# Unsteady Stokes Equations and the Fractional Leibniz Rule

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**Abstract**

This talk is dedicated to introduce a new inequality that involves an important case of Leibniz rule regarding Riemann-Liouville and Caputo fractional derivatives of order $\alpha \in (0,1)$. More specifically, we prove that for suitable functions $f$, it holds that

$$D^{\alpha}_{t_0,t}\big[f(t)\big]^2 \leq 2\Big[D^{\alpha}_{t_0,t}f(t)\Big]f(t), \quad \text{almost everywhere in } [t_0,t_1],$$

and

$$cD^{\alpha}_{t_0,t}\big[f(t)\big]^2 \leq 2\Big[cD^{\alpha}_{t_0,t}f(t)\Big]f(t), \quad \text{almost everywhere in } [t_0,t_1].$$

In the context of partial differential equations, the aforesaid inequality allows us to address the Faedo-Galerkin method to study the fractional version of the 2D Stokes equation on bounded domains $\Omega$

$$\begin{array}{rcll} cD^{\alpha}_t u - \nu\Delta u + \nabla p & = & f & \text{in } \Omega,\ t>0, \\ \nabla \cdot u & = & 0 & \text{in } \Omega,\ t>0, \\ u(x,t) & = & 0 & \text{on } \partial\Omega,\ t>0, \\ u(x,0) & = & u_0(x) & \text{in } \Omega. \end{array}$$

where $cD^{\alpha}_t$ is the Caputo fractional derivative of order $\alpha \in (0,1)$ and $f$ a suitable function. This is a joint work with Prof. Renato Fehlberg Júnior.

## References

[1] **P. M. Carvalho-Neto** and R. Fehlberg Junior, *Conditions to the absence of blow up solutions to fractional differential equations*, Acta Appl. Math. (2017)

[2] **P. M. Carvalho-Neto** and R. Fehlberg Junior, *On the fractional version of Leibniz rule*, (to appear Mathematische Nachrichten).