# The Swift-Hohenberg Equation under directional-quenching: finding heteroclinic connections using spatial and spectral decompositions

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In this talk I will discuss the existence of patterns (nontrivial, stationary solutions) for one-dimensional Swift-Hohenberg Equation in a directional quenching scenario, that is, on $x \leq 0$ the energy potential associated to the equation is bistable, whereas on $x \geq 0$ it is monostable. This heterogeneity in the medium induces a symmetry break that makes the existence of heteroclinic orbits of the type point-to-periodic not only plausible but, as we shall see, true. In this search we use interesting results of a memoir by Fefferman, Lee-Thorp, and Weinstein ("Topologically protected states in one-dimensional systems" - American Mathematical Society, 2017) to understand the multiscale structure of the problem, namely, how fast/slow scales interact with each other. In passing, we advocate for a new approach in finding connecting orbits, using what we call “far/near decompositions”, relying on information about the asymptotic spatial behavior of the solutions, on spectral and harmonic analysis. Our method is functional analytic and PDE based, relying minimally on dynamical system techniques and making no use of comparison principles whatsoever. Joint work with N. Yoshinaga
(Advanced Institute for Materials Research - Tohoku University).