Existence of positive solutions of Schrödinger equations with vanishing potentials

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Abstract

We show the existence of at least one positive solution of the equation

$$\begin{cases} -\Delta u + V(x)u = f(x,u), & x \in \mathbb{R}^N \\ u > 0 \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{1}$$

for $N \geq 3$ and assuming that $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a Caratheodory nonnegative function and $V : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative potential which can vanish at infinity. Here, we assume that f is superlinear at the origin and at infinity and has subcritical growth, and we give examples where f does not satisfy the classical condition of Ambrosetti-Rabinowitz nor monotonicity conditions.