Rightarrow \boxed{F'(s) = \frac{1}{s-2} - \frac{1}{s+2}} \quad 05$$

$$\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{s-2} - \frac{1}{s+2} \right\} = e^{2t} - e^{-2t} = -t\,f(t)$$

$$\boxed{f(t) = \frac{e^{-2t} - e^{2t}}{t}} \quad 05$$