We consider the problem $-\Delta u = g(u)|\nabla u|^2 + f(x,u)$ in Ω, $u > 0$ in Ω, $u = 0$ on $\partial\Omega$, where Ω is a smooth bounded domain in $\mathbb{R}^N$. Using a Kazdan-Kramer change of variable, this problem can be reduced to a semilinear problem without gradient term, which can then be approachable by variational methods. However various new difficulties arise, in particular with respect to the Ambrosetti-Rabinowitz condition. We investigate some of these difficulties.

In the second part of the talk, we consider the situation of a system. Using again a Kazdan-Kramer change of variables, this system can be transformed into a system without gradient term. In some cases, this latter system can be handled by an upper-lower solution approach, by variational methods, or by blow-up. The blow-up procedure leads to the study of new Liouville type theorems.

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