New trends in Fractional Regularity and Applications *

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Abstract

In this talk we are going to discuss some recent develpments on Fractional Regularity for solutions of Quasilinear degenerate equations. Our goal will be to link the nonlinear character of the differential operator with spaces of fractional order of differentiability, and to review and present some new and old results regarding the regularity of solutions to the sort of equations as well as the related a priori estimates. Special attention will be delivered for the case of a p-Kirchoff equation, cf. [1] and also to the (p, q)-Laplacian, cf. [2].

1 Introduction

In the past years, the phenomenon of fractional regularity has been addressed for a large class of linear and/or quasilinear differential operators, mostly, in terms of certain Besov spaces. As it turns out, for the the so-called p-Laplacian, this regularity is guided in the light of the Nikolskii class, the case where the interpolation parameter is infinite. Despite of its own interest, fractional regularity methods may be used as a tool for the investigation of some Partial Differential Equations which are not usually addressed in this manner. Thus, the purpose of the present paper is to exploit such methods in order to provide some results regarding existence and regularity of solutions to a class nonlocal elliptic equations which are linked to the p-Laplacian. This is done by means of explicit a priori estimates regarding Lebesgue and Nikolskii spaces, which are part of the present contribution. As a consequence, this approach allows a relaxation on some of the standard conditions employed in this class of problems.

Throughout the talk, we will addresses the gain of global fractional regularity in Nikolskii spaces for solutions of a class of quasilinear degenerate equations with (p, q)-growth and also we are interested to present some applications of this theory for nonlocal elliptic problems.

Indeed, we are interested in the effects of the datum on the derivatives of order greater than one of the solutions of the (p, q)-Laplacian operator, under Dirichlet's boundary conditions. As it turns out, even in the absence of the so-called Lavrentiev phenomenon and without variations on the order of ellipticity of the equations, the fractional regularity of these solutions ramifies depending on the interplay between the growth parameters p, q and the data. Indeed, we are going to exploit the absence of this phenomenon in order to prove the validity up to the boundary of some regularity results, which are known to hold locally, and as well provide new fractional regularity for the associated solutions. In turn, there are obtained certain global regularity results by means of the combination between new a priori estimates and approximations of the differential operators, whereas the nonstandard boundary terms are handled by means of a careful choice for the local frame.

More specifically, we aim to discuss the fractional regularity of solutions to the following class of degenerate elliptic equations

$$\begin{cases} -\alpha\Delta_p u - \beta\Delta_q u & = f \text{ in } \Omega \\ \qquad\qquad\qquad u & = 0 \text{ on } \partial\Omega \end{cases} \tag{$\mathcal{D}$}$$

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where $q \geq p > 2$, $\alpha > 0$, $\beta \geq 0$, and $\Omega \subset \mathbb{R}^N$ is an open bounded domain of class $C^{2,1}$. Indeed, our aim is to describe the effects of the parameters α and β, which control the ellipticity of ($\mathcal{D}$), and also the interference of the interplay between p, q, and the order of integrability of f on the spatial derivatives of order greater than one of the solutions to this class of equations, the well-known (p, q)-Laplacian operator.

Further, on the second part of the talk, we present an investigation on the existence and fractional regularity of solutions for

$$\begin{cases} -\left[a\left(\|u\|_{1,p}^p\right)\right]^{p-1}\Delta_p u + u &= f(x,u) \quad \text{in} \quad \Omega \\ \dfrac{\partial u}{\partial \eta} &= 0 \quad \text{on} \quad \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is an open bounded domain with smooth boundary $\partial\Omega$, Δ_p is the p-Laplacian operator

$$\Delta_p u = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right), \qquad \text{with } p > 2$$

and $a(.)$ is the so-called p-Kirchhoff, or Kirchhoff term, which will be assumed to be continuous and bounded by below.

References

[1] de Araujo, A.L.A and de Miranda, L.H. - On fractional regularity methods for a class of nonlocal problems. *accepted for Publication*, 2018.

[2] de Miranda, L.H. and Presoto, A. - On the fractional regularity for degenerate equations with (p, q)-growth. *accepted for publication*, 2018.