SEMINÁRIO DE EQUAÇÕES DIFERENCIAIS

Beyond DiPerna-Lions/Ambrosio: the Lagrangian approach for transport/continuity equation

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Resumo: In this presentation, we shall explore the theory developed after DiPerna-Lions (Invent. Math., 1989) and Ambrosio (Invent. Math., 2004), where they study solutions satisfying the so called "renormalization property". Loosely speaking, the aforementioned property states that composition of smooth functions with solutions of the transport equation are also solutions. For vector fields satisfying it, existence, uniqueness and stability of solutions of transport/continuity equation, and as a byproduct we have well-posedness of the associated flow.

Despite the renormalization property holding for $W^{1,p}$ (or more generally BV) vector fields, the property is quite rigid and many closely related spaces to BV do not satisfy it. Therefore, a new approach was developed by Crippa-DeLellis (Crelle, 2008) by first obtaining a well-posed flow for $W^{1,p}$ vector fields and then constructing a Lagrangian solution for the transport/continuity equations. The space for which the technique is applicable was further extended and now is much greater than BV with the very recent result of Nguyen (ARMA, 2021).

We shall give the broad strokes of the technique, highlighting the major results so far and commenting on new results by the author and their advisor Marcelo Santos with applications on Vlasov-Maxwell system and the generalized SQG equations.